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PREFACE

TO

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In writing the present book it has been the endeavour of the authors to furnish to students a simple, yet accurate, account of the theory and design of the modern dynamo. That the attainment of this aim is no light task we are only too well aware. Such a book, if it is to be of use to those who are either engaged or about to engage in electrical engineering as a profession, must combine both the theoretical and practical sides; if it is not to repel the general scientific reader, it must be free from excessive technicality; while, lastly, it should not be too mathematical, since it is intended for the learner, who with a general knowledge of electricity comes to the special study of dynamos. Most of all, however, is it important that, so far as it goes, its statements should be entirely accurate. If, therefore, we have attained something of the all-saving merit of accuracy, many defects on other scores may be forgiven, and our attempt to write a new book on so well-worn a theme cannot be pronounced to have been entirely vain. A completely novel treatment of the subject must be either erroneous or well-nigh impossible to devise: any writer on dynamos must necessarily draw from the same fountain-sources, such as the papers of Hopkinson, Kapp, Swinburne, and Esson, and even the best method of presenting much of this accumulated knowledge may have been worked out by some previous author. To such well-known writings we have

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CHAPTER XXI

THE WORKING AND MANAGEMENT OF DYNAMOS

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THE DYNAMO

CHAPTER I

INTRODUCTORY

1 THE nature of the dynamo is essentially dual, and furthermore it is dual in two ways: not only is it at once electrical and magnetic, but it also has a mechanical as well as, and in contrast to, an electro-magnetic aspect. Its theory is one among the many meeting-grounds of electricity and magnetism, and its design is based on two fundamental equations, of which one may be called the electrical equation, the other the mechanical equation, each also involving the facts and laws of magnetism. Lastly, this twofold nature finds its exact counterpart in the structure of the dynamo as a machine, for it may broadly be regarded as built up of an iron or magnetic portion and a copper or electric portion.

In general terms a dynamo may be defined as 'a machine in which mechanical energy is converted into electrical energy by means of continuous relative motion of an electrical conductor or conductors and a magnetic field or fields, such motion causing the conductor to cut or traverse the lines of force of the field.' Taking this as our text, our aim will be to establish it generally, and thence to deduce the principles which govern the design of dynamos, and determine the practical shapes which they take.

In so doing we have at the outset to explain certain technical terms which appear in the definition, and in the first place must consider what is meant by the expression 'electrical energy.' For the appearance of electrical energy there are required two factors, viz. an electric pressure, or electromotive force, and a current flowing under that pressure, and until both are present there is no electrical energy. If an electrical difference of pressure has once been established between two points forming the ends of an electrical conductor or system of conductors, then when those two points are further joined by any form of electrical conductor a current flows round the completed circuit, and continues to flow if the electrical pressure be maintained. A simple and often-quoted analogy from hydraulics will render this plainer : the electric pressure may be compared to a head of water, or a difference of level between two points, in virtue of which water will flow from the higher to the lower level, if the two are joined by a pipe.

When such a channel is provided, the body of water which previously only had potential energy will do work, as in turning a water-wheel, and will realise its potential energy. If, however, this development of power is to be continuous, a pump must be provided to raise the water from the lower to the higher level—in other words, to keep up the difference of level in virtue of which the water flows, and so does work. In exactly the same way a current of electricity flowing from a point of higher electrical level or potential to a point of lower potential does work, and electrical energy is present ; by its flow it tends to equalise the potential of these two points, and it will only continue to flow so long as there is some difference of potential between them. Hence, if there is to be a continuous flow of electricity we need an electrical pump or machine to constantly maintain the pressure in virtue of which the current flows. Such an electrical pump is the dynamo, which is therefore primarily and essentially a generator of electrical pressure ; its sole function is to generate an electric pressure or electromotive force, and subsequently to maintain it, when a current flows under that pressure. The fact of a current flowing continually makes it necessary that there should be a closed circuit which throughout its entire path conducts electricity, and which contains some source of electromotive force, or E.M.F. ; in the case with which we have to deal this latter is concentrated within the limits of the dynamo which forms part of the circuit. Hence any system containing a dynamo at work, whether lighting lamps, or charging accumulators, or driving motors, must consist of a closed circuit divisible into two portions on either side of certain points. In fig. 1 there is shown diagrammatically a closed electrical circuit round which a current is flowing and part of which is a dynamo in its simplest form; the meaning of the conventional symbols will be apparent later. The points A and D divide the entire circuit into two portions; in that to the left, which may be called the 'internal' portion of the circuit, viz. A R_a D, a certain action is set up which results in a difference of electrical pressure between A and D, A being at a higher potential than D, and therefore having a + or positive sign attached to it.

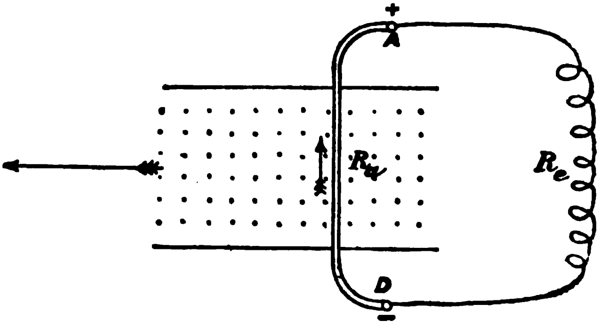

Fig. 1.

When A and D are further joined by the external portion of the circuit, or, as it is called for shortness, the 'external circuit,' viz. A R_e D, a current flows which would result in the equalisation of the potential throughout the whole circuit, and would then cease, were it not for the fact that R_a represents a dynamo which continuously keeps the potential of A higher than that of D. The two points A and D form at once the 'terminals' of the machine and the 'terminals' of the external circuit; the current flows from A to D in the external circuit, but within the dynamo itself from D to A, just as in the case of a voltaic battery, where the current flows from the positive to the negative pole outside it, but from the negative to the positive pole within the cell itself. If we replace the dynamo by a pump, and the external circuit by a pipe, through both of which water is kept continuously circulating, the hydraulic analogy will again help us; the dynamo does not, strictly speaking, generate electricity any more than the pump generates the water; it is, however, only after the production of an electrical difference of potential, and the consequent setting-up of a current, that the electricity becomes manifest, and both machines generate and maintain a certain pressure, in the one case hydraulic, in the other electrical. Neither is it the electricity that is consumed or lost in passage round the circuit, but the pressure under which it flows, just as the 'head' of the water is lost in its passage through the pipe, only to be renewed by the action of the pump. Further, the mechanical energy developed in any time in the one case is proportional to the quantity of water flowing round the system in that time and the total pressure under which it flows, i.e. to the product of the number of gallons passed and the pressure in pounds per square inch; the electrical energy in the other case is proportional to the quantity of electricity and the pressure, i.e. in the practical units of the C.G.S. system, to the product of coulombs and volts. But in both cases we are more concerned with the *rate* of development of the energy or the 'power' of the machine, which is proportional to the rate of flow of the water in gallons per minute or of the electricity in coulombs per second. The rate of flow of electricity is called the current, and one coulomb per second is an ampère of current. Consequently the power of the dynamo is measured in volt-ampères or 'watts,' and it is in such units that its value as a machine is reckoned, just as the mechanical value of a steam or other engine is reckoned, not in foot-pounds or in horsepower-hours, but by its horsepower or rate of doing work, without reference to any particular time. Again, referring to our system of fig. 1, it is evident that if we wish to make use of the water forced through the pipe external to the pump itself, as, for instance, if we wish to drive some form of water-motor, it is not the total rate of development of energy throughout the entire system that can be used, but only the rate of development of energy that is available outside the pump. Of the total work done by the pump on the water some portion is spent in overcoming the friction which resists the passage of the water through the pump's own channels and valves, so that the available power is less than the total power of the pump. Similarly the 'output' of the dynamo is its external power or its rate of development of *available* electric energy; that is, the product of the voltage between the points A and D, and the current flowing through the external circuit R_e; or, in symbols, if E_e=the volts at the terminals A D, and C_e=the current through R_e, the output of the dynamo is equal to E_eC_e.

Electrical energy is therefore the energy developed by a current flowing under an E.M.F., and the output of a dynamo or its rate of development of electrical energy external to itself is reckoned in units of power; the watt being but a small fraction of a horsepower, it is more convenient to express the output in 'kilowatts,' or units of 1,000 watts. Since the output is always proportional to the product of two factors, the volts and the ampères, the same output may be due to a great variety of different combinations; for example, 10,000 watts, or 10 kilowatts, may be the electrical power of either 10 ampères flowing under a pressure of 1,000 volts, or of 100 ampères flowing under 100 volts, or again of 2,000 ampères under a pressure of 5 volts. These three illustrations will serve to emphasise the important difference between machines of high voltage and small currents such as are used for arc lighting or transmission of power over considerable distances or for the distribution of electricity on the alternating current system, and machines of moderate voltage and current, used for the direct lighting of incandescent lamps, or finally machines supplying large currents at low voltages such as are used for electro-plating, electro-depositing, and other chemical or metallurgical processes; although their power may be the same, the construction of the machines is very widely different, owing to the different natures of the work for which they are respectively suited.

Now by the principle of conservation of energy it is impossible that any form of energy can be absolutely created; in any machine it can only be transformed from one form to another, and consequently in the case of the dynamo in which electrical energy is developed it is necessary to ask what is the form in which energy is supplied to the dynamo. The answer is given by our definition, which states that in the dynamo it is mechanical energy which is converted into electrical energy. Hence the output of the dynamo is electrical energy, while the input is mechanical energy. With the latter as supplied by a steam or gas engine, turbine, waterwheel, or other prime mover, all are familiar; in any such machine motion is produced against a resistance, and our definition requires that there should be in the dynamo continuous motion of one portion relatively to another portion. The movable portion of the dynamo may be driven by means of a belt and pulley, by rope gearing, or by friction gearing; or it may be coupled directly to the main shaft of the prime mover, as on to the crank-shaft of a steam-engine or the shaft of a turbine; but in all cases when electrical energy is developed, there is resistance to the motion, and consequently mechanical energy is consumed only to reappear in another form. The rate at which mechanical energy is supplied to the dynamo is usually measured in horsepowers, and the relation between the two units in which rates of development of mechanical and electrical energy are respectively measured is such that one horsepower is the mechanical equivalent of 746 watts, whence it follows that 1⅓ horsepower is nearly equal to one kilowatt.

But in no machine can the conversion of energy from one form to another be carried on without some loss, by which is meant not the actual disappearance or annihilation of energy (which is an impossibility), but its appearance in a useless form or in useless places. To take the case of a centrifugal pump, the rate at which mechanical energy is supplied to it is measured by the brake horsepower spent on its shaft, but the rate at which useful energy is developed is determined by the rate of flow of water under the pressure created by the pump at its delivery outlet; some of the work done on the shaft of the pump is expended in heating its bearings by friction, and of the actual hydraulic work, as has been already pointed out, a portion is expended in driving the water through the channels of the pump against the resistance of fluid friction. Both are inevitable losses, and closely analogous to them are the losses in a dynamo; apart from the loss by friction in its bearings, we have seen that there is a necessary loss from its total rate of development of electrical energy, due to the absorption of work in driving the current through the electrical resistance of the dynamo itself. In consequence therefore of these mechanical and electrical losses, if the rate of supply of mechanical energy to the shaft of the dynamo be one horsepower, its output must be something less than 746 watts. Nevertheless, of all machines yet invented, the dynamo may now rank as one of the most perfect transformers of energy. The 'efficiency' of the dynamo or the ratio between the useful energy obtained from it and the energy supplied to it in another form is very high, since in all but very small machines it is practically and commercially possible to obtain as much as 90 per cent. of the mechanical energy supplied, returned at the terminals of the dynamo in its new form of electrical energy, and in large machines even higher percentages are frequently attained. Much improvement cannot therefore be expected on this score, and although the dynamos of the future may be cheaper to construct, they will scarcely be more efficient.

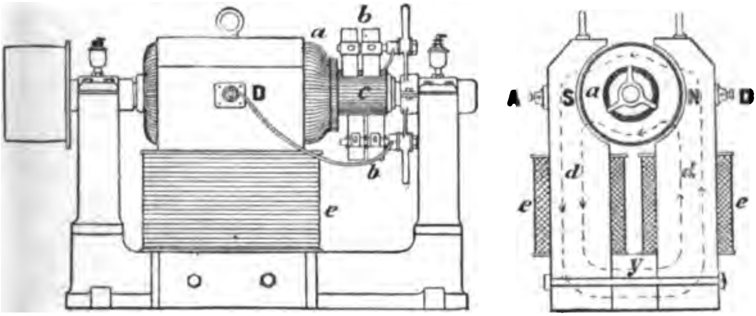

FIG. 2.—Two-pole continuous-current dynamo.

Let us now consider the action of the typical dynamo of fig. 2, which represents in elevation and section a two-pole belt-driven machine such as is used for supplying current to incandescent lamps; dynamos the same or similar in design have been and are now constructed by many makers in perhaps larger numbers than those of any other class. We have said that any dynamo consists of two structural portions, between which relative motion is produced: in this machine the stationary portion consists of an iron horseshoe magnet, *dd*, round the limbs of which are wound coils *ee* formed of many turns of insulated copper wire;

its two pole-pieces, N and S, are bored out so as to nearly embrace the circumference of the revolving portion *a*. The latter consists of a cylindrical iron structure rigidly mounted on a steel shaft which is supported at either end by bearings and at the one end carries a pulley; on it is wound a number of turns of insulated copper wire lying close together all round the circumference, and passing on the outside longitudinally across from one end to the other: these turns are connected at one end to another smaller cylindrical structure known as the 'commutator,' *c*, on which rest two pairs of stationary 'brushes,' *b b*, each pair being connected electrically to one of the two terminals of the machine, A,D. The action of the machine is shortly as follows. By sending an electrical current round the coils, *e e*, we make the iron horseshoe strongly magnetic, or, as it is termed, 'excite' a 'magnetic field' between its poles. By means of a belt passing over the pulley the shaft, and with it the cylinder, is caused to rotate, and so move relatively to the magnet; at once an electromotive force is set up in each of the copper wires under the pole-pieces, the general effect of which may be broadly summed up by saying that a difference of electrical potential is produced between the points A D, just as was the case in the simple dynamo of fig. 1. Now if an external circuit be applied to these terminals this electrical pressure will cause a current to flow round the completed circuit, passing on its way through the copper wires themselves. Directly such a current flows we have electrical energy developed, and therefore mechanical energy must be supplied to the shaft through the pulley; in other words, as soon as the circuit is closed, and a current flows, the revolving portion offers resistance to the motion, and work has to be done in overcoming this resistance. But what is the 'magnetic field' which we have spoken of as being created by passing a current through the coils encircling the magnet? This question we have next to consider.

CHAPTER II

THE MAGNETIC FIELD

Let us take an ordinary straight bar magnet and place it across an upright piece of wood, so that it is supported in the air away from any iron by means of the prop at its centre. Let us next approach it with a small 'magnetic needle,' or piece of steel magnetised along its length, so that one end is a north pole and the other a south pole; we may use for the purpose either a little compass needle delicately pivoted by a jewel or agate cap on a sharp-pointed support, so as to be free to move in a horizontal plane, or, better still, a magnetised needle suspended by its centre of gravity at the end of a thread, so as to be free to set itself in any plane, horizontal or otherwise. Now, when such a small needle is brought up near to the bar magnet, it will be found that at whatever point in the neighbourhood of the magnet it is placed it will set its length or magnetic axis in a definite direction, and if thrust aside from that position of equilibrium it will return to it and to no other. The exact direction will vary according to the place where the needle is held, but for each place there is one particular direction in which it will always set itself. And this is true whether it be held above or below, to the right or to the left of the bar magnet; it always takes up its definite position, with its north pole pointing more or less directly towards the south pole of the large magnet, and its south pole towards the magnet's north pole. We see from this that the space surrounding the bar magnet is in some way different from the same space when the magnet is removed, for then the needle will always point to the earth's north pole. Our first aim must therefore be to explore the surrounding space, and to arrive at some method by which its peculiar properties can be adequately described. To do this scientifically we require to examine its effects upon a single magnetic pole; and, consequently, the small needle which we used above, and which had two poles, one at either end, is not strictly the exploring instrument which theory demands. Now in nature it is absolutely impossible for the two poles to be separated so that a north pole should exist by itself and apart from a south pole, or *vice versâ*; yet by certain devices we may so arrange matters that the one pole of the exploring needle is so far from the bar magnet as to be practically uninfluenced by it, and thus we can make a practical approximation to the ideal requirement of a single free magnetic pole.

Suppose that we place our bar magnet, still supported at its centre, in the middle of a large glass vessel, away from the sides, and fill up the vessel with water to the level of the magnet (fig. 3). Let us then magnetise a very thin and long piece of steel, such as a knitting-needle, and fix it vertically with one of its ends, say its north pole, in a piece of cork; if we now place the cork in the water, with the needle projecting downwards like the mast of an overturned boat, the whole will float, with the top of the cork just level with the water, and the north pole of the needle projecting through it so as to be level with the bar magnet. The other, or south end of the needle, will then be so far away from the bar magnet that we may regard it as out of reach of the influence of the latter, and may therefore consider that we have a single free north pole floating on the water; for the cork, being non-magnetic, will not affect the result in any way whatever. Let us now bring our exploring boat close up to the north pole of the magnet: as soon as we loose our hold it will start off on a voyage to the south pole, and, when it has reached it, will there come to rest. In its

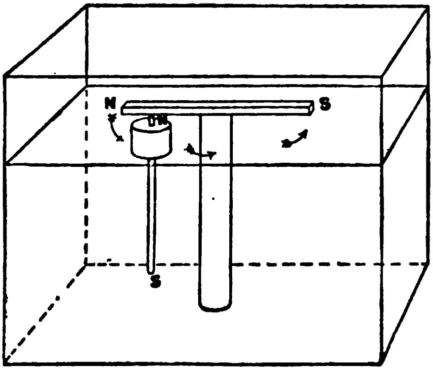

FIG. 3.

passage from the north to the south pole it will describe a certain curved path, as shown by arrows on the top of the water in the diagram. If we take the boat back and place it at another part of the north end of the bar magnet, it will start off again, but along another path, perhaps further out from, or perhaps nearer to, the bar, according to the position of the point from which it starts. The same will be repeated also on the other side of the bar magnet; further, if between the experiments we gradually turn the bar magnet over on its side, we shall find that similar curves are invariably described by our floating north pole, until after having turned it through 180°, we see that there is no difference between any of the four sides of the bar magnet as regards its action on the exploring pole.

If we place the boat far away from the magnet, near the side of the vessel, perhaps it will not move at all; yet this is only because the magnetic force which tends to drive the boat may not be strong enough to overcome the resistance of the water to the boat's motion, and if we take the needle out and can magnetise it more strongly, we shall find that the boat will start on one of its voyages to the south pole in whatever part of the water it is placed, except very close to the centre of the bar.

We thus find by experiment that a magnet influences a magnetic pole brought into its vicinity by acting upon it with a certain force, which at any point tends to move it in a particular direction: this force is not always in the same straight line, but gradually curves round, so that a free pole moving under it traverses a definite curved path between the north and south poles of the magnet. The space round the magnet where we can detect this effect is called the 'magnetic field,' or simply the 'field' of the magnet. Further, this field has no fixed limits in space, since the distance at which the magnetic force can be detected depends upon the sensitiveness of the instruments which we employ to detect it.

Let us next trace out the various paths traversed by the N. pole of our boat: they were all described on the horizontal plane of the surface of the water, but relatively to the magnet they were traced in every plane which contains its length, since the magnet was gradually rotated round lengthwise without in any way affecting the character of the curved paths. If, therefore, we combine together all the paths which we have seen described, they will form a complete system of lines encircling and enveloping the magnet on all sides, and all curving round from the north to the south pole. Fig. 4 gives a view from above of a bar magnet in the centre of a system of lines described in every plane containing its length, and shows how they would form a kind of spheroidal cocoon of threads stretching from one pole to the other. Now what are these paths so traced out? They are 'lines of force,' being the lines or paths in which a single magnetic pole placed in the field at any point will, if left free and unconstrained, be forced to move from the one pole to the other. A tangent drawn to a line of force at any point gives the actual direction of the magnetic force at that point; this direction is continually changing, and the lines are curved, but the simple pole as it follows the curves is assumed to be uninfluenced by anything of the nature of centrifugal force. Since our exploring boat would start moving from any place near the magnet (except just at its centre, where there is no magnetic force), it is evident that the number of such 'lines of force' is really infinite, yet it will be sufficient to mark down a number large enough to give us a clear idea of the various directions of the force. And this the more so because it will be found that if the lines are traced out very accurately, no two lines ever cut each other, but that they always run side by side, more or less converging or diverging, but never intersecting; in fact, it would be impossible for two or more lines to intersect, since at any point in the field a free single pole can only move in one particular direction.

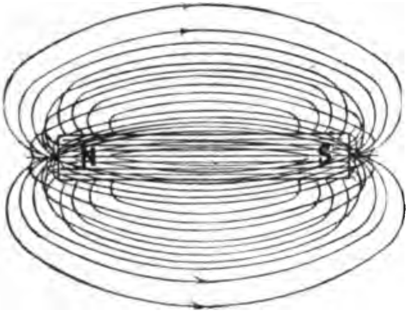

FIG. 4.

But not only have the lines a certain position in space, they have also a certain 'sense,' as it is termed; by this is meant direction in the further sense that a free north pole set down on any line will move in one direction along it, while a free south pole will move in the opposite direction. It is conventionally agreed that the positive direction of the lines is that in which a north pole would move, and hence in our first experiments we chose a north pole to place in our exploring boat; the direction thus given to the lines by a north pole is usually indicated on them by arrowheads. Consequently all the lines of force are regarded as emanating from the north pole of a magnet and 'flowing' as a stream round to the south pole through the surrounding air. Further, what is their relation to the fixed directions which, as our first experiment showed, were imposed on a small compass needle when brought into a magnetic field? If we eliminate the effect upon it of the earth as a magnet, it will be found that at every point its magnetic axis is set tangentially to the direction of the curved lines of force; hence the picture given to us by our system of lines of force enables us to predict the definite position which a magnetised needle will take up at any point in the space surrounding the bar magnet. Fig. 5 shows the position and direction of a few lines separated out from the number which traverse the field of a magnet, and further shows a small compass needle setting itself tangentially to their direction. If the needle be small, and at some distance from the magnet, it does not move as a whole along the lines of force, as did our single magnetic pole, because its two poles are nearly equally affected by opposite forces, and the one tends to move in the one direction as strongly as the other in the opposite direction. Yet if placed athwart the lines the needle is acted on by a couple tending to rotate it until its axis coincides as nearly as possible with the line of force; it therefore sets itself tangentially to the direction of the lines at that part of the field where it is placed. Its north end will point along the line in the same direction as that in which a free north pole would move. By the north end or

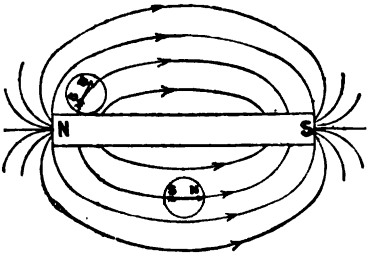

Fig. 5.

pole of a magnet we shall throughout mean that which points to the geographical north, or, as it is sometimes termed, is north-seeking; and when a compass needle sets itself along lines of force, the end which points towards the geographical north, if acted on by the earth's field alone, will indicate the positive direction of the lines of force, or point down the lines in that direction which will lead us to the south pole of the magnet.

We have now seen that the condition of space which we call a magnetic field can be usefully represented by 'lines of force'; to sum up, they themselves show the path and direction in which a free magnetic pole will move when placed in the field at any point, and a tangent to them shows the position which a small piece of magnetised material, such as a compass needle, will take up under the action of the forces on its two poles.

The reader will probably know the way in which the field of a magnet is illustrated by means of iron filings sifted on to a sheet of glass or paper held over a magnet. The little pieces of iron become magnetised by induction, as it is termed, along their greatest length, so that one end forms a north pole and the other a south. They then resemble our compass needle, and consequently set themselves end-on along the direction of the lines of force at any part of the field; where the plane of the paper cuts across the direction of the lines the filings stand up erect, and give, as it were, a view of the lines in section; where the directions of the lines fall in the plane of the paper they follow along them in chains, the north pole of each little magnet clinging to the south pole of its next neighbour. Thus in the case of a bar magnet laid horizontally on its side, the filings collect in crowded tufts above the poles and stand up nearly vertically; from the edges they radiate outwards, and gradually curve round, forming more or less closed chains in the horizontal plane of the paper. Meanwhile there is one point on which the picture given by the iron filings must not be allowed to mislead us. The filings arrange themselves at the sides of the magnet in curves which are to a certain extent broken and separated off by intervals in which there are no filings. But we know that an infinite number of closely contiguous lines of force could have been traced out by means of our exploring north pole, and consequently that there are in reality no gaps in the magnetic field corresponding to any gaps between the lines of the iron filings. From their mutual action on one another the magnetised filings tend to fall into lines when the paper is gently tapped, and to complete those lines, leaving interlinear bands, rather than to remain spread uniformly over the surface, even though their magnetic axis be set along some line of force.

Yet what at first sight seems a defect in the picture given by the iron filings may serve as a hint to suggest a method by which we may still further map out a magnetic field so as to fully exhaust its properties. At any point in a magnetic field the force acting on a pole or on a magnet as a whole, not only has a certain position and direction, but also a certain magnitude ; in other words, the magnetic field has a certain intensity or strength. This 'strength of field' must again be estimated by reference to such a simple magnetic pole as previously gave its direction, and for this purpose the pole must be of unit strength. Unit pole is a magnetic pole of such strength that if it is placed in air at unit distance from another such pole of equal strength there will be unit force acting between the two, repelling or attracting them according as they are of the same or opposite sign. Consequently on the C.G.S. system of units a unit north pole, placed at a distance of one centimetre in air from a south pole of equal strength, will be attracted with a force of one dyne. The strength of a field at any point is estimated by measuring the force with which a unit pole would be acted upon at that point. A field is therefore of unit strength if a unit pole placed in it is acted upon by a force of one dyne. The field of a bar magnet increases in strength the nearer we approach either of its poles, as is shown by our north pole being strongly attracted or repelled when close up to the south or north pole of the magnet.

Can we, then, also represent the strength of a field by the same geometrical picture of magnetic 'lines of force'? The experiment with the iron filings suggests a method by which this can be done. It has been stated that there are no real gaps in the magnetic field, but that the force pervades the whole of it, varying, it may be, in strength in different places, yet continuously shading off from one value to another without any sudden jumps or breaks ; yet the lines of force indicated by the filings are crowded together near the poles where the force is strongest, and gradually widen out until where the force is weakest they are comparatively widely scattered. Now suppose that we cut the lines by a plane at right angles to their direction at any point; obviously we should have more lines per unit area on such a transverse plane if it is taken near to the poles than if it be placed at some distance from them; consequently the density of the lines or the number of lines traversing any unit area in a plane perpendicular to their direction might be made to serve as a guide to the relative strength of the force at different parts of the field.

If, therefore, out of the infinite number of lines of force that could be drawn at any part of the field we select a certain number of marked lines, we could represent the magnitude of the force acting on a unit pole placed in any part of the field by the closeness or sparseness of such selected lines at that part; and if each selected line represents a force of one dyne acting on a unit pole, we can then express the strength of the field by stating how many such lines there are per unit area of cross-section on a plane normal to their direction. Unit field will have one line per unit area of such a plane, because unit pole placed in it would be acted upon by unit force, or on the C.G.S. system one line per square centimetre will represent a field acting with a force of one dyne upon a unit pole.

In fig. 6 let each of the small squares represent in perspective one square centimetre of area on a plane at right angles to the direction of the lines of the field: out of the infinite number of these as represented by the shading, four selected lines stand out, and if C.G.S. lines, as they are called, indicate a field of such strength that in it unit pole will be acted upon by a force of four dynes. By such unit lines the intensity of a field can be numerically expressed; *e.g.* a field whose intensity or strength is 1,000 in C.G.S. measure means one in which the number of C.G.S. lines per square centimetre on a plane normal to its direction is 1,000, and in which, therefore, a unit pole would be acted on by a force of

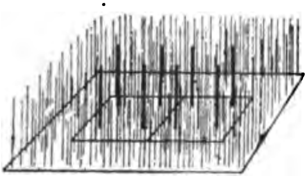

FIG. 6.

1,000 dynes. Thus by means of a definite system of lines all the properties of a magnetic field can be accurately represented, its direction and its strength mapped out and marked. A uniform field will be graphically represented by lines running parallel to one another and spaced at equal distances, as in figs. 13 and 20; since, then, wherever the unit area be taken on a plane normal to the direction of the field, the same number of lines traverse it. A field of varying intensity, when plotted, will appear with a varying density of lines, yet always so spaced that the number passing through unit area on the normal plane is equal to the strength of the field in dynes when acting on unit pole at that place. To express its strength, so small an area of cross-section must be considered that the density of lines within that area may be regarded as uniform. Fig. 21 shows in section and plan portions of two fields whose intensities gradually decrease as we move towards the central gap between them, the direction of one being opposite to that of the other.

Henceforth the unit pole will disappear from our pages, and all magnetic phenomena will be treated by means of the imaginary conception of 'lines.' When these lines of the field are spoken of almost as if they had actual existence in Nature, it must be remembered that they are only a device for expressing clearly and concisely certain physical facts, and their meaning must always be sought for by reference back to the free unit pole of north polarity. We may compare the method by which they are pictorially indicated to that by which light or shadow is represented in line engraving: a strong field can be represented by a shading of closely-packed but separate lines, just as is a dark shadow, although both field and shadow have no structure and are perfectly continuous, the one in nature and the other in the picture which the engraving represents in black and white. The conventionality of the method is shown by the fact that we sometimes have to deal with fractions of a line; for example, the intensity of the earth's field in England is such that it would act on a unit pole with a force of ·47 or about half a dyne; such a case would be represented by half a line per square centimetre, or, as this could not be pictorially represented, by one line passing through every two square centimetres of area on the normal plane.

CHAPTER III

THE MAGNETIC CIRCUIT

In the preceding chapter the magnetic field existing outside a straight bar magnet has been mapped out by means of lines of force which were found to emanate from its north pole and enter into its south pole ; but it still remains to ask, What is the *internal* condition of the bar magnet, and can we still further extend the method to its interior? Obviously we cannot make any exploring experiment within the mass of the iron, even with the approximation to a single free pole which we previously used, and the question can therefore only be decided by theory as explanatory of all the phenomena connected with the subject; and to help us to form such a theory we must next consider the magnetic field surrounding a conductor which is conveying an electric current.

In the year 1819 a Danish physicist, Oerstedt, discovered that when a magnetic needle was brought near a wire carrying an electric current, it was deflected into a definite position relatively to the wire ; in other words, between the conductor conveying an electric current and the magnet he found that there existed a certain magnetic force which, if the conducting wire were stationary, and the needle movable, would cause the latter to set itself in a definite direction : for example, if the conductor conveying the current is a long straight wire held horizontally over a compass needle, the latter tends to set itself at right angles to the length of the wire ; and, further, the direction in which its north pole points when it has so set itself depends upon the direction in which the current is flowing in the wire.

Hence we see that it is not only permanent magnets of steel which have a magnetic field ; an electric current is also surrounded by such a field, and upon this fact depends the whole principle of dynamo-electric machinery. The simple experiment of Oerstedt was in fact the first instance that had been observed of the conversion of electrical energy into mechanical work, and as such illustrated the principle of the electric motor; for the needle as it moves could be made to do mechanical work.

In order to map out more definitely the magnetic field due to an electric current flowing in a straight wire, we will again have recourse to the picture given us by iron filings. Let us pass a straight wire through a hole in a piece of cardboard or glass plate held at right angles to the vertical wire, send a current through the wire, and sprinkle iron filings on the plate; as in the similar experiment with the bar magnet, the filings will arrange themselves in certain lines, which in this case are concentric circles having the wire as their centre (fig. 7). We can slide the plate up and down, while still keeping it in a plane perpendicular to the axis of the wire, and there will be no alteration in the shape of the figures. Now the circles thus formed show us the position and distribution of the lines of force of the field, and by examination of them we find that a rectilinear portion of an electric circuit, such as a straight wire, surrounded by air, and traversed by a current of which the return path is a considerable distance off, is surrounded by concentric circular lines of force, in a plane perpendicular to the axis of the wire, and having their centre in its axis.

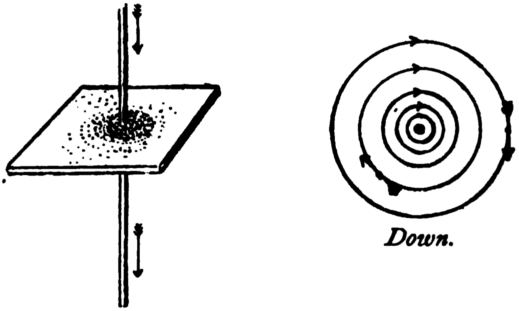

FIG. 7.

To the right of fig. 7 the position of a few of the lines of force surrounding a straight conductor is shown more clearly, together with a small compass needle setting itself tangentially to the lines; the current is supposed to be flowing downwards through the plane of the paper, and the direction of the lines as marked by arrowheads is seen to be clockwise. By means of the compass needle, whose north pole will always point in the positive direction of the lines, it will be found that the direction of the lines is related to the direction of the current, just as the direction of rotation of an ordinary right-handed screw is associated with the direction of its forward or backward movement as viewed from its head to which the screw-driver is applied. For example, in fig. 8, if the screw is to be driven in from right to left, it is rotated in a clockwise direction, as seen from the end A, and similarly with a current flowing from right to left the positive direction of the lines or the direction in which a free north pole would be impelled round and round the conductor is also clockwise; if the direction of the current is reversed, or the screw is being drawn out from left to right, its direction of rotation is counter-clockwise, and so also is the direction of the lines.

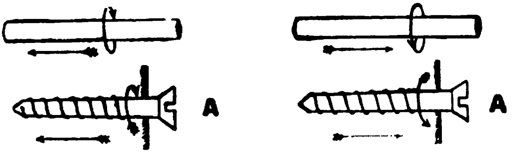

Fig. 8.

The position and direction of the lines of force surrounding a straight conductor have now been given, and their distribution is again shown by the iron filings: these are crowded together more closely near the wire, and thus show that the strength of the field is greatest immediately round it, and gradually diminishes as the radial distance from the wire is increased. Further, if the strength of the current through the wire be increased, the rings of iron filings will crowd together still more closely in all parts of the field, though still being relatively scattered at some distance away; in short, the strength of the field varies directly as the strength of the current, and inversely as the radial distance from the wire.

One more peculiarity, which is as yet entirely new to us, remains to be noticed : it is that the lines of force are in this case closed or endless curves, entirely surrounding the axis of the wire. When any line is traced along its entire course we eventually arrive at the same point as that from which we started to trace it.

Next let us, as in fig. 9, bend the wire round so as to form a closed loop, the ends being twisted together and insulated from one another, so that the current passes round the entire loop ; at once it will be seen that if the lines of such a current-loop retain the general character of the lines due to a straight conductor, they will, all of them, pass right through the loop, entering into one face of it and issuing outwards from the other, and such is in reality the case. The field of such a loop can be explored by means of a small compass needle, and we shall find that it not only takes up a definite direction when outside the loop, but also when placed inside it; and as so traced the lines are found to pass through the loop, and thence gradually to expand into curves which are larger and larger as we pass from the centre of the loop outwards until they become too indistinct to be traced.

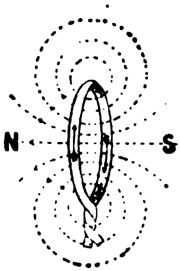

FIG. 9.

Again, therefore, the lines are closed curves, interlinked with a closed electric circuit, like a number of rings threaded on to one common ring. By again applying the rule of the screw to the modified case of our circular conductor, it will be found that the direction of the lines as they pass through the loop is associated with the direction of the current round the loop, just as is the movement of a right-handed screw with the direction of its rotation, and this leads us to the following extremely simple and convenient rule : Curve the right hand round the outside of the loop, keeping the palm towards its axis, so that the direction of the flow of current is from the wrist to the tips of the fingers ; then the outstretched thumb will point along the positive direction of the lines within the loop, the face from which they issue being therefore on the same side of the hand as the thumb.

Now, since all the lines enter into one face of the loop and issue from the other, the loop may be said to have a south and a north face, just as a bar magnet has a north and a south end ; and the loop will, in fact, act in every respect exactly as if it were a thin circular magnet magnetised transversely along its shortest axis ; it will repel or attract other magnets according as the face which is presented to them is of the same or opposite polarity, and if suspended so as to be free to move, it will set itself in a plane perpendicular to the magnetic meridian, so that its north side faces the geographical north.

Thus, for the circular electric current we might in imagination substitute a thin disc of magnetised iron or steel, or, as it is termed, a 'magnetic shell' coinciding in form with the loop, and having one face a north pole and the other a south pole. The external effect of such a magnetic shell would be identical with that of the current-loop ; their external fields would be the same, only in the case of the iron magnet we could not trace the passage of the lines *through* its substance by means of a compass needle.

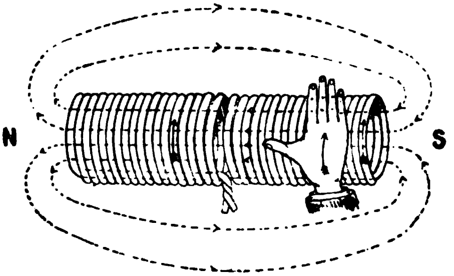

FIG. 10.

Next let us place several such loops of current side by side in a straight line. This is best effected by coiling up one and the same length of insulated wire into a number of closely-contiguous turns, the ends being brought back through the centre of the hollow cylinder so formed (fig. 10). Such an arrangement is called a 'solenoid,' and when its field is explored by a compass needle the lines (a few of which are shown in the diagram) will be found to run right through the cylinder, issuing forth from and entering into one or other end according as the current flows in one or other direction round it. Again, by the application of the *right* hand curved round the outside of the helix with the palm towards the axis, and so placed that the current flows from wrist to finger-tips, the direction of flow of the lines through the solenoid is given by the fact that the thumb points to the north pole or end from which they issue. Now, just as a single loop was found to be equivalent to a magnetic shell, so a solenoid is equivalent to a bar magnet; as regards its external field, it will set itself north and south when suspended freely, and exhibit directive, attractive, and portative effects just like a bar magnet.

Now let us insert into the hollow space within our solenoid a cylindrical 'core' of unmagnetised iron or steel, extending from the one end to the other, and examine the effects thereby produced; although entirely insulated from it electrically, it will be found that a north pole is developed at the one end of the iron, and a south pole at the other, so that the whole arrangement still acts externally just as did the simple solenoid, which itself was like a permanent bar magnet, only now its attractive and directive force is very much increased. Owing to the electric current circulating round the turns of the solenoid, the iron becomes magnetised by 'induction,' as it is termed, polarity being induced in it by the magnetic field in which it was placed. We have, in fact, an 'electro-magnet' whose magnetism depends upon the presence of the 'exciting current' circulating round it; when this current is interrupted the iron almost or entirely ceases to attract; as soon as circuit is again made, at once the iron becomes magnetised; it can therefore be made a magnet at will. The external field of the excited electromagnet is closely analogous in its distribution to that of the solenoid before the iron core was inserted, the only difference being in the greater strength of the field of the electromagnet; and, further, it is closely analogous to the field surrounding a permanent bar magnet as previously mapped out. Now we have seen that in the simple solenoid without any iron core the lines of force were closed curves which passed right through its interior; it is therefore reasonable to assume that if we could explore the interior of the iron core placed inside the solenoid, the lines would be found to traverse its substance from end to end in a manner exactly similar to that in which they traverse the hollow solenoid; and, further, it is reasonable to assume that the same is also the case with the permanent bar magnet. This view is, in fact, the basis upon which the whole of the modern theory of magnetism rests; its complete justification and establishment would lead us so far afield that the reader must rest content with the above proof by analogy.

The assumption which is here made needs, however, to be rendered more definite and explicit. When a piece of unmagnetised iron is placed in a magnetic field, and so magnetised by 'induction,' every small portion of it, wherever situated, has a definite magnetic axis, which is the direction of magnetisation at that place. Now, if a transverse cut be supposed to be made in the iron at right angles to the direction of magnetisation at any place, and the free unit north pole of theory be brought to the place in question within the infinitely thin air gap formed by the cut, it would be acted upon by a certain force, which when expressed in dynes is known as the 'magnetic induction' at that place. This force can, as before, be represented in direction and magnitude by C.G.S. lines, so spaced that the number traversing a square centimetre of the transverse section is equal to the number of dynes of force which act on the unit north pole. By continuing our imaginary exploration with a unit north pole the 'induction' at any part of the transverse section which we have made in the iron could be mapped out, and by means of a number of such sections similarly explored we should arrive at a complete system of lines of force within the iron, mapped out by a particular and definite process: as so mapped out they are called 'lines of induction,' since they indicate the direction and strength of the 'induction' at any point. The origin of this term is rendered more evident if, as above, a piece of unmagnetised iron is considered in which magnetism is induced, but the fact conveyed by the technical expression, 'the magnetic induction,' and the entire process by which the lines of induction are arrived at, are equally applicable to any piece of iron, whether it be an electromagnet or permanently magnetised, and they are, in fact, so applied. The density of such lines per square centimetre of area on a plane perpendicular to their direction, i.e. 'the induction' at any part of the iron, is symbolised by B, and its numerical value is the force in dynes acting on a unit pole when the above process is strictly adhered to. The 'total induction' through any area of cross-section is the total number of such lines that cross the area in question. Now the reason for the introduction of the above method of exploring the interior of a piece of magnetised iron with all its necessary precautions and exactitude is that the lines of induction so mapped out in the iron are the continuations of the lines of force which we previously mapped out in the field external to an iron magnet; these latter when traced by a small compass needle were found to spring from and end at the polar surfaces of the magnet; beyond that we were unable to trace them, although the evidence of the lines of force round and through a solenoid enabled us to guess their continuance through the mass of the iron itself. But now theory has shown us the method by which they can be traced through iron; there are as many lines of induction passing through the iron of a magnet as there are lines in its external field, and each line of induction in the iron finds its appropriate continuation in one of the lines of the external field as originally mapped out by our compass needle. These latter lines are, in fact, *lines of induction in the air*, and could have been mapped out by the same process as above described for lines within iron; and similarly the symbol B can be extended to express their density per square centimetre in air. Such a process leads, however, to exactly the same system of lines as we have previously discovered by simple experiment with a compass needle, so that the lines which have been described as encircling a straight wire conveying a current, or a loop, or a hollow solenoid, were all lines of induction in the air. Hence in air the precautions of the method are unnecessary, and it is simpler to start with the ready method of mapping out a magnetic field as first described. We have now, therefore, arrived at the following results: Every line of induction is a closed curve, which either loops round an electric current or currents (and in this case may be entirely in air or entirely in iron, or partly in the one and partly in the other), or at some portion of its endless path passes through a piece of permanently magnetised steel or iron. Wherever lines of induction issue out of iron into air or other non-magnetic material, there a north pole is developed; and wherever they enter from air into iron, there we have a south pole : within the iron their direction is from south to north, without it, from north to south. Lines of induction are lines of force according to our original definition, whether they are in air or iron, but in the latter they are lines of force mapped out by the particular method described. The expression 'lines of force within the iron' is, however, often confined to a system of lines mapped out by a different theoretical process, and from which lines of induction have to be distinguished. For the exact nature of and reason for this distinction it must suffice to refer to larger works dealing more fully with the subject.[1] Now in dynamos the E.M.F. of the machine is, as we have said, generated by certain movements of electric conductors through a magnetic field in air, for the conductors, though wound over or laid on iron, are in effect entirely surrounded by air or other non-magnetic material. Since, therefore, in the air no distinction need be made between lines of force and of induction, whatever the theoretical method by which they are plotted, and they are identical, the E.M.F. may indifferently be said to be due to movement through either lines of force or of induction. But in every dynamo the field is produced either by one or more electromagnets or by permanent steel magnets; hence the lines with which we have to deal have always passed through iron at some portion of their path, and are in every sense lines of induction. In future, therefore, we shall deal solely with lines of induction, and invariably speak of the E.M.F. of dynamos as dependent on 'lines of induction.'

Returning to the solenoid of fig. 10 we have already stated that the strength of its field is very much increased by the insertion of an iron core; in other words, the total number of lines of induction passing through the solenoid and spreading out to form the external field has been enormously increased by the presence of the iron core. This result can only be traced to some specific property of the iron as iron; if we insert a core of wood, or glass, or porcelain, or any non-metallic substance, no difference can be found as compared with the same solenoid encircling only air. The same, too, may be said if we try any of the metals, with the single exceptions of iron, cobalt, or nickel, and certain of their alloys and compounds; and even in the case of cobalt or nickel their effect though sensible to delicate measuring instruments is not to be compared with that of iron. But, more than this, the strength of field of an electromagnet with a core of soft annealed wrought iron can be made much more intense than that of any combination of permanent steel magnets of reasonable dimensions. It is owing to this fact that the magnetic field which must be present in any dynamo is now almost always produced by means of electromagnets, strongly excited only when the dynamo is in use; so long as the electrical engineer was dependent on permanent steel magnets for supplying him with a magnetic field, the powerful dynamos of the present day were impossible.

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[1] For a short epitome of the subject see Professor J. A. Fleming's *The Alternate Current Transformer*, i. 13, 14; also, for a detailed statement, Prof. J. A. Ewing's original papers in *Proc. R.S.*

But what is the strength of the field in the case of any given solenoid? To determine this question let us take a ring of wood of circular cross-section and wind upon it, and uniformly all round it, a number of turns of insulated copper wire through which an electric current can be passed (fig. 11); there is now no external magnetism and no poles, since the lines of induction form closed curves entirely within the loops of the solenoid, but by means of a ballistic galvanometer the number of lines passing through the solenoid can be measured in a manner which at this point we need not further describe. It will be found that the strength of the field is increased by increasing the strength of the current flowing through a given number of turns, or by increasing the number of turns through which a given current flows; it is, in fact, with a non-magnetic core simply proportional to the product of the current and the turns through which it flows, i.e. to the ampère-turns of the solenoid; and this product is called the 'exciting power' of the solenoid. For example, if a certain strength of field is produced by an exciting power of 10,000 ampère-turns wound round the ring, it is indifferent whether these be made up by

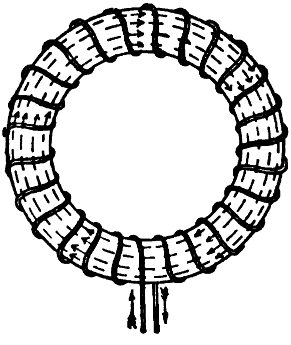


FIG. 11.

100 ampères flowing through 100 turns or one ampère through 10,000 turns. Further, with the given number of ampère-turns, the strength of the field is inversely proportional to the length of the circumference of a circle passing centrally through the interior of the ring-core. If, therefore, the strength of field within the solenoid or number of lines per unit area of its cross-section is symbolised by H, the general law is summed up by the equation

$$\mathrm{H} \propto \frac{\mathrm{AT}}{l},$$

where AT are the ampère-turns and l=the circumference corresponding to the mean diameter of the ring; or if l is in centimetres and H in C.G.S. lines per square centimetre,

$$\mathrm{H} = k \cdot \frac{\mathrm{AT}}{l},$$

where $k = 0{\cdot}4\pi$, or $1{\cdot}256$. The density of the lines within the solenoid is uniform over its area of cross-section, and hence the total number of lines, Z, passing through it is equal to the density of the lines per unit area multiplied by the area, or $\mathrm{Z} = \mathrm{H} \times a$, where a=the area in square centimetres encircled by the loops.

Now H is the strength of field within the solenoid when enwrapping wood or air or other non-magnetic material; but if for the ring of wood we substitute a ring of iron of the same shape and size, and wrap round it the same exciting power, we know that the number of lines and their density is much increased. Let B=this new value for the density of the lines; then $\mathrm{B} = \mu\mathrm{H}$, where μ is a numerical coefficient expressing, as it were, the multiplying power of the iron by reason of which the density of the lines is increased.

The great importance of the iron core may be estimated from the fact that the numerical value of μ may be as much as 2,000, i.e. by its presence the induction may be increased to 2,000 times the value which it previously had when the circular solenoid enclosed merely air or wood. Stated more generally, the value of μ depends in reality upon the nature of the material of which the core is made, and expresses a definite physical property of it, so that if H=the original strength of field within a given circular solenoid whose ampère-turns encircle air, and B=the new strength of field or induction when a core of a certain material in a certain condition takes the place of the enclosed ring of air, then for that material in the given condition μ is equal to $\frac{B}{H}$. For air, wood, brass, copper, and most non-magnetic materials μ is sensibly equal to 1, so that H is, in fact, the 'induction' in air for the given circular solenoid, or the value of B when the lines of induction are in air. For iron, cobalt, or nickel μ is greater than 1; but for bismuth, which slightly decreases the density of the lines, the value of μ is slightly less than unity.

For any substance, therefore, in the form of a ring wrapped uniformly round with a certain number of ampère-turns, the density of lines of induction passing through it per square centimetre of cross-section is $B=\mu H$, and the total number of lines of induction is

$$Z=B\times a=\frac{0{\cdot}4\pi AT}{\frac{l}{a}\cdot\frac{1}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Now the path of every line of induction is a closed loop or circuit, and in the above case they all flow round the ring in paths whose mean length is l: the area normal to their direction through which they pass is a, while μ is a numerical coefficient depending on the specific nature and condition of the material or substance through which they flow. Hence the denominator of the above fraction bears a close resemblance to the expression for the electrical resistance of a conductor, viz. $\frac{l}{a}\cdot\frac{1}{k}$, where k is the specific electrical conductivity of the metal of which the conductor is composed, or the reciprocal of its specific electrical resistivity; and considered from this point of view, the fundamental equation is seen to be of the same form as the well-known equation for Ohm's law, $C=\frac{E}{R}$. This suggests to us that an analogy might be drawn between the flow of a current through an electrical closed circuit and the flow of lines of induction through a magnetic circuit, that flow taking place in closed loops; and this analogy has been found to be not only useful by way of illustration, but sufficiently accurate to afford a ready and convenient method of treating the laws which govern the production of lines of induction by current-turns round an electromagnet. The path of the lines forms a 'magnetic circuit' closed upon itself and having a certain length and area; through this circuit there flows a stream of lines of induction propelled round by a 'magnetomotive force,' and their total number is the quotient of the magnetomotive force divided by the magnetic resistance, or (to give this property a more distinctive name), the 'reluctance' of the circuit. Thus, analogous to Ohm's law—

$$\text{electric current} = \frac{\text{E.M.F.}}{\text{electrical resistance}}\text{; we have}$$

$$\text{magnetic flow or number of lines} = \frac{\text{magnetomotive force}}{\text{magnetic reluctance}}.$$

The magnetomotive force is proportional to the number of ampère-turns wound round the circuit; by these is produced a certain difference of magnetic potential, which resembles the difference of electrical potential in a battery, since both are expended in driving the electric current or the magnetic flow against the resistance or reluctance of the circuit. The current or the flow of lines is, in fact, the rate of change of potential over the resistance or reluctance of the circuit. The magnetic reluctance of an entire circuit or any portion of it is proportional directly to its length, and inversely to its area. Further, all substances do not conduct the lines of induction equally well; consequently μ represents the specific conductivity of the material through which the lines pass when in the particular condition implied by the fact of such a flow of lines, or, as it is termed, its 'permeability.' On the C.G.S. system air has been determined upon as the substance of unit permeability, for which, under all circumstances, $\mu = 1$. The magnetomotive force of a circuit is measured by $0{\cdot}4\pi$ of the exciting power in ampère-turns; consequently if one C.G.S. line passes across from one side to the other of a centimetre cube of air, unit difference of magnetic potential must exist between the opposite faces of the cube, i.e. a difference of magnetic potential equal to $0{\cdot}4\pi$ of an ampère-turn. Hence when the path of the lines is entirely in air, as in our circular solenoid enclosing air, $B = H$, or the strength of the field without the presence of any iron.

From the conception of a magnetic circuit several very important consequences follow, which will here be briefly considered. The example of a magnetic circuit which we have considered in our primary equation has been a closed one of iron of the same nature, and further of the same cross-section throughout its length, and the path of the lines of induction was throughout confined to the iron, and had a length which may be considered as equal to the mean circumference of the ring. But a magnetic circuit may also be made up of different materials having different permeabilities; these, again, may have different areas of cross-section and different lengths, while further they may be arranged either so that each line of induction in its closed path traverses them in succession (i.e. they are 'in series') or so that the path followed by some lines is different from the path of others, either entirely or at some portion of its length (i.e. they are 'in parallel'). And it is in such cases that the analogy of the electric and magnetic circuits proves of use as a safe clue to a ready method of treatment. If a group of lines passes round an entire circuit, or through a portion of a circuit, which may be divided along its length into different sections having different lengths, areas, and permeabilities, their total magnetic reluctance is the sum of their separate reluctances: *e.g.* if $l_1\ a_1\ \mu_1,\ l_2\ a_2\ \mu_2,\ l_3\ a_3\ \mu_3,\ \ldots$ &c. are the respective lengths, cross-sections, and permeabilities of different portions of a circuit, all traversed in succession by a certain set of lines, their total magnetic reluctance R

$$= \mathrm{R}_1 + \mathrm{R}_2 + \mathrm{R}_3 + \cdots$$
$$= \frac{l_1}{a_1\,\mu_1} + \frac{l_2}{a_2\,\mu_2} + \frac{l_3}{a_3\,\mu_3} + \cdots$$

The reciprocal of the reluctance of a magnetic circuit is called its 'permeance;' it is related to the permeability just as the conductance of the electric circuit, being the reciprocal of its resistance, is related to its conductivity.

Next, if at any point in their path a group of lines separate and follow different paths or sections of the total area of their magnetic circuit, the law which they obey is exactly analogous to the law of electric circuits in parallel, viz. that the number of lines flowing through any section of the magnetic circuit is the quotient of the difference of magnetic potential between opposite ends of the section, and the magnetic reluctance of the section; thence it also follows that when there are several paths for the lines between two points between which a difference of magnetic potential exists, the total flow of lines is divided between the several paths directly as their permeances, or inversely as their reluctances, as determined by the respective lengths, cross-sections, and permeabilities of the different paths; and the joint permeance of the parallel paths is simply the sum of their several permeances.

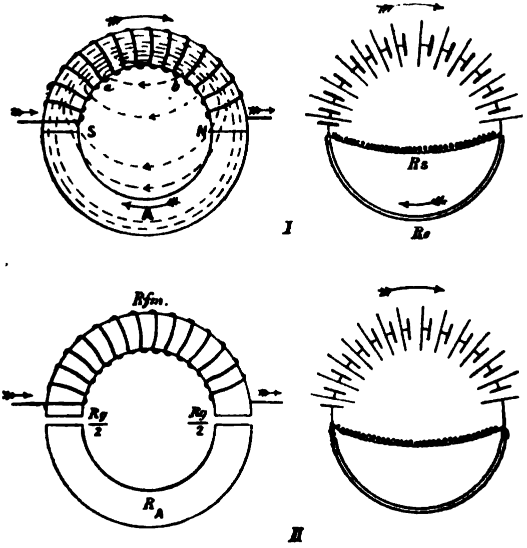


FIG. 12.

To illustrate these laws let us group our ampère-turns entirely on one side of a closed iron ring composed of a curved electromagnet and an armature or keeper A (fig. 12, I); the arrangement may be likened to a battery, shown at its side, of which each cell represents a current loop, and the ends of which are joined by a thick piece of copper wire R_e: there is a uniform rise of magnetic potential as we pass through the loops from south to north, analogous to the rise of electric potential through the cells of the battery, and since the ends between which there is a difference of potential are joined so as to form a closed circuit, there is a flow of lines or of current round the circuit which within the magnetic or electric battery is from the — or S. end to the + or N. end, but without is from the + or N. end to the – or S. end. But at this point there enters a very important difference: there are no insulators for magnetism, similar to the substances which are insulators for electric currents; consequently, if between any two points there exists a difference of magnetic potential, there will be a flow of lines along any path between those two points equal to the magnetic potential divided by the magnetic reluctance of the path, whether it be composed of air, iron, brass, or any other substance. The lines of induction will therefore not be confined entirely to the iron, as they were in fig. 11, but will pass across from the N. end of the magnet to the S. end through the air, as well as through the iron of the keeper. Air, gun-metal, brass, or manganese bronze, &c., are often spoken of and used as magnetic insulators, but it must be remembered that they are so only in a comparative degree; at the best their conductivity to magnetic lines is only some thousand times less than that of iron, and at the worst this difference may sink to less than one hundred times.

But, more than this, lines of induction will flow across the air between *any* points between which there exists a difference of magnetic potential; consequently there will be a flow from, *e.g.* *b* to *a*, since *b* is at a higher magnetic potential than *a*, i.e. from one portion of the magnet to the other, and such flow will increase as we pass away from the centre of the magnet towards its ends. If, therefore, we wish to keep up the analogy between the electromagnet and the battery, we might regard the latter as immersed in a conducting liquid or electrolyte, through which currents may flow across even from one cell of the battery to another in addition to the external current through the wire joining the terminals. Such currents would be a leakage as far as useful current in R_e is concerned, and in the same way the lines of induction in the air of fig. 12, I, and in similar cases, are frequently spoken of as magnetic leakage, stray lines, or waste field; but it must be remembered that such a point of view implies that only lines passing through the keeper or armature are useful and desirable. The whole illustrates the case of two or more magnetic paths joined together 'in parallel' to form a composite magnetic circuit. And this leads us to a second point which deserves our attention : it is that, owing to the non-existence of magnetic insulators, it is often impossible to define the exact limits of the magnetic circuit or the various paths followed by the lines, and consequently the equation of the magnetic circuit which demands a knowledge of the exact length, area, and permeability of the different portions of the magnetic circuit only admits of an approximately accurate application. In the case upon which our fundamental equation (1) was based, viz. of a circular ring-core with loops uniformly distributed round the whole of it, there is no doubt about the dimensions of the magnetic circuit and its permeance; all the lines are confined entirely to the iron, and the number flowing through any section across it is identical; there is, in fact, no external magnetism and no 'leakage' of lines out of the loops and across the air. The reason is that all points of the ring are at the same magnetic potential, since the magnetomotive force supplied by the loops over any length is only just sufficient to propel the total number of lines through that length; it may be likened to a closed loop of wire round which an E.M.F. is generated, this E.M.F. being uniformly distributed along the length of the wire; a current flows, but all points of the loop are at the same potential, since the E.M.F. developed in any length is just sufficient to drive the current through the resistance of that length.

But now in our new case of an iron ring, in which the winding is grouped on one side only, the total magnetic reluctance of the circuit is a complex combination of the internal reluctance of the magnet and the external reluctance of the armature; for in strictly scientific language the armature of an electromagnet is all that closes its magnetic circuit, whether air, or iron, or any other substance.

How, then, are we to calculate the reluctance of the air, since the paths of the lines through it are so manifold and differ so much in their length? They flow not only from the extreme ends, but across from all points of the magnet, so that the number of lines carried by the magnet, and therefore their density within the iron, is continually varying; at its centre they are a maximum, but thence they gradually leak out on all sides, especially towards the ends, so that a smaller number passes through the iron of the keeper than through the magnet. The calculation can most readily be performed by certain assumptions which are only approximately true. We must represent the whole by an electric battery between the terminals of which there are two paths, one of low resistance and one of high resistance R_s (fig. 12, I), placed as a shunt to the other. The second represents the reluctance of the air, which is very high as compared with that of the iron.

For the sake of simplicity, therefore, the reluctance of the air is regarded as being in parallel with the reluctance of the keeper, as if all the lines passed right through the iron of the magnet from one end to the other, and then at its ends divided into two groups, some going through the keeper and some through the air, these latter being the leakage lines, which are thus supposed to issue forth from and enter into the magnet only at its ends. On this assumption, then, if R_a and ρ are the reluctances of the two paths, afforded respectively by the keeper and the air, the external reluctance, being their joint reluctance in parallel, is $\frac{R_a \cdot \rho}{R_a + \rho}$, and this is in series with the internal reluctance of the magnet, R_{fm}. The total reluctance, therefore, of the entire magnetic circuit is

$$R_{fm} + \frac{R_a \rho}{R_a + \rho} = \frac{1}{P},$$

and the total number of lines produced by a given number of ampère-turns encircling the circuit is

$$Z = \frac{0{\cdot}4\pi AT}{R_{fm} + \frac{R_a \rho}{R_a + \rho}} = 0{\cdot}4\pi AT \times P$$

or, using the convenient abbreviation M.M.F. for magnetomotive force, magnetic flow = M.M.F. × permeance.

R_a is equal to $\frac{l_a}{a_a} \cdot \frac{1}{\mu_a}$, where l_a, a_a, and μ_a are the mean length, area, and permeability of the iron of the keeper. The reluctance ρ of the air-paths, or their permeance p, can be calculated by assigning a certain mean length of path and a certain area of cross-section to them; or we may determine experimentally the number of lines flowing through the middle of the magnet z_m, and the number flowing through the keeper, z_a; the difference between the two, or $z_m - z_a$, gives us ζ, or the number of lines which leak through the air.

On the same assumptions we can also calculate the total number of ampère-turns required in order to produce a given number of useful lines, z_a, through the iron of the keeper; just as the product of a current and a resistance through which it flows gives us the number of volts which must be applied to the ends of the resistance in order to produce the current, so the product of a number of lines of induction and a reluctance through which they flow gives us the magnetic difference of potential which must exist between the ends of the reluctance in order that the given number of lines may flow through it, *i.e.* M.M.F.$=$ZR. Hence the magnetic difference of potential which must exist between the ends of the armature in order to drive z_a lines through it is $z_a \times R_a = \cdot 4\pi AT_1$. But this is also the magnetic potential under which the leakage lines are assumed to flow; consequently if we multiply $\cdot 4\pi AT_1$ by p, the permeance of the air, the product is the number of leakage lines, which we call ζ; the sum of z_a and $\zeta = z_m$, or the number of lines flowing through the magnet, and the magnetic potential required to drive this larger number through the magnet is $z_m \times R_{fm} = 0 \cdot 4\pi AT_2$. The total magnetomotive force required is therefore $\cdot 4\pi(AT_1 + AT_2) = 0 \cdot 4\pi AT$, and thence the required ampère-turns are obtained directly.

Next let us hold the keeper at a little distance away from the magnet, so as to interpose in the magnetic circuit two equal air-gaps, one at either end of the keeper, with parallel faces and having a certain definite length (fig. 12, II).

At once the total number of lines of induction produced by a given number of ampère-turns wound on the magnet is enormously decreased, owing to the high reluctance of the two air-gaps. The effect may be repeated in the case of the battery if we interpose at each end of the external portion of the circuit a short length of wire having a very high specific resistivity but of the same diameter as our original external resistance: these will reduce the current in the external resistance, and the current through R_s will bear a much larger proportion to the current through R_e than formerly, since the two resistances R_e and R_s have become more nearly comparable. In the same way the number of lines through the armature is greatly decreased by the interposition of the two air-gaps, but the magnetic reluctance of the rest of the surrounding air remains the same as before, and the number of lines therein will be the same if the magnetic difference of potential at the poles of the magnet remains the same; the proportion, therefore, which they bear relatively to the number in the armature or to the number in the magnet is very much increased. Before, almost all the lines passed round the magnetic circuit entirely through iron; now, a much larger proportion go partly through the surrounding air. The two air-gaps being in series with the iron of the keeper, the total reluctance of the three is the sum of their separate reluctances. Hence, if the reluctance of one air-gap be $\frac{R_g}{2}$, and they are both equal, their total reluctance R_g and R_a must be substituted for R_a in the equations for the number of ampère-turns or lines of induction. It might be thought that it would be easy to calculate R_g, since the exact length of each air-gap can be measured, as also the exact area of the parallel faces of the iron where the circuit is divided. But here again a caution is required; the lines to a great extent pass straight across from face to face of the gap in the iron, but at the edges they spread outwards, and so pass across by curved paths, which form a kind of 'fringe,' gradually shading off in density. Consequently the exact area and length of their paths is *not* known, and has to be determined experimentally, or an allowance for the fringe has to be made theoretically.

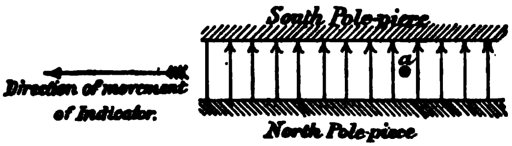
Now this last case is the one which bears most directly on the magnetic circuit of the dynamo. For the production of the E.M.F. of the machine we require a magnetic field, and this must be in air, since a system of electrical conductors has to be moved through it; further, we require as many lines as possible, within a small space, and produced with a reasonable expenditure of electrical energy in the coils through which the magnetising current flows. We must therefore have recourse to an electromagnet with an iron core in order to produce the field. From our equation it follows that, if we wish to obtain as many lines as possible round a whole magnetic circuit or through any portion of it with a given exciting power, we must make the length of the path under consideration as short as possible, its area large, and its permeability great. Iron at once presents itself as the most permeable substance, and would be used throughout were it not for the fact that we cannot have a closed magnetic circuit entirely of iron, since there must be one or more air-gaps in it. We shall, however, endeavour to minimise the effect of these as much as possible by making their length short and their area large; the rest of the magnetic circuit will be as short as possible, and composed of iron of high permeability. How these requirements are worked out in practice will form the subject of a later chapter.

CHAPTER IV

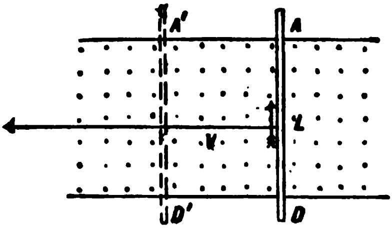
THE PRODUCTION OF AN E.M.F.

It has been stated in our introductory chapter that the dynamo is primarily a generator of an electric pressure or E.M.F. whence results a difference of potential at certain definite points which are the 'terminals' of the machine. The first fundamental equation upon which its theory is based must therefore deal with the origin and maintenance of its E.M.F., or, in other words, with the 'voltage' which forms one of the two elements of its output.

Let us suppose that we have before us a magnetic field mapped out by lines of induction, so as to at once exhibit the direction and strength of the field (fig. 13); how it is produced we are not concerned here to inquire. In this field let there be placed an electrical conductor, such as a straight piece of wire, shown in section at *a*, and let relative movement take place between field and conductor. Such relative movement, it is evident, may be divided into two kinds, according as the conductor does or does not 'cut' across the lines of induction along its length. The field of lines in fig. 13 may be likened to a mass of impalpable and invisible threads passing straight across from one pole-piece to the other, somewhat like a brush thick-set with straight filaments. Now between and through these threads a needle-like conductor may be passed in any direction without in any way disturbing them so long as the direction of movement is wholly in the plane which contains at the same time the length of the conductor and the direction of the lines, i.e. in our diagram the vertical plane which contains the conductor. If the direction of movement coincides with the length of the conductor, then the conductor merely passes end-on through the lines of the field; if the direction of movement coincides with the direction of the lines, the conductor merely slides up and down the lines; or its motion may be compounded of both, partly endwise through the field and partly sliding along the lines; in none of these cases is it caused to intersect the lines by passing athwart them or 'cutting' them transversely with its length. On the other hand, if the direction of motion does not fall wholly in that plane which contains the length of the conductor and the direction of the field—*e.g.* let it be in a horizontal plane as in fig. 13—then it will



Sectional Elevation.



Plan on a Plane normal to the field.

FIG. 13.

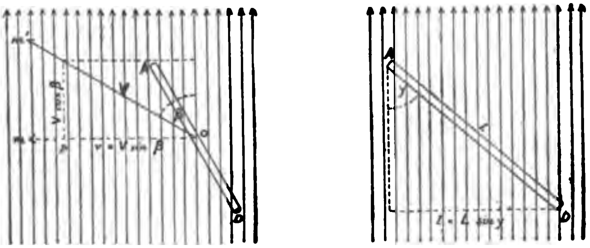
be seen that the conductor 'cuts' the lines of induction along its length.

The above description and its pictorial representation might seem to ascribe to the lines a physical existence, each one being separate and distinct from every other, whereas we know that the magnetic field is in reality *structureless.* Yet, although structureless, it has one property which is its direction, and by reason of that property the whole of the above is true of a conductor moved through it; no question of the avoidance of the separate lines by the moving conductor is in reality involved, since between the pole pieces the field is everywhere present. Owing, however, to the field having direction, any movement is divisible into the two kinds described above, according to the way in which its direction is related to the direction of the field and the length of the conductor.

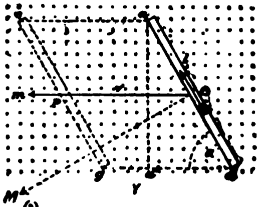
Now, in the first case, if an electrometer be applied to the ends of the moving conductor, no difference of state can be detected in it due to such motion; in fact, it is perfectly indifferent whether the magnetic field is surrounding the conductor or not. But let the movement be of the second kind; at once an entirely new phenomenon arises: the electrometer applied to its ends shows a difference of potential between them during the motion, and only so long as it lasts; in other words, an E.M.F. is set up or 'induced' in the conductor along its length in virtue of which one end of it is raised to a higher potential than the other. The conductor may now be appropriately termed an *inductor*, and there is a tendency for electricity to flow from the one end of it to the other; if these two ends are electrically joined so as to complete a closed circuit of conducting material, as in fig. 1, that tendency will be called into action, and a current will flow.

The dependence of the induction of an E.M.F. in all such cases of relative movement upon the 'cutting' of the lines of the field may now be expressed in general terms. In a conductor surrounded by a magnetic field, and moved in any direction relatively to it, take any two points, A and D, and consider the portion of it lying between those two points as a line which has a certain length, L, between the ends A, D. Let it be moved in a given direction with velocity, V; then it is required to find out whether this movement induces any E.M.F along this line (which we will call the line of action) in virtue of which a difference of potential is set up between A and D.

Upon a plane normal to the direction of the lines of the field project both the line of action and the direction of movement; then if these two projections are inclined to each other at any angle other than 180° an E.M.F. will be induced along the line of action by movement in the given direction; if, however, they fall in the same straight line, there may be E.M.F.'s induced along other lines of action between other points, but at least there is no E.M.F. induced along the line of action A D with which we are immediately concerned. The reason of this is not far to seek; it is that no lines of induction are cut along the line A D by the movement unless the two projections enclose some angle other than 180°. In fig. 14 (3) let *a d* and *o m* inclined to each other at the angle α be respectively the projections of the line of action and the direction of motion on a plane normal to the direction of the field, and in those lines let *a d* $(=l)$ and *o p* $(=v)$ be respectively the projections of the length L and velocity V. Then the completed parallelogram *a d g e* is the projection of the actual area traversed in unit time on the normal plane, and since it is an *area* in that plane it encloses lines of the field which have been directly traversed or 'cut' across by the conductor along its length A D. Consequently, since lines are cut by the movement, there is an E.M.F. induced along the line of action A D. For the induction of an E.M.F., therefore, the line of action must present



(1) Side Elevation from Y. (2) End Elevation from M.



(3) Plan on a Plane normal to the field

FIG. 14.

a certain broadside projection on a line at right angles to the direction of the movement, as in fig. 14 (3), where *a d′* is the projection of *a d* on a line at right angles to *o m*. If the direction of motion is in the plane which contains the direction of the field and the length of the conductor between A D, no lines are cut by the movement, and therefore no E.M.F. is induced along the line under consideration, for if the direction of motion in this plane be in the same straight line as the length of conductor, it moves endwise through the lines, and no area is traversed; while if the direction of motion be inclined to the length of conductor, there is an actual area swept through; but the projection of this on the normal plane is again a straight line, so that, again, there is no E.M.F. induced along the length A D.

The question of whether any lines of the field are or are not cut by a given length of conductive material when moved in a certain direction may not always be easy to determine at first sight, but the process of projection described above affords an infallible rule by which it can always be determined. It matters not what the material may be of which the conductor is made: whether it be copper, or iron, or German silver, in all alike an E.M.F. is set up when a conductor and a field of magnetic induction are so moved relatively to one another that the conductor is caused to 'cut' the lines of induction. Nor does it matter whether the conductor is moved across the field, or the lines of induction across the conductor by movement of the magnet between whose pole-pieces the field exists; or, finally, both field and conductor may be moved in opposite directions. In all cases where the conductor cuts the lines, either by its own movement or by reason of the moving magnetic field, it becomes the seat of an induced E.M.F.

The above is a simple statement of one aspect of the great fact of electromagnetic induction which was discovered by Faraday in 1831, and which has resulted within the short space of sixty years in the present marvellous development of electrical engineering. There are other aspects of the same fact, but they do not lend themselves so readily to the treatment of our subject, viz. the dynamo; and there are also many other important practical applications of Faraday's discovery, in especial the transformer used on the alternating system of distribution of electricity. But from all these the dynamo is separated by the requirement of our definition that there should be relative motion of field and conductor, that is, an actual movement in space of the one or the other, or both. Of the reason *why* an E.M.F. is induced when such movement causes the conductor to traverse the lines of the field, no more need be said, since behind the fact of such induction the engineer in his study of the dynamo seldom has occasion to go, even if the physicist has succeeded by his researches in reaching a still more ultimate explanation.

Now an E.M.F., like a mechanical force, is only completely described when not only its line of action has been given, but also its 'sense' or direction along that line, and further its magnitude or quantitative value. Since, therefore, the altered state of the conductor produced by its cutting of the lines of the field consists in the generation of an E.M.F., the complete law of the induced E.M.F. must deal with these two points.

In a conductor moving across a magnetic field, let us again take any two points, A and D, and consider that portion of it lying between those two points regarded as ends; then under all circumstances and in every case the induced E.M.F. generated along the length A D is simply proportional to the *rate* at which the lines of the field are cut along that length, and just as before we saw that the nature of the material of which the conductor is composed made no difference to the fact of the creation of an E.M.F., so likewise it does not in any way affect its magnitude. The greater the number of lines cut in a given time, or the shorter the time taken to cut a given number of lines, the higher is the E.M.F.

In the case of any phenomenon which can be expressed as a 'rate,' *e.g.* velocity as a rate of motion, a distinction must be made, if the rate be variable, between its average and instantaneous values; and such a distinction applies to the case of an E.M.F., since its magnitude is dependent on the rate at which the lines of the field are cut. The average rate of cutting to which the average value of the E.M.F. is proportional is the ratio of the number of lines cut in any time to the time taken to cut them; in symbols, if z is the total number of lines cut by the length of conductor under consideration in time t, the average E.M.F. induced between the ends of that length is

$$\text{E (average)} \propto \frac{z}{t}.$$

But the actual rate of cutting at any instant may vary to any degree on either side of the average value; at one moment it may be greater, at the next less, or it may vary continuously and regularly. Hence, to approximate to the actual rate of cutting when this is variable we must take the number of lines cut in a short space of time and divide them by that small time. The smaller this period of time is, the more accurately the rate of cutting is obtained, and to be completely accurate, the actual rate of cutting at any instant is the infinitely small number of lines then being cut divided by the infinitely small time in which they are cut; or, in the language of the differential calculus, the actual E.M.F. induced at any moment is

$$\text{E (instantaneous)} \propto \frac{dz}{dt}.$$

So far the induced E.M.F. has only been spoken of as proportional to the rate of cutting of lines, or $E = k\frac{z}{t}$, where k is some constant whose value depends on the system of units according to which the field is mapped out in lines and time is reckoned. On the C.G.S. absolute system k is unity, the system being so arranged that one line cut per second generates the absolute unit of electromotive force; but this unit being inconveniently small, the practical unit of the 'volt' is one hundred million times greater, and consequently 100,000,000 C.G.S. lines cut per second produces one volt. If, therefore, z be reckoned in C.G.S. lines, and t in seconds, and E is to be measured, as is usual, in volts, $k = 10^{-8}$,

or $$\text{E (average)} = \frac{z}{t} \cdot \frac{1}{10^8} \quad . \quad . \quad . \quad . \quad (2)$$

The C.G.S. unit line is therefore as inconveniently small as the absolute unit of E.M.F., and it has been proposed to adopt as the practical unit of induction a line which shall be the equivalent of 10^8 C.G.S. lines, the result being to eliminate the constant 10^8 from the fundamental equation.[1] In the following pages, however, C.G.S. lines will be adhered to, the divisor 10^8 being sometimes omitted where such omission can easily be made good by the reader.

Now the number of lines cut in any time is a question ot the virtual area swept through by the moving inductor, when its length and its path are projected on to a plane perpendicular to the direction of the lines of the field, and of the density of lines within that area.

Take the simplest case of a straight inductor moved parallel to itself across the lines of a uniform field. First let the plane containing the length of the inductor and the direction of motion be at right angles to the direction of the field, and let the direction of motion in that plane be at right angles to the length of the inductor. Such a case is represented in fig. 13, where the field has a vertical direction, the inductor A D is horizontal, and is moved in a horizontal plane, its direction of movement being at right angles to itself.

Then, under these circumstances, its length and path are already in a plane normal to the direction of the field, and no process of projection is required, the virtual area to be obtained by projection being identical with the actual area swept through by the inductor in any time; if L is the inductor's length which is actively traversing the lines, and V its velocity, the area traversed in unit time is $LV = AA'D'D$; and if B equal the number of lines per unit area, the number cut in unit time is BLV. Hence, on the C.G.S system, the value of the E.M.F. induced in the inductor along its length AD is

$$E = BLV, \quad \ldots\ldots\ldots \quad (3)$$

where L, V, and B are respectively in centimetres, centimetres per second, and C.G.S. lines per square centimetre.

The 'sense' or direction in which the E.M.F. acts along the length A D is in fig. 13 from the observer, so that the end A is at a higher potential than the end D; and if the two ends were joined by a wire, a current would flow through the inductor in the direction shown by the arrow.

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[1] At the 1895 meeting of the British Association, it was recommended that 10^8 C.G.S. lines should be called a 'weber,' and should form the practical unit of magnetic field; it has, however, been objected that the American Institute of Electrical Engineers have already given the name of a 'weber' to 1 C.G.S. line.

In this, the simplest case of electromagnetic induction, it will be seen that the directions of the field, of the length of inductor, and of the movement are all at right angles to each other, and form co-ordinate axes, as indicated in perspective in fig. 15; further, the direction of the induced E.M.F. is most easily remembered and discovered by means of the following rule. Place the *right* hand outstretched in the line of the inductor, so that the thumb points along the direction of the field, i.e. towards the S. pole, and further so that when relative movement takes place the lines of induction pass across the hand from the palm to the back; then the induced E.M.F. is directed from the wrist to the tips of the fingers (as shown in fig. 15). It will be found that, according to this rule, if it is the inductor which is moved while the field is stationary the palm of the hand must face in the direction towards which the movement is made, as in the case illustrated; but the direction of the E.M.F. would be exactly the same if the field were moved across the inductor in the opposite direction, the inductor itself remaining stationary, in which case it is the back of the hand which must face in the direction of movement. Or, again, both field and induction may be moved in opposite directions; but in all cases the lines of induction must by the movement traverse the right hand from the palm to the back, if we wish to discover the direction of the E.M.F. by the above rule.

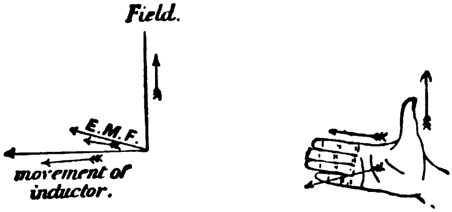


FIG. 15.

Let us next suppose that the three directions of field, inductor, and movement are not all inclined at right angles to each other. It will now be necessary to project the latter two on to the plane at right angles to the direction of the field, and first let the projections of the length of inductor and the velocity with which it is moved on the normal plane, i.e. l and v, not be at right angles to each other, but inclined to each other at any angle, a, as in fig. 14 (3). Then, as was shown before, the virtual area traversed on the normal plane in unit time is the area of the parallelogram $adge = ad' . v$. But $ad' = l \sin a$, or the projection of l on a line at right angles to v. Therefore the area of the parallelogram $= l \sin a . v$, and if B equal the number of lines per unit area, it encloses $B . l \sin a . v$ lines. Hence the rate of cutting, being the number of lines cut in unit time, is also $B . l \sin a . v$. And to this the E.M.F. induced along l is equal, or

$$E = B . l \sin a . v.$$

Again, by the rule of the right hand, when laid outstretched along the projected length of the inductor, so that the projected path of movement by which the lines traverse the hand is from the palm of the hand across to the back, it will be seen that the E.M.F. is directed from the wrist to the tips of the fingers. From this more general case it follows that when a is any other angle than 90° the area of the parallelogram $adge$ must necessarily be smaller than the area of the rectangle $l . v$, which would be traversed if the two projections were at right angles to each other, and this in the proportion of $\sin a$ to 1; while if $a = 0°$ or $180°$ no lines are cut, and therefore, as required by previous statements, no E.M.F. is generated along the length l.

Next let β = the angle at which the direction of movement is inclined to the direction of the field (fig. 14 (1)). Then the motion can be resolved into two components, $v \sin \beta$, in a plane normal to the field, and $v \cos \beta$ in a plane parallel to the lines of the field. In virtue of the latter or vertical component of the velocity, the inductor only slides along the lines, and has no E.M.F. induced in it; but $v \sin \beta$ is the projection of the actual velocity on the horizontal plane, and therefore may be substituted for v in the previous equation; or

$$E = B . l \sin a . V \sin \beta.$$

Lastly, let γ = the angle at which the length of the inductor is inclined to the direction of the field (fig. 14 (2)). Then the projection of its length on the normal plane is $l = \text{L} \sin \gamma$, and therefore $\text{E} = \text{BL} \sin \gamma \sin \alpha \text{ V} \sin \beta$.

We can now formulate the complete law of the induction of an E.M.F. in a rectilinear inductor when its movement through a magnetic field is one of simple translation. Let the length L of the inductor be inclined at any angle, γ, to the direction of the field; then the projection of its length on a plane normal to the direction of the field is $\text{L} \sin \gamma$. Next let the direction of motion be inclined at any angle, β, to the direction of the field; then the projection of its velocity on the same plane is $\text{V} \sin \beta$. Lastly, let these two projections of length and velocity be inclined to each other at any angle, α; then the virtual area traversed on the normal plane in unit time is $\text{L} \sin \gamma \text{ V} \sin \beta \sin \alpha$; the number of lines cut per second is $\text{BLV} \sin \alpha \sin \beta \sin \gamma$, and the induced E.M.F., or rate of cutting, is

$$\text{E}=\text{BLV} \sin \alpha \sin \beta \sin \gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The direction of this E.M.F. along the line of action is shown by applying the right hand to the projections in the normal plane.

When the angles α, β, and γ are all right angles, the action is evidently most direct, and reaches a maximum value for a given length of inductor, velocity, and density of lines; hence the simple case in which these directions formed co-ordinate axes, with which we started, is also the case which we should aim to reproduce in the dynamo wherein a voltage has to be set up. For example, in fig. 14 the best direction of movement would be along the line shown dotted at O M in a horizontal plane; if so moved the case becomes identical with that shown in fig. 13. It might appear that the cutting action would be greater if the direction of movement were kept unchanged along the line O *m*, and the inductor were rotated into a position at right angles to O *m*. But any such supposed advantage is purely imaginary, and suggested merely by the representation of the lines as dots on the plan. It must be remembered that the field is in reality structureless, as is shown by the fact that in the end elevation from M the lines appear at equal distances just as in the side elevation from Y, and so long as the direction of movement is at right angles to the length of the inductor, and both are in a plane normal to the direction of the field, it is a matter of indifference what is the exact direction of movement in that plane. Further, when this is the case, it will be found, not only that the E.M.F. is a maximum along the length of the inductor, but also that, whatever be its shape or size, there is no other line of action along which an E.M.F. is induced, except the direction of its length. It is often said that the direction of the induced E.M.F. is at right angles to the field and to the direction of movement; but this is only the particular case of the line of greatest action, and there may be several directions in an inductor in which E.M.F.'s are induced, and several different values for these E.M.F.'s. This is due to the fact that an inductor in nature must necessarily have a certain width; the effect of this has been so far left out of sight, and the line of action considered has been simply the length of the inductor. We are, however, already provided with a guide in our fundamental statement of the law of electromagnetic induction, and by it the case of an inductor having width has now to be considered.

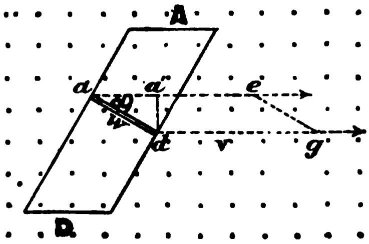


FIG. 16.

In fig. 16 let A D be an inductor of width w, moved in a horizontal plane across a vertical field: in order to find out what E.M.F., if any, is acting between the points $a\,d$, we have merely to take a thin lamina of its substance joining those two points and treat it as an inductor of infinitely small width, just as the inductor of previous diagrams was treated. Its length is w, and it cuts the lines of the field along its length at a rate of $B\,v\,w \sin \alpha$ lines per second, for the area which such a thin lamina traverses in unit time is $a\,d\,g\,e = v \,.\, a'd = v\,w \sin \alpha$. Hence in any inductor having width whose direction of motion is not at right angles to its length, there is an E.M.F. set up across it from side to side, so that if the two points *a* and *d* were joined by a wire, a current would flow across it.

Again, when the inductor has width, as A D in fig. 14, and it is required to find the E.M.F. acting between its two ends A and D, in strictness it would be necessary to mentally divide it along its length into a number of very thin laminæ lying side by side ; the area traversed by any of these would in any time be equal, since they move with equal velocity in the same direction, and therefore with a field of uniform density the E.M.F. induced along each would be equal, and they may all be regarded as in parallel. But if the one edge of the whole inductor were moving in a field of different density, as in fig. 17, where *a b* is moving in a stronger field than *c d*, then *a b* will induce a greater E.M.F. than *c d*. Consequently, unless the thin laminæ are electrically separated by some insulating material, a current will flow round the inductor as a whole in the direction shown by arrows. The importance of this in the manufacture of dynamos will be seen subsequently.

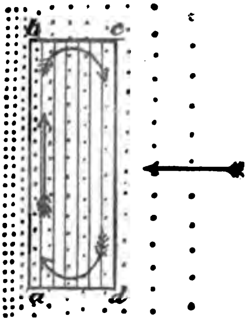


FIG. 17.

To determine the value of the E.M.F. induced by movement through a field of varying density, it is necessary to consider an area swept over by the inductor so small that the density of lines may be taken as having a uniform value over it ; and in the same way, if the velocity of movement varies, so small a period of time must be considered that it may be treated as sensibly constant throughout it.

Up to this point the active portion of the inductor has been usually regarded as straight, although this is not essential, and in fact it may have any shape ; for example, suppose that it be bent up into a curve in a vertical plane, as in fig. 18, the field being vertical ; then the projection of the actual length on a horizontal plane will be a straight line. Whenever this is the case the E.M.F. induced per unit length along its projection is equal throughout all portions of that length, and thence by transference to the actual length of the bent inductor we can discover the rate of production of E.M.F. along its actual length. For instance, in fig. 18 the E.M.F. induced in each inch of the projected length *a d* is equal, but along the actual length A D, an inch length is more active in the middle of it than at either end. If the length of the inductor be crooked or curved in a horizontal plane, this merely complicates the question of the virtual area swept through as projected on that plane. For example, in fig. 19, if the bent inductor there shown be moved parallel to itself across the field, the virtual area traversed is at once found by multiplying the velocity by the straight line *l* joining its ends A and D, and *l* may be regarded as its virtual cutting length, along which the E.M.F. is uniformly induced. But if, at the same time as it is moved along, the inductor be twisted about itself through an angle

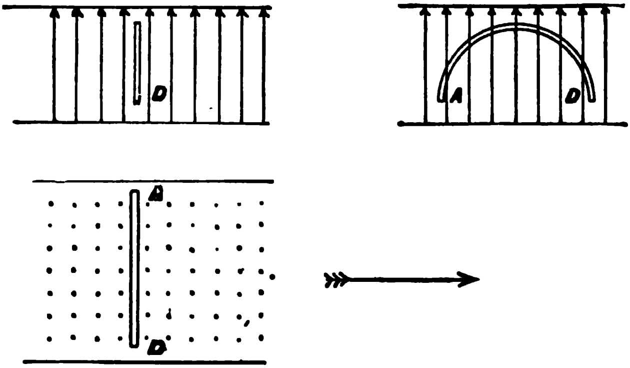


FIG. 18.

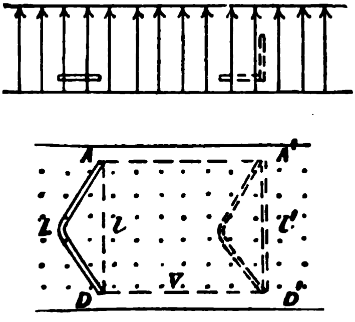


FIG. 19.

of 90°, at the end of its path, the projection of its length L will be l', and consequently the area traversed is the area of the irregular figure A L D D′ l' A′; since this is greater than the area of the figure A l D D′ l' A, the average E.M.F. induced during the movement will be correspondingly greater.

The first fundamental equation of the dynamo which deals with its voltage has now been determined, and may be conveniently brought to mind by the equation of the C.G.S. system, which gives the E.M.F. in volts, or

$$E = \frac{Z}{t} \times 10^{-8} \tag{I}$$

with its special form

$$E = BLV \times 10^{-8} \tag{Ia}$$

By means of these equations, and the methods by which they are reached, any case, however complicated, of E.M.F. set up in conductors by movement in a magnetic field may be solved; the inductors may be crooked or straight, moving with variable or uniform velocity in a field of uniform or variable strength; they may, too, be portions or not of a complete closed circuit and conveying currents or not; all that is necessary is to resolve each case into its elements to which our primary equations can be applied. For example, in the case of a complicated closed circuit it must be mentally split up into small elemental portions, and then for every small portion we have only to determine what is its rate of cutting across lines; for this is at once the E.M.F. set up in that portion due to its movement. The E.M.F. may be aiding a current or it may be opposing it; still there is that E.M.F. generated in that element of the circuit under consideration. Then by summing up all these E.M.F.'s throughout the whole of the circuit, we arrive at the resultant E.M.F. acting throughout the entire circuit. In so doing, however, two rules must be invariably followed: we must pay attention to the direction of these E.M.F.'s along the length of the inductor, and if we fix upon a certain direction round the circuit as positive, i.e. the direction in which the current round it is flowing, then all E.M.F.'s in that direction must be considered as + or positive, and all in the opposite direction as — or negative, and we must add together the E.M.F.'s algebraically, with due regard to their signs; secondly, we must consider whether any or all of the inductors into which we have divided the conducting circuit are in parallel, since then the potential difference existing at the ends of any one is also the potential difference between the ends of all alike; and although the internal E.M.F. in one or more may be higher than that induced in the others, yet so far as the supply of current to the rest of the circuit is concerned, it is only this potential difference which is effective.

In order to illustrate the application of the above laws, let us consider the case of a four-sided loop moved horizontally in a direction at right angles to two of its opposite sides, across a uniform magnetic field, the direction of which is vertical (fig. 20). The two sides *a b*, *c d* will act as inductors, while in the two ends *b c*, *a d*, which are parallel to the direction of motion, no E.M.F. is generated, since they do not cut any lines. In *a b* and *c d*, the induced E.M.F.'s act in the same direction as viewed by an observer at either end, and are of the same value, since the lines are cut by each at the same rate. But the two E.M.F.'s oppose each other round the loop, and consequently if the circuit be completed from *a* to *d*, they will exactly neutralise each other, and no current will flow round the loop.

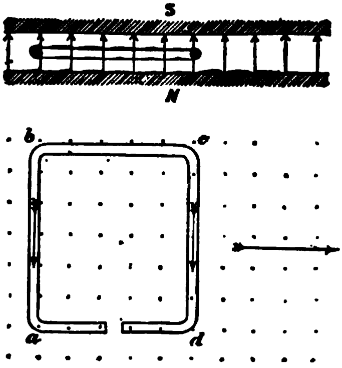


FIG. 20.

But now let us move the same loop in the same way in a field of varying intensity (fig. 21); then the E.M.F. in *a b* or E will be greater than that in *c d* or *e*, and therefore the resultant E.M.F. acting round the loop as a whole is $E - e$, which is proportional to the difference in the number of lines cut in unit time by *a b* and *c d*, *e* being a back E.M.F. opposing the direction of the current which would flow on closing the side *a d*. If we make *c d* move in a field the direction of which is the reverse of that through which *a b* is moved (as will be the case if in fig. 21 we move the loop onwards, so that the right-hand side enters the opposite field, shown on the plan by crosses), the two E.M.F.'s, though in opposite directions, as viewed by an observer at either end, will assist each other round the loop; in other words, the two inductors are in series, and their total E.M.F. is the sum of the two E.M.F.'s, or E+*e*.

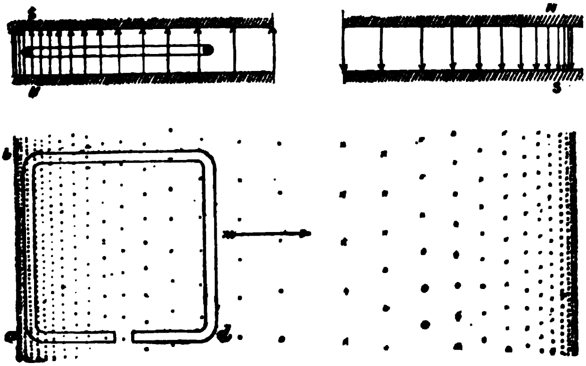


FIG. 21.

Next let us join *a* and *d*, and move the closed loop through the same field as in fig. 20, while we apply an external circuit to the ends *b c*, *a d* (fig. 22): the two E.M.F.'s which before balanced each other now unite to send a current through the external circuit, but being in parallel, the total E.M.F. acting is that of either of them. The arrangement, therefore, does not add to the voltage, but simply doubles the ampères that can be carried without damage resulting to the

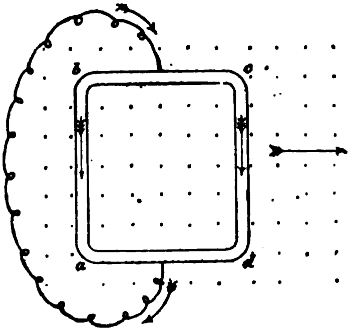


FIG. 22.

inductors owing to the heat generated by the passage of the current.

Lastly, let us rotate the loop about an axis passing horizontally through its centre (fig. 23). Now in this case the two sides, *a b* and *c d*, again act as inductors, while the ends are inactive; since the two inductors at any moment are cutting the lines of the same field in opposite directions, their E.M.F.'s assist each other round the loop, so that the total E.M.F. is the sum of the two; and if the loop is closed either immediately or by the interposition of an external circuit in *a d*, a current will flow, due to this total E.M.F.

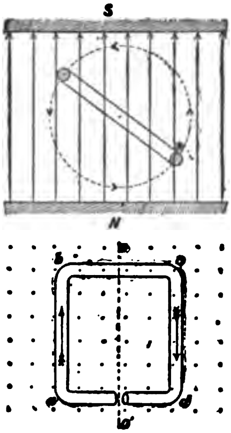


FIG. 23.

This last case leads us again to the important distinction, which has been before noted, between the average and instantaneous values of the E.M.F. If the number of lines which are being cut at any instant is the same throughout any period of time, then for so long the E.M.F. is constant, and the average and instantaneous values coincide. Such a constant E.M.F. may be due to movement with uniform velocity through a uniform field, or to movement with varying velocity through a field of varying intensity, the two varying inversely; in either case the rate of cutting lines is constant. But if the rate of cutting varies, either from movement with varying velocity through a uniform field, or from movement through a field whose varying intensity is not counterbalanced by an inversely varying velocity, then the instantaneous value of E.M.F. may differ greatly from the average value, since at any moment it corresponds to the actual rate of cutting at that instant. The average value for any given movement of an inductor is always proportional to the total number of lines cut, divided by the time taken to cut them; but when the rate of cutting is variable, the whole conditions of the case can best be represented graphically by means of a curve in which instantaneous values of E.M.F. are plotted, intervals of time being represented by abscissæ or lengths measured along the horizontal axis, and corresponding values of the E.M.F. by vertical ordinates. For example, in fig. 24 let the vertical lines represent a uniform magnetic field across which a straight inductor, shown in section at A, is moved parallel to itself with uniform velocity; then the curve of E.M.F. due to motion from A to B will be represented by a straight line, at some height above the horizontal axis, which is fixed by the scale on which volts of E.M.F. are plotted. The rate of cutting does not vary, and the E.M.F. is constant; while if at B the motion is instantly reversed, and the inductor travels back to A at the same speed, a constant E.M.F. of equal value is obtained, but in the opposite direction. We thus arrive at the pair of rectangles shown to the right in fig. 25. If, however, the inductor is rotated with uniform velocity in a circle about O, the rate of cutting will vary at every instant; as it describes the semicircle CED, at the first instant it slides along the lines, and the E.M.F. is zero; gradually the E.M.F. rises until it reaches a maximum at E, where the motion is entirely at right

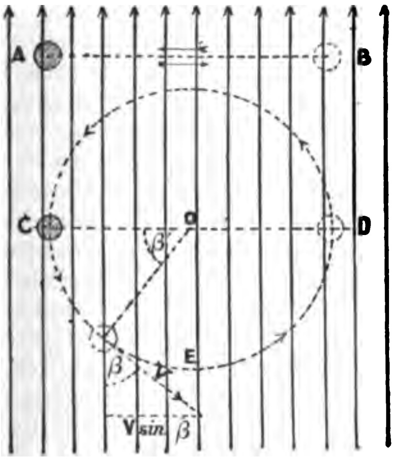


FIG. 24.

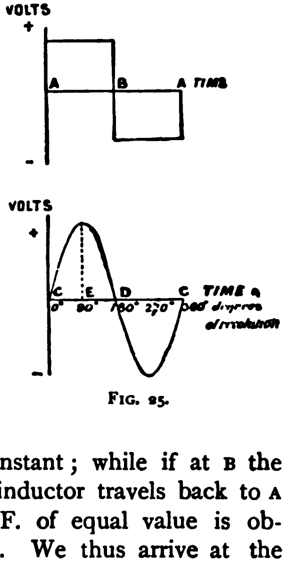


FIG. 25.

angles to the lines. From this point it falls gradually to zero again at D. Now what is the actual rate of cutting at any moment? The actual velocity v at any moment is in a direction tangential to the circumference of the circular path moved through; in fig. 24, if v represent it in magnitude and direction, its projection on a horizontal plane is v sin β; but the angle β between the direction of the velocity and the direction of the field is equal to β' or the angle through which the inductor has been rotated, reckoned from its position when its motion was entirely parallel to the lines; hence the horizontal component of the velocity in virtue of which the inductor cuts the lines is equal to v sin β'. Viewed from a position vertically above or below, the inductor would appear to move with ever-quickening velocity from C up to the middle position at E, where it reaches a maximum, and thence with falling velocity as it passes on from E to D. If we plot on CED the values of this apparent velocity v sin β', we obtain a simple sine curve whose maximum value is v. But the rate of cutting is directly proportional to this varying velocity, and therefore the curve connecting E.M.F. and time must itself be a simple sine curve like the curve connecting time and projected velocity. We have now, therefore, determined the exact shape of the curve (fig. 25); the instantaneous E.M.F. is a sine function of the angle through which the inductor has turned, reckoned from its position in a plane normal to the field and passing through the axis of rotation; and its value is BLV sin $\beta' \times 10^{-8}$. As the rotation is continuously maintained with uniform velocity, the E.M.F. acts first in one direction, then in the opposite direction, or is alternating. When β' is 0°, 180°, or 360° its value is zero; when β' is 90° or 270° its value is a maximum, first in a positive and then in a negative direction. If the time taken to move from C to D be the same as that taken to move from A to B, the *average* value of the E.M.F. in the former case or the average height of the curve must be the same as the value in the latter case, or the height of the straight line above the horizontal axis.

Let Z be the number of C.G.S. lines cut by the inductor as it moves from A to B or from C to D, and n be the number of revolutions per second; then the number of lines cut in the half-revolution from C to D is Z, and the time taken by this half-revolution is $\frac{1}{2n}$ seconds; therefore the average E.M.F. is

$$\frac{Z}{\frac{1}{2n}} \times 10^{-8} = \frac{2Zn}{10^8} \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The E.M.F. induced at any moment in the inductor is $BLV \sin \beta' \times 10^{-8}$. But $V = 2\pi r n$, where r is the radius of rotation $= \frac{CD}{2}$, and $BL\, 2r = Z$; hence the instantaneous E.M.F. at any moment $= \pi Z n \sin \beta' . 10^{-8}$; the maximum value of this is $\pi Z n\, 10^{-8}$: whence it follows that the maximum value is $\frac{\pi}{2} \times$ average value.

Now when an inductor passes through a cycle of changes, such that it returns to exactly the same condition as existed at the commencement of the cycle, the passage through the cycle is termed a 'period.' Each complete revolution of the inductor C in fig. 24 is such a period, yielding a complete wave of E.M.F. The number of complete periods per second is known as the 'periodicity' or 'frequency' of the alternating E.M.F. or the inductor which yields it, and in the above case is equal to n, or, if $N =$ the number of revolutions of the inductor per minute, to $\frac{N}{60}$.

CHAPTER V

THE MAGNETIC PULL

THE principle on which the primary function of the dynamo depends and an electric pressure is produced at certain points or terminals has now been discussed, and has led to the establishment of the fundamental equation which determines the first of the two factors that go to make up its output, i.e. its voltage. But so far the chief result contemplated in our diagrams of a simple inductor has been this electric pressure at its ends; no current could flow along it so long as the ends were shown open and the circuit was incomplete; no electrical energy was present, and therefore no mechanical energy was absorbed by the movement in order to be converted into any such new form. Hence the inductor of fig. 13 is only potentially a dynamo. But now suppose that the inductor wire forms portion of a closed conducting circuit, the remainder of which is external to the magnetic field, and is a simple electrical resistance of R_e ohms, such as a piece of wire (fig. 1); then when the inductor is drawn across the field, an E.M.F. is set up between its two ends, A and D; and these being points in a closed electric circuit, containing no other source of E.M.F. besides the inductor, a current of electricity will flow through the whole circuit. The value of the current in ampères will be equal to that E.M.F. in volts divided by the resistance of the entire circuit expressed in ohms; or in symbols

$$C = \frac{E_a}{R_a + R_e},$$

where E_a and R_a are respectively the E.M.F. induced in the inductor and its resistance. In short, the inductor has now become a dynamo.

But when a current is allowed to flow through the inductor, a distinction must be made between the internal E.M.F. developed in its length, or the internal portion of the circuit, and the difference of potential which exists at its terminals, A, D. The latter, which we will call E_e, is the actual 'voltage' of the inductor-dynamo, which when multiplied by the current passing through it gives the output, and it is less than the internal E.M.F. by the number of volts required to drive the current through the resistance of the inductor.

The analogy of the centrifugal pump will render this clear. We have said that the whole of the power developed by it is not available outside of it for raising water, and the reason is the same as in the case of the dynamo. The flow of water is the same inside and outside the pump, but some portion of the total pressure produced in the pump is lost in overcoming the frictional resistance of its own ports and channels to that flow, and to this loss of pressure is due the fact that the available power for lifting water is less than the total power developed in the pump.

The terminal voltage of the simple inductor-dynamo is therefore

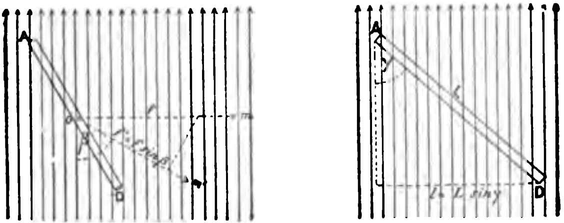
$$E_e = E_a - CR_a,$$

and from this it will be seen that on open circuit when no current flows $E_e = E_a$; but as the current is increased the difference between the two continually increases. The volts represented by CR_a are practically useless, being spent over the internal resistance; when multiplied by the current their product, or C^2R_a watts, is a certain rate of development of electrical energy; but this energy simply appears as heat generated in the inductor which is not only useless but costly and inconvenient to dissipate. The smaller R_a is, the less will be this loss, and the more nearly E_e will approximate to E_a. For this reason the inductive portion of the modern dynamo is almost invariably composed of copper; since with the exception of silver (which is too expensive) copper is of all substances the best conductor of electricity, and therefore for a given length and sectional area of inductor its resistance is less than that of other metals.

Directly, however, that we allow a current to flow in our inductor, an entirely new phenomenon comes to light. It now for the first time requires mechanical force to be applied in order to move it across the field. Magnetic field and electric current have interacted, and there is a definite resistance to the motion, which must be overcome by mechanical energy supplied from without.

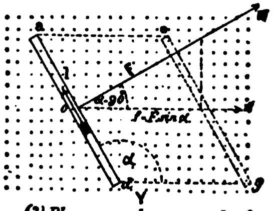
The study of this second phenomenon will follow a course closely analogous in every particular to the previous study of the principle of electro-magnetic induction. Let us place in a magnetic field a simple straight conductor A D (fig. 26) conveying a current whose direction of flow coincides with its length, and is from D to A; the rest of the circuit which must necessarily exist we will for the moment disregard. Let it occupy any position in the magnetic field, and finally let us take any line of action which we please, along which we wish to discover whether there is any mechanical force acting on the conductor and tending to move it in that direction. As, in the case of electro-magnetic induction, the direction of the field and of movement were given, and any line of action forming a length within the conductor was considered with reference to the existence of an E.M.F. along it, so now with direction of field and of flow of current given, any line of action is considered with reference to the question whether any force does or does not act along that line.

In the first place, whatever the relative positions of the lines which represent the directions of field and length of conductor, it is evident that they fall under one or other of two heads: either the length of the conductor is parallel to the direction of the field, or it is not. Now, in the first case, the conductor conveying the current is entirely unacted on by any force due to the external magnetic field in which it is placed, and it has no tendency to move in any direction. But in the second case it becomes the seat of a mechanical force which, unless resisted by an equal and opposite force, will cause it to move in a definite direction. This second case is equally well expressed by saying that, if there is to be a mechanical force acting on the conductor, the projection of its length on a plane normal to the direction of the field must be a line, and not a point, and whenever it has length or is a line, then there is interaction between the magnetic field and the conductor. In fig 26 let A D be a conductor whose length does not coincide with the direction of field in which it is placed, and let *a d* be the projection of its length on a plane normal to the field. At right angles to *a d* draw a line O M. Then the conductor A D is acted upon by a force distributed throughout its entire length (since it is entirely immersed in the field), and the direction of this force at each portion of its length is in a plane normal to the field and parallel to O M; in other words, if it is free to move, the conductor will move parallel to itself through the field owing to the action of forces, which may be summed up and represented by one force at right angles to its length and in a plane normal to the field, as indicated by the line O M.



(1) Side Elevation from Y

(2) End Elevation from M



(3) Plan on a plane normal to the field

FIG. 26.

We have next to determine the 'sense' of this magnetic force or pull, and, further, its magnitude. In fig. 26, if the direction of the lines of the field be vertically upwards, and the current flows in the conductor from D to A, the direction of the pull on the conductor will be from left to right towards M, i.e. exactly opposite to the direction of movement which would produce an E.M.F. directed from D to A. Hence if, instead of the right hand, we now place the *left* hand outstretched along the length of the conductor, so that the direction of current is from the wrist to the finger-tips, while the thumb points in the direction of the lines of the field, the direction of the mechanical pull on the conductor will be across the hand from the back to the palm; in other words, the palm will face the direction towards which the conductor tends to move. The force is a mutual action between the conductor and the field, so that if the conductor is held fast, and the pole-pieces between which the field exists are free to move, the direction in which they will move is of course opposite, i.e. from the palm of the hand across to the back. Next, as regards the magnitude of the force, if l be the length of the conductor A D as projected on to the plane normal to the field, then the force acting on A D at right angles to its length, and tending to move it parallel to itself along O M, is proportional to the product of the strength of current, intensity of field, and the projected length of the conductor, or

$$F \propto cBl.$$

But $l = L \sin \gamma$ (fig. 26 (2)).

$$\therefore F \propto cBL \sin \gamma,$$

from which it is again evident that the conductor is only acted on by the magnetic pull when its length is inclined at some angle to the direction of the field; since if it is parallel to it, $\gamma = 0$, and F is zero.

The above forms the statement of the fact of electro-dynamic action parallel to our previous statement respecting electro-magnetic induction. As, in the latter case, there were aspects other than that in which it was presented, so also are there other ways of presenting the second fundamental fact, but, again, none are so suitable in connection with the theory of the dynamo. Both are laws, ultimate as regards the subject in hand, and not only verifiable by direct experiment, but also affording a sure groundwork of theory on which to deal with any case that may arise.

The closely analogous nature of the two laws is already apparent in our first statement; if the projection of the conductor or inductor on the normal plane be a point, it is entirely unacted on by any pull from the magnetic field, and, further, no E.M.F. can be created in it by movement in any direction parallel to itself. But this analogy extends still further. When inclined to the direction of the field, the actual force acting on the conductor due to a magnetic pull is always $F \propto cBL \sin \gamma$ along the line O M; but the whole or part of this may be balanced by other external forces, and we have undertaken to discover the force acting on the conductor along *any* line of action which we please to consider. This we are now in a position to do, since all that is required is to resolve the force F along the given line of action, in order to find the component acting along it, which may or may not be unbalanced. In fig. 26 (1) let O *m′* be the line of action under consideration, its projection on the normal plane being along O *m*; then the component of the force F acting along the line O *m* is

$$f = F \sin \alpha,$$

where α is as before the angle which the projection of the line of action makes with the projected length of the conductor (fig. 26 (3)). Further, the component of f along the line O *m′* is

$$f' = f \sin \beta,$$

where β is the angle which the line ot action makes with the direction of the field (fig. 26 (1)). Hence the force acting on the conductor along the given line of action O *m′* is

$$f' \propto cBL \sin \alpha \,.\, \sin \beta \,.\, \sin \gamma. \quad . \quad . \quad . \quad (6)$$

The expression we have now obtained is exactly analogous in form to the expression previously arrived at for the E.M.F. induced in a rectilinear conductor when moved in any direction parallel to itself, and it may be put into similar words. With any position of the conductor relatively to the line of field and the line of action, either the projections of the conductor's length and the line of action are in the same straight line, or they are inclined to each other at some angle other than 180° : in the first case there is no magnetic force acting on the conductor and tending to move it in the given line; in the second case there is a certain magnetic force or component of a force acting along that line. Whenever a conductor is subjected to a magnetic pull, the line of action along which the force is a maximum is at right angles to the projected length and in the normal plane, and consequently, as we have said, it is in this direction that the conductor, if free to move, will travel parallel to itself. There is no tendency for it to rotate so as to alter the angle γ. For a given length of conductor, however, this force is a maximum when the conductor is itself at right angles to the field or in the normal plane, as shown in fig. 27, and this maximum value of the force is $F \propto BLc$, or

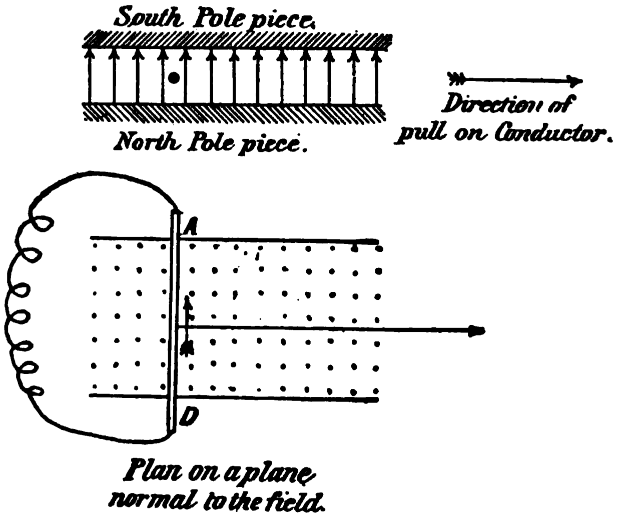

South Pole piece.
North Pole piece.
Direction of pull on Conductor.
A
D
Plan on a plane normal to the field.

FIG. 27.

$$F = kBLc,$$

where k is again some constant depending on the system of units employed. On the C.G.S. system, the force being measured in dynes, c in absolute units of current strength and B in C.G.S. lines per square centimetre, $k=1$. If c be in ampères, $k=10^{-1}$, and we have

$$\text{(II)} \qquad F\ (\text{dynes}) = BLc \times 10^{-1} \quad . \quad . \quad . \quad . \quad (7)$$

which forms the second fundamental equation of the dynamo. If F be measured in lbs. and L in inches, while B and c are retained in C.G.S. units and ampères respectively, the equation becomes

$$F\ (\text{pounds}) = 57 \times 10^{-8}\ BL''c \qquad (8)$$

One feature still remains to be considered. When the conductor of fig. 26 is unconstrained and moves parallel to itself in the direction of the line O M, it passes broadside through the field, and for a given distance moved through, the number of lines cut is a maximum. Further, since the force F has no component in a vertical direction or along the length of the conductor, it is evident that there is no tendency for the conductor to move up and down the lines or end-on through them. It is essential, therefore for the existence of force along any line of action that the conductor, by movement in that direction, should cut lines of the field along its length.

Now in the simple but fundamental case of fig. 27 two corollaries attract our attention most forcibly. By comparing fig. 27 with fig. 13 it will be seen that the direction of the magnetic pull on the conductor, when a current flows through it from D to A, is exactly opposite to the direction of movement which would induce an E.M.F. in it from D to A. And this feature is equally present in all other cases (compare figs. 26 and 14), and is embodied in the rule that, in determining the direction of an E.M.F. along a given line of action, the *right* hand is used, while in determining the direction of magnetic pull on the conductor the *left* hand is used. Suppose that the conductor of fig. 27 conveying a current from D to A is free to move under the action of the magnetic pull; then at once it cuts the lines of the field, and the induced E.M.F. as shown by the application of the right hand, is from A to D; it is therefore a *back* E.M.F. opposing the flow of the current and reducing its strength. Similarly in all cases in which a conductor is left free to move under the magnetic pull on it, it always cuts lines along its length, and consequently always induces an E.M.F. along its length, and we now see that this E.M.F. is so directed as to oppose the flow of current to which the movement is itself due. That it must so oppose the current is evident if we consider what would be the consequences were it to assist the flow of current; the strength of this latter would be increased, the pull would proportionately increase, and the conductor would move with ever-increasing velocity, which in turn would induce a continually increasing E.M.F. in the same direction as the current; and this process would go on for ever until both the E.M.F. and current were infinitely great. We should thus obtain an indefinite amount of electrical energy without the expenditure of any in another form, which is impossible by the law of the conservation of energy. Hence the E.M.F. induced by the free motion of a conductor carrying a current, when placed aslant in a magnetic field, must oppose the current, and this necessary fact might be used to determine the direction of movement which will ensue for a given direction of current in the conductor.

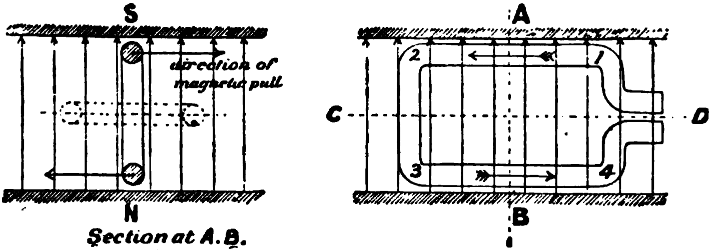
Next let the current in A D (see fig. 27) be itself due to movement of it through the magnetic field; that is, an E.M.F. is being generated in it, acting from D to A. For this to be the case the direction of movement must be towards the left. But the direction of the force acting on it, due to its being a conductor carrying a current from D to A, is towards the right, and is therefore directly opposed to the movement which produces the E.M.F. Hence the movement of the conductor is resisted by a mechanical force in the opposite direction, and this fact may conversely be used to determine the direction of the E.M.F. along the length of the inductor. For, suppose a current to flow under the induced E.M.F., the direction of the two, therefore, being the same; then this direction must be such that there shall be a force acting on the inductor opposing the movement which induces the E.M.F. In all cases, therefore, of electro-magnetic induction the direction of the induced E.M.F. must be such that a current flowing under it tends by its electro-dynamic action to stop the motion which produces the E.M.F. This general statement was formulated by Lenz in 1834, and is known as Lenz's law.

At once we begin to have an insight into the reason why an expenditure of mechanical energy is required, if a current is allowed to flow in our inductor of fig. 1, under the influence of the E.M.F. induced in it. By the principle of the conservation of energy, no new form of energy can be made to appear, save by the transformation of some other form. In the case of the electric motor which is contained in the first of the above two corollaries, a current is passed through a system of conductors placed in a magnetic field; they are thus subjected to a mechanical pull, which will cause movement, and so give off work, electrical energy being transformed into mechanical, and the conductors becoming also inductors, or the seat of an induced back E.M.F. In the case of the dynamo which is embodied in the second of the two corollaries, a current flows in the direction of the induced E.M.F., so that the system of inductors becomes also a system of conductors; the movement which produces the E.M.F. is resisted by a mechanical force, and consequently mechanical energy is transformed into electrical energy. The equality of the two is easily shown: *e.g.* in fig. 27 let c be the strength of current flowing through our inductor under the E.M.F. produced by movement of it towards the left with uniform velocity V, all units being on the C.G.S. absolute system. The rate of development of electrical energy throughout the circuit is equal to the product of the E.M.F. and the current flowing round it, i.e. $= cE$. But $E = BLV$; therefore electrical energy is developed at the rate of $cBLV$ ergs per second. Now with the current flowing in the direction shown in the inductor, it is acted on by a force at right angles to its length, and tending to draw it towards the right. The magnitude of this force is $F = cBL$. But this force is overcome through a length V in unit time; mechanical energy has therefore to be expended at the rate of FV ergs per second. But this is equal to $cBLV$, which is identical with the expression for the rate of development of electrical energy.

Or, again, take the case shown in fig. 14, and compare it with fig. 26; the E.M.F. produced by movement of the inductor in the direction $O\,m'$ is equal to $BLV \, . \sin\alpha \, . \sin\beta \, . \sin\gamma$ in the direction from D to A. Let this produce a current of strength c round the circuit when closed; then the inductor is acted on by a magnetic pull, $F = cBL \, . \sin\gamma$, in the direction O M (fig. 26). But the whole of this is not overcome by the force which produces the movement; it is only the component of F along the line $O\,m'$, which acts as a resistance to the movement, and this is equal to $F \sin\alpha \, . \sin\beta = cBL \, . \sin\alpha \, . \sin\beta \, . \sin\gamma$. The product of the resistance overcome, and the velocity V is therefore again equal to the rate of development of electrical energy, or cE.

But by what mechanism, it is naturally asked, is this resistance to a movement inducing an E.M.F., and which only arises when a current flows, brought about? It is by the interaction of the magnetic field due to the current, with the inducing magnetic field. It has been shown that a conductor conveying an electric current is itself the centre of a magnetic field, and it is through its magnetic properties that the current can react on the original field. This interaction can be treated in several different ways: these consist in attributing either to the magnetic lines themselves or to the closed conducting circuit certain properties, by means of which all the phenomena attendant upon the superposition of two or more magnetic fields can be correlated and brought into harmony. Of such methods two principal ones may be mentioned; but it must be remembered that the ultimate nature of the action is not more explained by them than by the primary law under which the subject was first presented.

Take the case of a rectangular loop of wire round which a current flows, and placed so that its plane is parallel to the direction of the field between two external pole-pieces (fig. 28). Then it follows from our electro-dynamic law that if the direction of the current round the loop be as shown by the arrow, the side 1–2 will be subjected to a force tending to move it towards the right, as seen from D, while the side 3–4 will be subjected to an equal force tending to move it towards the left; the ends 2–3 and 1–4 will while parallel to the field have no force exerted on them, and when in a plane normal to the field, or in any intermediate position, the force tending to move 2–3 in one direction will be counterbalanced by the force tending to move 1–4 in the opposite direction. The whole loop will therefore be acted upon by a couple tending to rotate it in a clockwise direction, and the position which it will take up if free to move is one in which its



Section at A.B. seen from D.

FIG. 28.

plane is normal to the direction of the field, as shown by dotted lines. When the loop is vertical and is subjected to a magnetic pull, the lines that would be due to the current alone would pass through it from right to left, as seen from D. Such a supposed field must, however, be compounded with the original vertical field, and the result of this composition is that the lines of the actual field in nature are twisted out of their direct path. When the loop has rotated into its position of rest, the lines that would be due to the current alone coincide in direction with the lines of the original field, *i.e.* pass vertically upwards through the loop. Hence, as the loop rotates, the lines of the resultant field shorten their path, and further become a maximum. Thus not only may the lines of a compound field be said to tend to shorten themselves, but every closed circuit conveying a current tends to set itself so that the lines embraced by it are a maximum.

Again, it has been shown that a loop of wire conveying a current may be likened to a magnetic shell, the two faces of which are respectively N. and S. Consequently, in fig. 28 the loop tends so to set itself that it presents its N. face to the S. pole of the external magnet, and its S. face to the N. pole-piece, since unlike poles attract, like poles repel. If the loop be forcibly rotated in an anti-clockwise direction, an E.M.F. will be set up which would be the cause of an alternating current; but with such a direction of rotation the N. face of the loop would be continually presented to the N. pole of the magnet, and therefore there will be repulsion resisting the motion.

Let us now consider the case of an iron cylinder along the outside of which and parallel to its length are arranged a number of conductors conveying currents, and placed in a magnetic field between two pole-pieces (fig. 29). The direction of the lines in the air-space between the iron poles and the iron cylinder will be nearly radial, except for a small fringe of lines near the gaps between the pole-pieces. Let the direction of the currents in the one half of the conductors (marked with a dot) under one pole-piece be towards an observer, looking at the cylinder from its end, and in the other half (crossed) away from the observer. Such is the case of the armature of the machine in fig. 2. By application of the left hand it will be found that each conductor is subjected to a pull exactly tangential to a circle passing through the centre of the conductors, save in those which are nearly or quite vertical. The directions of a few of these forces are shown on the diagram. We may consider the whole as a number of loops all tending to set themselves vertical, so as to embrace all the lines of the field, while the ends of the loops will exert neither an upward nor a downward pull. The total result of the action will be that, unless otherwise constrained, the whole system of conductors will rotate in a clockwise direction. Now, if the above represents an electric motor, the conductors will so rotate, and therefore induce in themselves an E.M.F. opposed to the flow of current.

But if the conductors are rotated in an anti-clockwise direction, they will induce E.M.F.'s in the directions which the currents are supposed to have, and the whole becomes a dynamo; hence the mechanical resistance to the rotation which arises when a current flows under those E.M.F.'s takes the form of a tangential drag, acting on each of the conductors round the circumference; and this must be overcome by the mechanical force applied to the shaft of the armature. This tangential drag is removed and put on again twice in each revolution as the

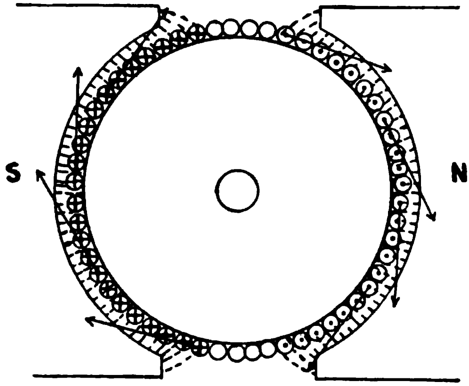


FIG. 29

conductor passes over from under one pole to the other; it is, however, always in the same direction relatively to the rotation.

Assuming the induction B in the air gaps to be practically uniform under the pole-faces, if l=length in cm. of each conductor within the influence of the field or the length of the iron cylinder, the force on each conductor within the polar field is $\text{B}lc \times 10^{-1}$ dynes, where c=the current in ampères carried by the conductor. If τ=the total number of conductors each carrying the same current c, and ϕ=the angular width of the polar field in radians, the number of conductors within one polar field at any moment is $\tau \times \frac{\phi}{2\pi}$, and the total tangential pull acting round both sides of the armature is

$$\text{F}=\tau \,.\, \frac{\phi}{2\pi} \,.\, \text{B}lc \times 10^{-1} \text{ dynes} \quad . \quad . \quad . \quad (9)$$

and this acts at radius r in cm. drawn to the centre of the conductors. The torque is therefore

$$\text{T}=\tau \,.\, \frac{\phi}{\pi} \,.\, \text{B}lcr \times 10^{-1} \text{ dyne-centimetres.}$$

Now the area of the field under one pole-face is $2\pi rl \times \frac{\phi}{2\pi}=rl\phi$, whence if Z_a=the total number of lines that pass into the armature from one pole or pass out of it into the other pole, $\text{B} = \frac{\text{Z}_a}{rl\phi}$. Substituting this value, the torque is

$$\text{T}=\frac{\tau \,.\, c \,.\, \text{Z}_a}{\pi} \times 10^{-1} \text{ dyne-centimetres, or } \frac{\tau \,.\, c \,.\, \text{Z}_a}{4{\cdot}26 \times 10^8} \text{ pound-feet} \,.\, (10)$$

Multiplying these by ω the angular velocity in radians per sec., we obtain the power of the machine either in ergs per second, or in foot-pounds per second.

In ordinary dynamos of modern construction the strength of field in the air-space through which the inducing system of conductors is moved is usually about 5,000 C.G.S. lines per square centimetre. Consequently the force acting on each conductor to resist its motion per foot of active length is ·0342 lb. for every ampère of current flowing through it, or nearly 3½ lbs. for 100 ampères. The shaft and framework of the dynamo must be strong enough mechanically to withstand the drag on the inductors, and they must be prevented from slipping on the armature. The second fundamental equation has now, therefore, introduced the mechanical element which is required in order to complete our first view of the dynamo as a machine, and it can be readily seen why the knowledge both of the electrician and of the mechanical engineer is required for the design of the dynamo; neither can be neglected, as indeed the very word 'dynamo-electric' foreshadows.

This leads us to the next question, viz. What determines the amount of current which may be taken out of any dynamo? We have seen that when once an electric pressure or electro-motive force has been generated by the simple inductor (fig. 1) the current which will flow through it is simply determined by the resistance of the inductor itself (R_a), and that of the external circuit (R_e), or

$$C = \frac{E_a}{R_a + R_e},$$

and consequently for given values of E_a and R_a it is a maximum when the external resistance is negligibly small, as will be the case if the terminals of the inductor are joined together by a short piece of stout copper, or, as it is termed, if the dynamo be short-circuited.

The machine is most effectively short-circuited if its terminals are brought into actual contact, so that in fig. 1 A and D are one and the same points: this secures the maximum strength of current for the E.M.F. induced round the circuit, and the magnitude of the E.M.F. itself depends on the speed of movement which can be maintained by the prime mover.

But for every ampère of current flowing through the inductor our second equation shows that a definite force is brought into play which resists the motion. Hence, theoretically, the maximum rate at which electrical energy can be developed by a dynamo is solely determined by the horsepower of the steam-engine, or other prime mover employed to overcome the resistance to the motion, when taxed to its utmost.

But for practical purposes it is not simply a maximum rate of development of electrical energy which is required, but a maximum rate of development *in the external circuit*, since here alone is it useful; it is the output or the product E_tC_t which has to be a maximum. When the machine is short-circuited, although the rate of internal development of energy is a maximum, there is *no* output, for the number of volts used in driving the current through the internal resistance of the inductor, or CR_a is equal to E_a, its internal E.M.F., and therefore E_t is zero. Hence in practice it is necessary to maintain at the terminals of the machine a certain voltage suited to the purpose in view, and also suited to the design of the machine as a generator of electric pressure.

But so long as the mechanical energy at our disposal suffices to produce such speed of movement as will maintain the required voltage at the terminals, the output of the dynamo may be indefinitely increased by lessening the external resistance, and so increasing the current. However, in practice, certain other considerations, apart from the question of the mechanical power at our disposal, limit the current which can safely be taken out of the dynamo, and one of the most important of these is the heating of the inductor by the passage of the current. Unless the heat which is continuously generated at the rate of C^2R_a watts can be dissipated quickly enough, the temperature of the inductor will rise so high that we run the danger of fusing it or at least burning its insulation. Given, therefore, sufficient horsepower applied to a dynamo constructed to give a certain terminal voltage, the greatest current which it will be able to *continuously* maintain is in general determined by the rate at which the heat produced in it can be dissipated, and so indirectly by the relation existing between the electrical resistance of its armature and the area and effectiveness of the surface exposed to the cooling of the air.

The two equations of the dynamo have now been discussed, and expressed symbolically, i.e. on the C.G.S. practical system

(I) $$E = BLV \times 10^{-8}$$

(II) $$F = BLc \times 10^{-1}.$$

The first deals with the creation of electrical pressure by means of electro-magnetic induction; the second with the absorption of mechanical energy, when an electric current flowing under this pressure interacts with the magnetic field by movement through which the E.M.F. was induced. Both are embodied practically in the dynamo and underlie it in all its manifold types.

CHAPTER VI

SELF-INDUCTION

ONE of the consequences which ensue when a current is permitted to flow through our inductor wire, viz. the magnetic pull on it, has now been considered; another of great importance remains still to be traced. It has been said that the inductor, in which we wish to calculate the E.M.F. induced by movement in a magnetic field by the rules of Chapter IV., may or may not be carrying a current, according as it is or is not part of a closed electrical circuit. Now, suppose that it is carrying a current: when this is the case, we have spoken of the superposition of its own magnetic field upon that already existing. We must now, therefore, ask, What is the effect of the co-presence of these two fields as regards the E.M.F. induced in the inductor? In actual nature at any point in space there can never be more than one magnetic field as mapped out by lines of induction; but this magnetic field may for purposes of investigation be regarded as the resultant of two or more fields; and hence it is legitimate to consider these as acting separately on the inductor, and so arrive at the resultant E.M.F. set up in it. In the case here to be considered we may regard the actual field in which the inductor is immersed as made up of two sets of lines—one due to an external source of magnetisation, and one due to the current in the inductor itself; each produces its own proper effect, which can be considered entirely apart from the effect of the other.

Now, when the current flowing through a conductor is a steady current in one direction and of constant value, the field due to it remains unvarying in intensity and position relatively to the conductor. Hence, if it be a steady current which we have supposed to be flowing in our conductor, then, even though it be moved in space, as, in fig. 24, by rotation round the axis of a spindle, it carries its own field along with it, and therefore can never by its own movement cut *its own lines*. At any point lying in its path, and through which it either will pass or has already passed, the number of lines transversal to its length may be more than the number due to the external field within any given area, but it cannot either cut or have cut this increased number, since it carries along with it its own lines, which have so increased the original field. But a further possibility still remains, and this is that the lines due to the current may by their own movement cut the conductor; and this does, in fact, take place whenever the current begins, or ends, or varies in strength. When a current starts to flow in a conductor the magnetic field surrounding it has to be called into existence; and we may picture to ourselves this process by means of the conception of the circular lines, shown in fig. 7, being thrown off from the central axis of the conductor and gradually expanding outwards like the circular ripples caused by the dropping of a stone into still water. As the ring-lines first formed expand outwards and pass laterally across the substance of the conductor out into the medium surrounding it, new ones are continually being formed at the centre, themselves in turn to expand outwards. Whenever the current increases in strength, more rings are thrown off from the centre of the conductor, and the field external to it becomes more crowded; while if the current decreases in strength all the rings contract inwards and some pass into the conductor and disappear by absorption at the centre. The magnetic circles behave like elastic rings stretched outwards by some internal pressure acting all round them. Finally, when the current ceases to flow, the rings, being in a state of strain, and now being left unsupported, as it were, by the falling current, collapse inwards, cut the conductor transversely and disappear entirely. All such changes are not instantaneous, but take a certain time—short it may be, but still a definite period of time.

Now here we have a case of relative motion of field and conductor by reason of which the lines cut the conductor at a certain rate; consequently the conductor conveying a current of varying strength must also act as an inductor, and an E.M.F. is set up in it proportional to the time-rate at which the lines, expanding or contracting, cut the conductor. Such cutting is not due to any movement of the conductor itself, but to the variation in the strength of the current flowing in it, which causes its own magnetic field to vary, and the E.M.F. so produced is called the E.M.F. of 'self-induction,' as being induced by the conductor upon itself. Since the circular lines of force surrounding a solitary rectilinear conductor are in a plane perpendicular to the direction of the flow of the current at any point, the line of action of the E.M.F. which they produce must necessarily be the axis of the flow of current in the conductor. Its direction along this line of action has next to be considered. In fig. 7 let the current passing downwards through the plane of the paper be increasing in value, then circular lines having the direction shown are expanding outwards. Now, if we place the outstretched right hand along the conductor, so that the thumb points in the direction of the lines of induction, and so that their radial direction of expansion passes outwards across or through the hand from the palm to the back, i.e. with the palm facing inwards towards the centre, it will be found that the direction of the self-induced E.M.F., being by the rule of p. 47 from the wrist to the finger-tips, is opposed to the direction of the current, and therefore resists the increase of its strength. If, however, the strength of the current is decreasing, and the circular lines are therefore contracting inwards, the palm of the hand must face outwards, and the self-induced E.M.F. is in the direction of the current, and tends to keep up its strength. Again, therefore, the self-induced E.M.F. resists the change in the current by which it is itself produced, and this is in fact the universal law which governs its nature.

Consider the case of a loop of wire (fig. 30) which is rapidly pushed up to the pole of a magnet from some position outside its field; by the cutting of the lines of the external field an E.M.F. is set up in the direction of the arrow on the loop: this increases in strength, since more and more lines are cut as it moves through the denser field near to the pole. Under the action of the E.M.F. in the closed loop a current begins to flow; but as soon as this current begins, lines of induction due to it spread outwards from the conductor, and form loops linked with or passing through the loop of the electrical circuit. A second field is thus superposed on the first, and the direction of the lines of this second field as they pass through the loop is exactly opposite to that of the first: this is roughly indicated in the diagram by the dotted lines running counter to the full lines of the original field. It follows that the rising current tends to crowd the loop full of lines opposite in direction to those of the external field, or, in other words, it tends to reduce the field strength through the loop, just at the time when it is being increased by the motion. Consequently the rise of current in the loop is not so rapid as it would be if the current had itself no magnetic effect, or we may say that the current by reason of the magnetic qualities of its circuit reacts on the field strength. At any moment the resultant field in which the loop is moving has a certain value and distribution, and the rate at which the lines of this field are cutting the conductor loop gives the E.M.F. to which the current strength at that moment is proportional; but this actual field may also and more conveniently be resolved into two—the original field, and the current's own field, the effect of the two being considered separately. At any instant the lines induced by the current on itself are increasing at a certain rate or being thrown off round the loop and cutting it, but the effect of this opposes the effect due to the cutting of the external field. If we place the right hand along the conductor on the inside of the loop, it is the direction of the two sets of lines which is different; if we place it on the outside, it is the direction of movement of the lines which is different. Hence the movement of the second field, due to the fact that the current is rising, induces a counter or back E.M.F. opposed to the rising current, just as was previously the case with the simple straight conductor; but now further in this case where the change in the current strength is due to change in the E.M.F. induced by movement through the external field, the latter as it rises in value is continually opposed by a counter E.M.F. of its own creating, and the current cannot

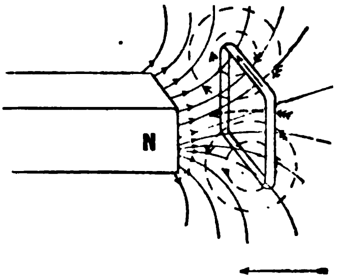


FIG. 30.

rise in strength as rapidly as the E.M.F. imposed by the external field rises.

If the movement of the loop is suddenly reversed, and it is made to recede from the magnet, the direction of the E.M.F. will be reversed, and so also eventually will the current. But not at once; the lines of its own field have to collapse upon the wire, to be succeeded by fresh lines passing in the reverse direction through the loop. The collapse of the former lines inwards and the expansion outwards of the new lines both cause a self-induced E.M.F. in the same direction as that in which the current was flowing: this for a time tends to counterbalance the new E.M.F. impressed on the loop by the external field, and hence the current only gradually sinks to zero and finally becomes reversed in direction.

From the universal law that the self-induced E.M.F. always opposes the change in the current strength of which it is itself the effect, the real nature of self-induction opens out before us. It is a certain property attaching to a circuit or to a conductor regarded as forming a possible part of a circuit in virtue of which no change in the current flowing through it can be instantaneously produced. If a steady electromotive force of E volts be applied to the ends of a conductor of resistance R, the current does not instantaneously rise to its full value, as given by Ohm's law, viz. $C=\frac{E}{R}$; it takes time for it to reach this value, since at the very moment when a current starts to flow, it calls into existence a back or opposing E.M.F. dependent on its own rate of increase, and this checks the rise of the current. Similarly if the E.M.F. be instantaneously removed, the current shrinks to zero, not instantaneously, but by a gradual decrease of strength. The reason must be sought in the fact that the current is indissolubly connected with the presence of a magnetic field due to it, and when the current is started, or stopped, or altered in value, this field has also to be created, destroyed, or altered. The analogy of the electrical property of 'self-induction' to the mechanical property of 'inertia' at once strikes us; in virtue of the inertia which attaches to any mass of matter, a definite velocity cannot be given to it instantaneously, or destroyed instantaneously, nor can it be instantaneously altered to another value; a fact which is most familiar to us in the case of the starting from rest, or stopping, or altering the speed of, a heavy body such as a flywheel. When, however, we apply the term electro-magnetic inertia to the property of self-induction, it must be clearly understood that the current itself has no quality analogous to inertia; it is only to the current as producing a magnetic field that the property attaches. For with the same current flowing round a circuit, the self-induction can be altered very greatly by any change which affects the magnetic field surrounding the circuit; and it is therefore a truer view to regard every circuit, or portion of a circuit, as possessing a definite property other than its electrical resistance, but which in conjunction with its electrical resistance determines the strength of current flowing through it at any instant under any E.M.F.; and this quality of the circuit is briefly called its 'inductance,' since upon it depends the E.M.F. of self-induction. Obviously the inductance of a circuit, or portion of a circuit, will be dependent on the magnetic conditions which determine the number of lines of induction connected with it, when a given current is flowing, and hence is governed, not only by the geometrical form or shape of the conducting path, but more especially by the presence of iron within or near it.

Two simple cases will, however, serve to indicate the nature of the property which we call 'inductance,' and for this purpose let us first consider a single loop of wire surrounded by air, such as fig. 9, in which case the lines of induction due to a current round it traverse a medium of constant permeability. As we have seen, they all pass through the loop, and are linked or looped with the electrical circuit once, so that when the E.M.F. causing the current is removed from the loop, and the current is diminished to zero, each line vanishes into the wire, cutting it once; and this process takes place with a certain time-rate of disappearance, depending on the rapidity of the change in the current. The inductance of such a loop is equal to the number of linkages of lines with the wire when unit current is flowing round the loop; so that if $z =$ the total induction through the loop when $i =$ the strength of current in it, the 'inductance' of the loop is $L=\frac{z}{i}$; and this quantity is so related to any change of current strength that if the latter be varied, and the rate of change be symbolised by $\frac{di}{dt}$, the E.M.F. of self-induction is $e_s = -\mathrm{L}\frac{di}{dt}$, the minus sign signifying that if the strength of current be increased, the direction of the self-induced E.M.F. is negative or opposed to its increase.

Next let us place several such loops side by side, so as to form a helix or solenoid, as in fig. 31. Again, we know that the lines of induction pass in the main right through the coil; but this is not true of all the lines, since some leak out at the sides of the solenoid, and so only pass through a few of the loops. In a solenoid, therefore, a line may be linked more than once with the conducting circuit, and the greater number are linked with or thread through all the loops of the solenoid. In the diagram, where batches of lines are indicated by single dotted curves, 100 lines are linked with all four loops, and 50 with two loops, so that the total number of linkages is $(4 \times 100) + (50 \times 2) = 500$; if the current ceases to flow, each of the 100 lines vanishing by contraction cuts through the electrical circuit four times, and each of the fifty lines cuts through two loops; consequently the effect of such alteration must depend upon the number of times that the lines are linked with the wire, and not merely upon the total number of lines of induction. If X=the number of such linkages when unit current is flowing, the inductance of the solenoid of many loops is $\mathrm{L} = \frac{\mathrm{X}}{i}$, and again the E.M.F. of self-induction is $-\mathrm{L}\frac{di}{dt}$. If L is in C.G.S. measure, e_s is given in absolute units of E.M.F.; but if L is in practical units of inductance (for which the name 'henry' has been lately proposed, in honour of the American scientist, Prof. Joseph Henry), e_s is given in volts when i is reckoned in ampères, the practical unit of the henry being 10^9 times the absolute unit.

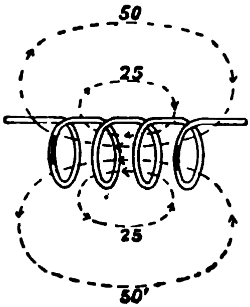


Fig. 31.

But now if we insert an iron core into the solenoid, or in any way modify the case by the presence of iron near the solenoid, the inductance is no longer a constant quantity, but varies with the strength of current under consideration, and the variations in its values can only be adequately represented by some graphical method. More than this, the term 'inductance,' when applied to circuits embracing iron or other magnetic media, admits of more than one definition, and these result in different values under the same circumstances. In dynamo-electric machines iron is almost invariably present, and in consequence the inductance of circuits forming part of them is often a subject of no little intricacy. Yet even in such cases it often suffices as a first approximation to consider L as the ratio between the E.M.F. of self-induction and the time-rate of change in the current strength which produces it, and further as being a constant quantity.[1]

It is evident from a consideration of the case of an increasing current which is directly opposed by the self-induced E.M.F. that the latter cannot be the cause of the flow; there must be another E.M.F. in the same direction as the current, which is greater than the back E.M.F.; further, that the actual E.M.F. causing the flow of current is the difference between the two. We must therefore, in cases where the current is altering in value, distinguish carefully between (1) the *impressed* E.M.F.; (2) the E.M.F. of *self-induction*; and (3) the *resultant* E.M.F., which immediately causes the flow of current, and the value of which at any moment is equal to the algebraic sum of the values of the other two. In any conductor forming part of a closed circuit and conveying a current there must be a certain E.M.F. 'impressed' upon it, and to which the current through it is ultimately due; this impressed E.M.F. may be a difference of potential applied to it from without, or existing within its own limits, as in a battery; or, again, it may be an E.M.F. generated in it by movement relatively to a magnetic field, as in the loop of fig. 30, in discussing which we have already distinguished between the E.M.F. impressed on it by its movement through the external field and the E.M.F. of self-induction. The general law by which the three E.M.F.'s are related is that if e_i=the impressed E.M.F. at any moment, and e_s=the self-induced E.M.F. at the same moment, the resultant E.M.F. to which the current is then proportional is

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[1] For further treatment of the subject the reader may be referred to *The Alternate Current Transformer* (Fleming), Chapters II. and III.; also to a paper, 'Inductance and its proposed Unit, the Henry' (Kenelly), reprinted in the *Electrical Engineer*, January 2 and 9, 1891.

$$e=e_i\mp e_s,$$

the proper algebraic sign depending on the question whether the current is increasing or decreasing in strength. At any instant, and under all circumstances, the current flowing in any conductor is $c=\frac{e}{\text{R}}$, as given by Ohm's law; but it is only when the current is constant in direction and value, and therefore $e_s=0$, that $e=e_i$, and therefore $c=\frac{e_i}{\text{R}}$.

Take the case of a steady E.M.F. such as that of a battery impressed on a simple electrical circuit of resistance R which contains no other source of impressed E.M.F. As soon as the circuit is made, a current begins to grow, evoking a back E.M.F. opposed to its growth; hence $e=e_i-e_s$ or $e_i=e+e_s$.

Now to what does this division of the impressed E.M.F. into two portions correspond? The answer is that two different kinds of work are being done. In the case of a motor to the armature of which electrical energy is being supplied at the rate of EC watts, the impressed E.M.F. is divisible into two portions, the one equal to the E.M.F. which is required to drive the current through the resistance R_a, i.e. equal to CR_a, and the other equal to the back E.M.F. e developed by the motor, or $\text{E}=\text{C}\text{R}_a+e$: the work done is similarly divisible into two portions, one corresponding to C^2R_a, which appears as heat, and the other corresponding to $e\text{C}$, which appears as mechanical work done in turning the motor against the resistance of its load. In exactly the same way, the rate of expenditure of energy when an impressed E.M.F. causes a current whose strength is varying to flow in a circuit of resistance R may be divided into two portions; the one corresponding to the rate at which energy is being dissipated in the circuit in the form of heat, and the other to the rate at which energy is being stored up in the magnetic field. If e_i, e_s and c be the values at any one and the same moment of the impressed E.M.F., self-induced E.M.F., and the actual current then flowing, the one portion is equal to $c^2\text{R}$, the other to ce_s, but the total rate of expenditure of energy is ce_i; consequently when the field is being created

$$ce_i = c^2\text{R} + ce_s\,;$$

whence $$e_i = e + e_s.$$

The creation of a magnetic field, therefore, demands energy which is absorbed in the process; but when once the field is established, i.e. when a steady current is set up, no further expenditure of energy is required to maintain it. That a certain amount of energy is in reality absorbed in the creation of the magnetic field round the conductor is evident from the fact that, although so absorbed, it is not irrecoverable, but is as it were stored up and can be liberated. For suppose e_i to be instantaneously withdrawn; then the self-induced E.M.F. tends to keep the current flowing, and does actually do so, since the current only falls to zero after a certain period of time. During this time work is being done in the circuit, the energy stored up in the magnetic field reappearing as heat, *e.g.* in the spark which occurs when the circuit is opened. The same is true if the impressed E.M.F. is gradually withdrawn, for the current falls less rapidly than the impressed E.M.F.; and in general, when the magnetic field is being destroyed $e_i = e - e_s$. The analogy of a flywheel revolving between bearings is exact; when being started, work has to be done upon it, which is greater than the work absorbed by friction by the amount which is required to impart a certain angular velocity to it. When once its speed has been got up, and is kept constant, all the work done is spent in overcoming its frictional resistance; but when the turning force is withdrawn, the kinetic energy possessed by the wheel keeps it rotating until it is itself absorbed by the friction, and during this period work is done by the wheel against its frictional resistance.

In the above case of a simple circuit to which an E.M.F. is applied, it is easy to see that energy is stored up when a magnetic field is created, and, since the strength of the latter depends on the magnetic qualities of the circuit, that the inductance is a property belonging to the circuit or the conductor to the ends of which a difference of potential is applied. But in the case of a conductor in which an E.M.F. is generated by movement through a magnetic field such as the loop of fig. 30, the magnetic effect of the current shows itself by modifying or *reacting* on the external field, and such expressions as the inductance of the loop and the energy stored up in its field are only legitimate owing to a mental separation of the actually existing field into two component parts. It is not that the loop first reacts on the field, and then cuts the resultant field by its own movement; the resultant field moves relatively to the conductor, partly by reason of the latter's own movement, and partly by reason of change in its own current, which affects the strength and distribution of the actual field; to this movement of the resultant field relatively to the conductor the resultant E.M.F. is due; and if we were to consider this resultant field, we must not also credit the loop with self-induction or inductance: it must then be considered as a circuit possessing only resistance. It is, however, simpler at the outset to treat the conductor as possessing inductance, and so to deal with the external field unmodified by reaction of the current on it.

Two different causes have now been considered, either of which may cause a variation in the strength of the resultant E.M.F. acting on a conductor; if e_i is steady and constant, the circuit may be made, altered in resistance, or broken; that is, the resistance of the circuit may be varied within the limiting values R and ∞; or while R remains constant, e_i may itself be varied in direction or value, or both. The first case deals with the growth and decline of current in a circuit to which a steady E.M.F. is applied. The second case is the one with which we are chiefly concerned in the study of dynamos, for upon it depends the theory of alternators. As before, e_s always opposes a rising current and assists a falling current; but now, as distinct from the former case, we have also to consider its relation to the varying e_i. Reverting to the rotating inductor of fig. 24, it was shown that the impressed E.M.F. varied not only in direction but also in value according to a certain function of the time, and these variations were graphically represented by a single-valued curve which during one complete period gradually rose from zero to a positive maximum, then as gradually fell to zero, and lastly repeated both rise and fall on the negative side of the zero line. Now if the circuit of an alternator formed out of one or more such rotating inductors is closed, as in fig. 41, and a current is allowed to flow, how will the current vary in strength? If the circuit have no inductance, but be composed entirely of a certain ohmic resistance, there will be no self-induced E.M.F., and consequently the current will alternate not only after the same law as the impressed E.M.F., but exactly simultaneously with it; its value at any moment will be given by dividing the impressed E.M.F. at that moment by the resistance of the circuit, so that the curve which represents to one scale the varying impressed E.M.F. will also represent to some scale the variations of the current. But such a case of a circuit with no inductance is only possible theoretically; physically it will always have some inductance, although it may be so small as to be negligible. We have therefore to consider the effect of self-induction on the current curve; and since this latter will always be identical in shape and law with the resultant E.M.F. curve, we require to compare together the curves of impressed E.M.F. and of resultant E.M.F., and until we have done so we cannot assume that they will be identical; they may not be coincident in phase, their maximum values or heights may not be alike, or they may even be dissimilar in shape. The necessity of this caution will be apparent from the following considerations.

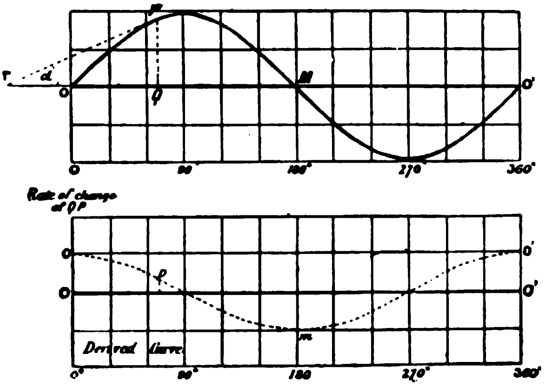


FIG. 32.

In fig. 32 let O P M O′ be any single-valued curve; in this curve take any point, P, and through P draw a line P T tangential to the curve and making an angle α with the horizontal axis O O′. Then it can be shown mathematically that the rate at which the ordinate Q P is increasing or decreasing with respect to O Q is represented by the trigonometrical tangent of the angle of inclination α; or, as it is termed, the 'slope' of the tangent at the point P represents the rate of change of Q P. Let us now plot a second curve, *o p m o′*, whose ordinates at any point represent the slope of the tangent at the corresponding points on the first curve, the horizontal axis

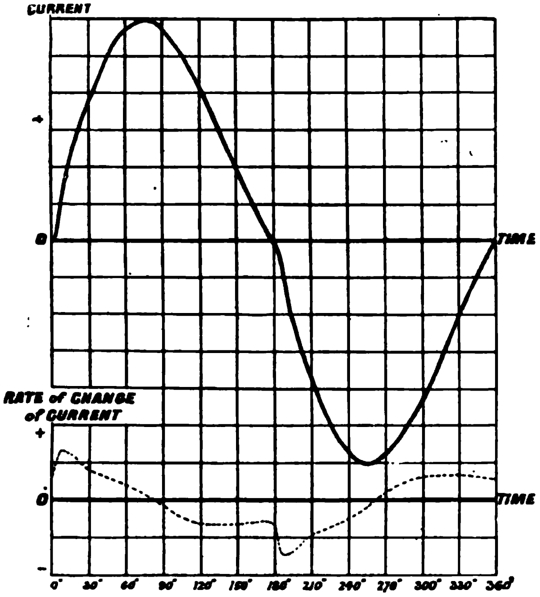


FIG. 33.

being retained unchanged; then the ordinates of the second or derived curve so obtained will represent the rate of change of the ordinates of the first curve at the same points along the horizontal axis.

Now we may reasonably assume that the current curve of an alternator will alternate, and rise and fall somewhat after the same fashion as the impressed E.M.F. curve. Let us suppose, therefore, that in fig. 33 the thick line is the curve representing graphically the strength of current of an alternator for a complete period, time being represented by abscissæ measured from the origin O on the left-hand side of the diagram. This curve need not be symmetrical or expressible by a simple mathematical function, the sole assumption being that it is single-valued and divisible into two half-periods, during which it is in opposite directions From this curve can be obtained a second derived curve (shown by a dotted line below), the values of which represent to some scale the slope of the tangents to the first curve; the two curves of fig. 33 will therefore represent respectively the strength of the current at any instant and the rate of variation of its strength at any instant. It should be noticed how greatly the two derived curves of figs. 32 and 33 differ from each other in shape and character, although the two original curves both show a wave-like form; this difference is due to the fact that their rates of change are different at different points of time, an almost imperceptible difference in the shape of the two originals completely altering the shape of the derived curves. Further, in fig. 32 the original curve is a sine curve, and therefore symmetrical, whereas in fig. 33 the two sides of each half-wave are not identical, the fall being shown more gradual than the rise. Since the original curve of fig. 32 is a sine curve, it follows from its mathematical nature that its derived curve is also a simple periodic curve exactly analogous to its original, and, further, a sine curve whose phase precedes the phase of the original curve by 90°. As the point P in fig. 32 is moved away from the origin up the curve, the angle α gradually diminishes in value; at the top of the curve, $\alpha = 0$ and $\tan \alpha = 0$; therefore the rate of change of the ordinate is zero; but as the point P descends to the zero line, $\tan \alpha$ increases until the rate of change reaches a maximum at O or M. The crests and hollows therefore of the derived dotted curve do not occur at the same moment as those of the original curve, since the higher the ordinate Q P, the less is its rate of variation, but are shifted forwards (towards the left) by one quarter of a complete period, or 90°. Something of the same nature is seen in the derived curve of fig. 33, but a comparison of the two curves will show that the maximum ordinate of the dotted or derived curve does not exactly correspond in point of time to the zero of the original, and that there may be several crests and hollows in the half-period of the derived curve, even though the original curve only shows one, since the rate of rise or fall of the latter need not vary after any simple law.

Now the varying current produces a flow of magnetic lines of induction looped or interlinked with its conductors, and this flow also varies in intensity; for upon the strength of the current depends the number of lines of induction due to itself, through the same magnetic circuit, and also the magnetic circuit presented to the loops of current may itself be varying at any instant in length or nature. Consequently, analogous to the two curves of current and rate of change of current there must be two other curves; the one representing the number of lines induced by the current itself through the magnetic circuit, which is at any instant presented to the current loops, as connected with the time during a complete period; and the other representing the rate of change of this total induction with respect to time, and derived from the former. In fig. 34 let the full line be the curve of self-induced lines plotted on the same horizontal axis of time as the current

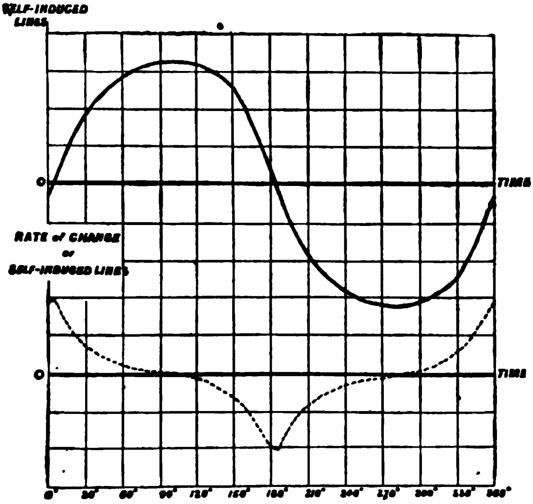


FIG. 34.

curve of fig. 33; these two are not by any means necessarily of the same shape, since the number of self-induced lines may not vary directly as the current under all circumstances, nor need the phase of the current curve and induction curve be the same; there may be a lag of magnetisation behind the magnetising current in respect of time, and, as will be shown hereafter, where a portion or the whole of the magnetic circuit through which the self-induced lines pass is composed of iron, there must be such a lag, although it may be very slight. Hence in fig. 34 the curve of lines is shown passing through zero at an instant slightly later than that at which the current curve passes through zero. From the curve of self-induced lines we can derive the second dotted curve of fig. 34 representing to some scale the rate of change in the number of self-induced lines at any moment; and this, as we know, may be unsymmetrical if the original curve is unsymmetrical. This second curve represents at any instant $\frac{dz_s}{dt}$, where z_s is the total number of self-induced lines; but the E.M.F. of self-induction is proportional to $-\frac{dz_s}{dt}$, the negative sign indicating that if z_s is increasing by reason of an increasing magnetising current, the E.M.F. of self-induction is negative and acting to oppose the increase of the current. If, therefore, we invert the derived curve of fig. 34, it will also represent to some scale the E.M.F. of self-induction at any instant, and at the same time indicate the direction of this E.M.F. When the number of self-induced lines is increasing in the positive direction, or decreasing in the negative direction, the E.M.F. is negative or in a direction opposing a positive current; and when they are decreasing in the positive direction, or increasing in the negative direction, the self-induced E.M.F. will be in the positive direction. Hence, if the curve of lines shows, as in fig. 34, a continuous increase of lines, and then a continuous decrease of lines, although the rate of this increase or decrease may be very different at different points, the curve of self-induced E.M.F. will pass through zero when the total number of self-induced lines is a maximum, and for a whole period will be divisible into two portions, below and above the horizontal axis.

In fig. 35 let the dotted curve E_s be the dotted curve of fig. 34 when inverted, being, therefore, to some scale the curve of the self-induced E.M.F. in volts, and upon the same horizontal axis of time let us reproduce by the thick line (E) the current curve of fig. 33. It will be seen that, roughly speaking, the phase of the self-induced E.M.F. lags 90° behind the phase of the current, but owing to the slight lag of the induction behind the magnetising current, the curve of self-induced E.M.F. does not reach its maximum until after the current curve has passed through its zero.

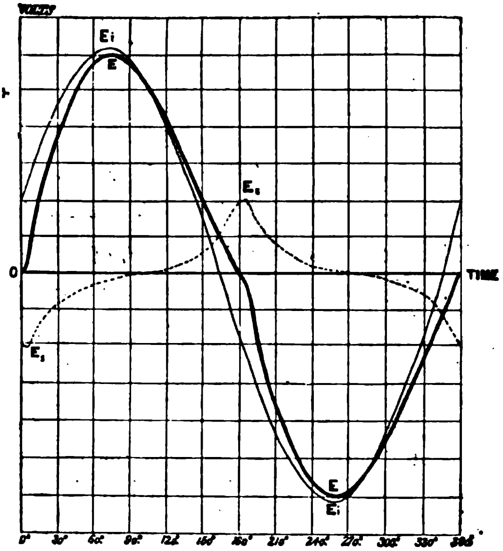


FIG. 35.

When the current is rising in the positive direction or descending in the negative direction, the self-induced E.M.F. is negative in direction, so as in both cases to oppose the change of the current strength. But the thick-line current curve will also represent the curve of resultant E.M.F. provided that the scales be so chosen that the same height represents indifferently either one ampère or one ampère multiplied by the resistance of the circuit or portion of the circuit under consideration. In fig. 35 this is supposed to be the case, so that the volts of resultant E.M.F. can be read off the curve E. We are now, therefore, in a position to deduce the curve of the impressed E.M.F., which must have acted on the portion of the circuit under consideration in order that with the assumed conditions the curve of resultant E.M.F. should have the shape shown in the diagram. Since we have obtained curves which determine for us the signs at any moment of e and e_s, we can write our equation of p. 84 as $e_i=e-e_s$, and have only to subtract the ordinates of the self-induced E.M.F. curve from the ordinates at the same points of the resultant E.M.F. curve and plot their difference as a third curve, due regard being paid to the algebraic sign of the ordinates; the third thin-line curve so obtained (E_i) will be the required curve of impressed E.M.F. in volts. Or if we had been given the curves of E_i and E_s we could have deduced the curve of E. Two striking facts now come to light: the first is that the result of combining the curve of self-induced E.M.F. with the impressed E.M.F. curve may very materially modify the shape and form of the resultant E.M.F. curve as compared with the latter; for example, the curve of impressed E.M.F. shown in fig. 35, and deduced logically from the previous curves, is strictly a sine curve, yet the curve of resultant E.M.F. is not a sine curve, and is not even symmetrical on its ascending and descending sides. Secondly, while the self-induced E.M.F. always opposes a rising e_i, it does not always assist a falling e_i. From this, two important consequences follow: (1) Since the self-induced E.M.F. does not always assist a falling e_i, but does always assist a falling current or falling resultant E.M.F., the curve of resultant E.M.F. cannot coincide in phase with the impressed E.M.F. curve, but, on the contrary, the current curve lags behind the curve of impressed E.M.F. by a certain time or angle depending on the inductance of the circuit. (2) So long as the self-induced lines continue to rise, however slowly, there is some self-induced E.M.F. opposing the impressed E.M.F.; but at the instant when they are at their maximum, and the self-induced E.M.F. is zero, the curve of resultant E.M.F. cuts the curve of impressed E.M.F. Where this point of intersection is, entirely depends upon the curve of self-induced lines, and this depends, not only on the curve of magnetising current, but also on the magnetic circuit through which at any instant it is inducing lines. Hence, if the curve of self-induced lines continues to rise after the current curve has begun to fall, owing to a more than proportionate decrease in the reluctance of the magnetic circuit, the curve of resultant E.M.F. will cut the impressed E.M.F. at a point after it has reached its own maximum, as in fact is shown in fig. 35. If, however, the maximum number of self-induced lines coincides in time with the maximum strength of the current, the highest value of the resultant E.M.F. will be its point of intersection with the curve of impressed E.M.F. Further, a fixed magnetic circuit always permits of an increase of the number of lines of induction through it, when the magnetising current is increased. If, therefore, the impressed E.M.F. is never constant, but *always* altering in value, the curve of self-induced lines never becomes a straight line (unless the magnetic circuit be altered so as to exactly counterbalance the changing current—a rare possibility); hence there is a definite self-induced E.M.F. at the moment when the impressed E.M.F. reaches its maximum, and the point of intersection must be subsequent to the point of highest impressed E.M.F.; in other words, E can never attain as high a value as E_t.

The whole may easily be illustrated by the case of a flywheel to which is applied a turning force, which not only alternates in the direction in which it tends to turn the wheel, but also varies in value from zero to a maximum strength, and thence, passing through zero, to a maximum in the opposite direction. Suppose that the wheel is rotating in the positive direction, and the turning force changes to the negative direction; the wheel cannot instantly change its direction, and it continues to rotate for some little time in its old direction; consequently its speed does not become zero until the turning force has acted for a certain time in the negative direction, and so for each change. The phase, therefore, of the wheel's velocity would lag behind the phase of the turning force. Further, since the turning force is never steady, but varies continuously, the flywheel would never reach the maximum velocity corresponding to a steady turning force equal in strength to the maximum value of the alternating force, since before reaching such a velocity the value of the turning force has already begun to decrease, and in just the same way the resultant E.M.F. lags behind the impressed E.M.F., and its maximum value is less than the maximum value of the impressed E.M.F.

As in the case of the loop of fig. 30, so now we may regard the resultant E.M.F. as that produced by relative movement between the inductors and the actual field which exists at any moment. This actual field will be the field due to an external magnetising current as modified by the reaction on it of the induced current; this will in general weaken the original field and shift the position where the rate of cutting is a maximum, thus accounting for the different height and position of the curve of E as compared with the curve of E_t. When so regarded, the alternator must, as pointed out before, be considered as inductance-less.

It has been stated previously in general terms that the rate of development of electrical energy throughout an entire circuit or in any portion of it is equal to the product of the volts of E.M.F. acting in it, and the ampères of current which flow through it under that pressure. But now, in the case of an alternator or an alternating current, we see that the voltage and strength of current are continuously varying after some periodic law, and the question therefore arises, What number of volts and what strength of current we are to multiply together in order to arrive at the power which is being developed. Obviously not the maximum values of E.M.F. and current, since at another portion of the period they both fall to zero: it must be some mean value of the two which will fairly represent their average magnitudes throughout an entire period. The principle on which this question is solved is the same as that which underlies our first simple rule; it must, however, be now expressed in more explicit terms. In any portion of a circuit the rate of development of electrical energy at any instant is equal to the product of the current which is then flowing in that portion of the circuit and the E.M.F. which is impressed upon it at that instant. Thus, in fig. 36, if the thick line represents the curve of current in ampères flowing through any portion of a circuit, and the thin line represents the corresponding curve of E.M.F. in volts which is impressed upon that portion, the rate of development of energy in watts at any instant, a, is equal to the product of the ordinates ab, ac, which represent the impressed volts and the ampères flowing at that instant. By thus multiplying together a number of simultaneous values of the impressed E.M.F. and current, and plotting their products along the same horizontal axis, we should obtain a third curve (shown with a shaded fringe in fig. 36), representing the instantaneous rate of development of energy in watts throughout an entire period, and the area which it encloses, being the product of power and time, represents work done. In so doing we must pay attention to the algebraic signs of the E.M.F. and current, all ordinates above the horizontal line being reckoned as + and all below as − ; and, further, if their product be positive, it must be plotted above the horizontal line as positive work ; if negative, below it, as negative work. Since the product of two quantities, one + and the other —, is negative, the product of two ordinates is negative unless both are either above or below the horizontal line, i.e. positive work is done only when current and impressed E.M.F. are in the same direction. Now, to what does this distinction between positive and negative work correspond? Again the flywheel analogy will help us : whenever the rotation of the flywheel is in the same direction as that of the turning force, the turning agent is doing positive work

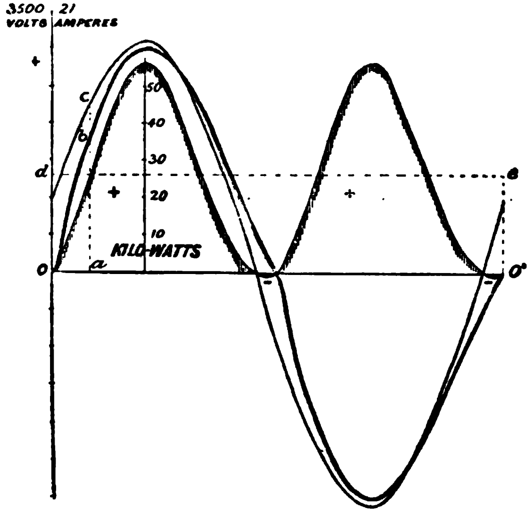


FIG. 36.

on the wheel; but whenever the direction of rotation of the wheel is opposite to that in which the turning force is endeavouring to rotate it, the flywheel is doing work on the turning agent, or negative work is done. In exactly the same way, whenever the current is in the same direction as the impressed E.M.F., positive work is done by the impressed E.M.F.; but whenever the current is flowing against the impressed E.M.F., negative work is done, the current being forced against the impressed E.M.F. The possibility of this is due to the fact that the positive work of an alternator is divisible into two separate portions; one part is expended in heating resistances or in forcing a current against a back E.M.F. due to causes external to itself, as in transformers, both being useful work so far as the alternator is concerned; the other part is work done on its own magnetic field. This latter is stored up, and, at a later portion of the periodic time, is again restored when the magnetic field does work on the circuit, this being negative work so far as the impressed E.M.F. is concerned. Consequently the total useful work done by the alternating current and E.M.F. in one complete period, or the total energy transformed from electrical energy into heat or other useful work, is measured by the net area enclosed by the fringed curve in fig. 36, when the two negative areas (shown black and marked −) are added together, and their sum subtracted from the two positive areas. It will be seen that the wavy character of the power curve is due to the fact that current and E.M.F. are continually varying, but the appearance of the negative work (shown black) is solely due to the 'lag' of the current curve behind the impressed E.M.F. curve, by reason of which the impressed E.M.F. and current can be in opposite directions; and this lag is solely caused by the inductance of the portion of the circuit considered, which therefore indirectly determines the respective amounts of the positive and negative areas. The greater the lag, the smaller the net amount of work done. If the lag of the current curve were to amount to as much as a quarter of a period, the entire current curve being retarded in phase by an angle of 90° as compared with the E.M.F. curve, so that the current value is zero when the impressed E.M.F. is a maximum, and *vice versâ*, then the two negative areas would be exactly equal to the two positive areas, and therefore the net work done in a period would be nil, the explanation being that the magnetic field gives back as much energy in the second and fourth quarters of a period as was previously developed in the first and third quarters by the source of the impressed E.M.F. On the other hand, if there is no inductance and no lag, there is no negative work done, since the phases of E.M.F. and current coincide. Both cases are impossible practically, but serve to indicate the theoretical limits to which practical cases approximate more or less closely.

Given, therefore, the two curves of impressed E.M.F. and current during one complete period, we can deduce a curve whose area represents the work done, and whose ordinates represent the power developed at any instant. The mean power or mean rate of development of energy will be the mean of all the values of the product of current and impressed E.M.F. taken over a sufficiently long time; since we have supposed that the E.M.F. and current vary periodically, so that the curves continually repeat themselves after each period, it will suffice to consider the mean of all the values during one period; so long as the conditions are unchanged any other complete period or periods will give the same result. The mean value of the power during one period will evidently be represented by the mean ordinate to the curve of power, i.e. an ordinate o *d* (fig. 36) of such a height that when multiplied by the length o o′ the area of the rectangle so formed, o *d e* o′, is equal to the net work done, or the difference between the areas of positive and negative work. This mean ordinate will give the mean power developed in the circuit or portion of a circuit considered.

The whole of the above is applicable, not only to any portion of a circuit, but also to the circuit as a whole; in the case of an alternator supplying energy to the external circuit, its total rate of development of electrical energy is equal to the mean ordinate to a curve formed by multiplying together simultaneous values of its induced E.M.F. and current, and its output or rate of development of energy in the external circuit is equal to the mean ordinate of a curve formed by multiplying together simultaneous values of the E.M.F. impressed on the external circuit from its terminals and of the current flowing in that circuit. It must be remembered that the E.M.F. impressed on the external circuit need not be identical with the curve of resultant E.M.F. for the external circuit, and in fact will only be so if the external circuit be a simple ohmic resistance without inductance.

The determination of either the total power or the output of an alternating dynamo, as so far described, is necessarily tedious : it involves a knowledge of the exact curves of E.M.F. and current —neither of which are usually known—the multiplication of their simultaneous values, and the discovery of the mean ordinate to the curve so formed. A simpler process is desirable, provided that such a process is accurate enough for practical purposes. In the previous explanation of the behaviour of alternating currents, the effect of self-induction has been expressed in more or less general terms, independent of the exact type of the curves drawn and assumed, but in the discovery of the simpler process now required, the laws which govern the shape of these curves must be considered further. It has been said that if the full-line curve of fig. 32 is a sine curve, it follows, from the mathematical nature of a sine curve, that the curve derived from it and expressing its rate of change will also be a sine curve of the same periodic time, but differing by 90° in its phase. If, therefore, the current curve in any piece of alternating machinery is a sine curve, and if its inductance be strictly constant throughout the whole periodic time, and the lines of induction due to the current are strictly proportional to the current, and follow simultaneously upon its variations, the curve of self-induced lines (fig. 34) will be a sine curve ; consequently the dotted curve derived from it, which, when inverted, represents the self-induced E.M.F., will also be a sine curve of the same periodic time, but differing by 90° in its phase. Further, if two sine curves be combined together, as was done on p. 93, the third curve so deduced is a sine curve of the same periodic time, but differing in phase from either of the two component curves, and therefore the curve of impressed E.M.F. (in fig. 35) will be such a curve. Fig. 37 shows the combination of two sine curves to form a third, and it is, therefore, the counterpart of fig. 35 on the supposition that the curves of current and E.M.F. are sine curves. From a comparison of the two figures it will be seen that the chief difference is that in fig. 37 there is a definite angle of lag (18° in the diagram) between the current and impressed E.M.F. curves. Not only does the former pass through its zero at a certain angle behind the zero of the impressed E.M.F., but it passes through its maximum with the same angle of lag. Further, the maximum ordinate of the current curve coincides with its point of intersection with the impressed E.M.F., and lastly the maximum value of the resultant E.M.F. is necessarily less than the maximum ordinate of the impressed E.M.F. The effect, therefore, of constant inductance in any portion of a circuit traversed by an alternating current is to shift the entire current curve backwards by a constant angle of lag, and to reduce the height of all the ordinates of the resultant E.M.F. curve as compared with those of the impressed E.M.F. curve. We thus obtain three sine curves, representing respectively impressed E.M.F., self-induced E.M.F., and resultant E.M.F. or current, mutually related to one another by simple mathematical laws, so that from any two the third necessarily follows, and from the mathematical nature of sine curves we can go on to deduce other facts respecting the three related factors. Now, the assumption that in any given alternator the three curves

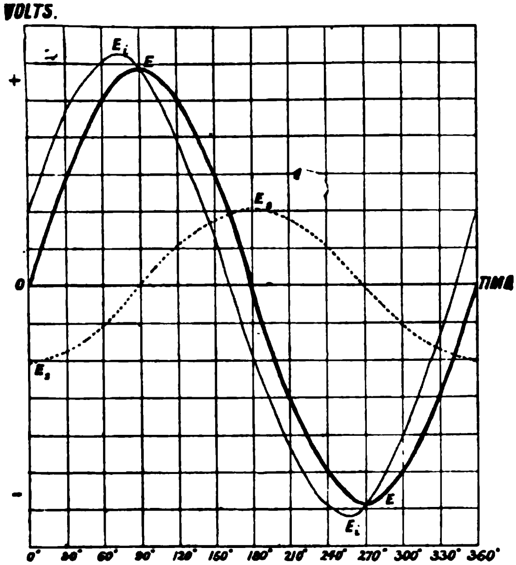


Fig. 37.

are sine curves is not by any means necessarily true, but experience has shown that in modern machines no great error arises in theory or practice from assuming that they *are* sine curves. The curves of impressed E.M.F. of modern alternators, so far as they have been experimentally determined, do closely resemble sine curves in their general nature, and so also but to a less degree do the curves of current under their normal conditions of working. Consequently we may assume that both approximate sufficiently nearly to sine curves to enable us to make use of the mathematical nature of sine curves in order to simplify any given problem and to supply a basis on which to treat it graphically.

Now, if we have two simple sine curves of equal period, but of different amplitude and phase, and simultaneous values of the ordinates of the two curves be multiplied together, it can be shown mathematically that the mean of all the products so obtained is equal to half the product of the maximum values of the two sets of ordinates multiplied by the cosine of the angle which expresses their difference of phase. Hence in fig. 36, if the two curves of E.M.F. and current be sine curves, the mean ordinate to the power curve can be thus directly obtained; if e and i be the impressed E.M.F. and alternating current at any instant, and E and I be their maximum values, the mean value of ei or the mean rate of expenditure of energy on the circuit during a complete period or any number of complete periods is $\frac{EI}{2}\cos\phi$, where ϕ is the angle of lag or difference of phase between the impressed E.M.F. and current. If, therefore, I_e and E_e be the maximum values of the current flowing in the external circuit of an alternator and of the E.M.F. impressed on the external circuit from its terminals, its output is $\frac{E_e \cdot I_e}{2}\cos\phi$.

But how are we to measure E_e and I_e, or indeed any alternating E.M.F. and current, the values of which are continually varying? Given a current fluctuating in value and flowing through a fixed resistance of R ohms, the rate at which electrical energy is transformed into heat over the resistance is at any instant equal to the product of the ohmic resistance and the square of the current strength at that instant. If, therefore, being given the curve (I) of an alternating current (fig. 38) during one complete period, we square the ordinates and plot these squares as a second curve on the same horizontal axis, the total area enclosed by this second thick-line curve o I^2 o′ is proportional to the total heat developed in the resistance in one period. But there is some one ordinate to this second curve whose height multiplied by the horizontal line o o′ gives a rectangle equal in area to the total area of the curve o I^2 o′. This mean ordinate or mean square is equal to the square of a certain current which would produce the same heating effect if it flowed for the same time through the resistance with unchanging strength. Consequently the square root of the mean square is the value in ampères of that steady current whose rate of production of heat in a given resistance is equal to the mean rate at which heat is actually developed by the alternating current in passing through the same resistance; and this square root of the mean square is called the 'effective current,' since, as regards heating, the alternating current is equal in effect to a steady current of that number of ampères. Now if the first curve o I o′ be a sine curve, as in fact is shown in fig. 38, it can be proved mathematically that the mean of the values of all its ordinates squared, i.e. the mean ordinate to the second curve o I^2o′, is half

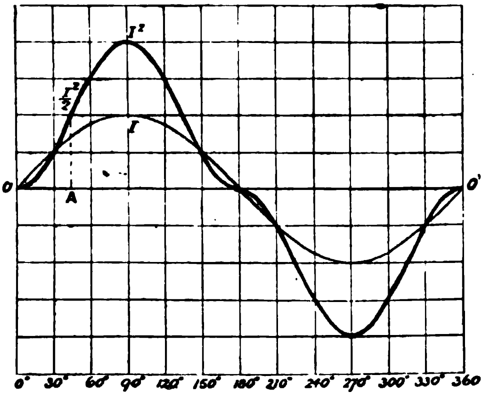


FIG. 38.

the square of the maximum value of the ordinates of the first curve, so that if I be the maximum value of the current, the mean square is $\frac{\text{I}^2}{2}$, and the square root of the mean square, or the effective current, is $\frac{\text{I}}{\sqrt{2}}$.

In a Cardew or other hot-wire voltmeter the heating effect of a current passed through it causes the needle to take up a definite position, which indirectly measures the rate of development of heat in the instrument. If, therefore, the current passed through the instrument is alternating, the position taken up by the needle measures the square root of the mean square of the current strengths, or the effective current. But if the wire through which the current flows has inductance so small as to be negligible—and this is in fact the case—the E.M.F. impressed on the wire from the terminals of the instrument is at any instant proportional to the current strength, so that the curve of impressed E.M.F. is identical in its law and phase with the current curve. Hence the position taken up by the needle of the voltmeter not only determines the square root of the mean square of the current, but also the square root of the mean square of the E.M.F. impressed upon the instrument. This square root of the mean square of the E.M.F is called the 'effective E.M.F.,' since it is that value of a steady, uni-directed E.M.F. which would when applied to a given resistance produce the same amount of heating as the alternating E.M.F. produces. Now if the Cardew voltmeter be applied to the terminals of an alternator supplying current to an external circuit, the E.M.F. impressed upon the voltmeter is also the E.M.F. impressed upon the external circuit; the reading of the needle gives us the number of volts which is equal to the square root of the mean square of the alternating E.M.F. impressed upon it, and if the curve of the impressed E.M.F. be a sine curve (as is usually assumed) we know that this is equal to $\frac{1}{\sqrt{2}}$ of the maximum value of the periodic impressed E.M.F. We have thus found $\frac{\text{E}_e}{\sqrt{2}}$, which is one factor in the expression for the output of the alternator. Other instruments are adapted for the measurement of the effective voltage besides the Cardew voltmeter, but all alike measure the square root of the mean square. In a similar way the effective current can be measured by a dynamometer or ammeter suitable for alternating currents, either instrument being placed in series with the external circuit. Its value on the sine-curve assumption is equal to $\frac{I_e}{\sqrt{2}}$, and the product of the two, effective E.M.F. and effective current, gives us $\frac{E_e I_e}{2}$; but to obtain the actual output of the machine it still remains to multiply this product by the cosine of the angle of lag. In actual working when the alternator is lighting lamps directly or is connected to fully loaded transformers the angle of lag between the external current and the E.M.F. impressed upon it is very small, and may in most cases be neglected, the output being therefore simply equal to the product of the effective E.M.F. and effective current of the external circuit or half the product of the maximum values of the impressed E.M.F. and current. In fig. 36 a very small amount of negative work has been shown, and the output of an alternator, to the external circuit of which the curves there shown apply, would be 26 kilowatts; a value not much less than would be obtained on the assumption that the curves were sinoidal, and that there was no lag of external current behind external E.M.F., i.e.—

$$\left(\frac{3100}{\sqrt{2}} = 2190\right) \text{volts} \times \left(\frac{18}{\sqrt{2}} = 12{\cdot}7\right) \text{ampères} = 27\tfrac{3}{4} \text{ kilowatts.}$$

CHAPTER VII

CLASSIFICATION OF DYNAMOS

THE variety of different forms which the dynamo may take is well-nigh endless, yet in all there may be traced the two structural portions which the dual nature of the machine requires. It will be advantageous here to repeat and enlarge our first description of these two portions.

There is, first, the 'field-magnet,' the function of which is simply to serve as the path for the lines of induction constituting the field, and which is therefore composed entirely of iron. It may usually be divided into three parts, which can be distinguished in fig. 2, viz. (1) the magnet 'cores' or limbs (*d d*), on which the exciting coils (*e e*) are wound or placed, and which lead the lines up to or away from the pole-pieces; (2) the 'yoke' (*y*) which joins together the limbs of the magnet; and (3) the pole-pieces themselves (N, S). These latter by reason of their particular shape cause the lines to issue forth into and pass through the air-gap in such directions and with such intensity as will best adapt them to be cut by the inductors.

Secondly, there is the 'armature' (*a*), which consists of a number of inductors, almost invariably of copper, which are systematically arranged and connected together in a definite and particular order; in the dynamo of fig. 2 they form portions of a large number of loops or turns of wire, each of which is entirely insulated from the structure upon which they are wound, and also from every other turn, save where the end of one loop is electrically joined to the beginning of the next. In the inductors, by reason of their movement relatively to the field, E.M.F.'s are generated, and by their grouping are added together in series or placed in parallel, or in general brought into practical use at certain definite points or 'terminals' (A D) to which the external circuit is applied. When we leave the sphere of theoretical diagrams such as those in Chapter IV. it is evident that the inductors must be supported in some way, and in most cases they are in practice arranged or wound upon a mass of iron, which itself forms part of the magnetic circuit, and is traversed by the lines of induction; it is this structure of iron which in strictness of language is the armature or 'keeper' of the magnet or magnets employed to produce the field. Thus, in the ordinary two-pole dynamo the cylindrical armature of iron, *a* (fig. 2), may be likened to the 'keeper' placed between the two poles of a simple horseshoe magnet; but from its intimate connection with the electrical conductors which it supports, the term has now been transferred to the system of inductors, the iron portion itself being called the 'core' of the armature. Finally, the central support on which the conductors are wound or laid is now termed the 'armature-core,' whether it be made of iron or porcelain or any other substance, magnetic or non-magnetic.

In the diagrams of Chapter IV. the principle of electromagnetic induction was mostly exhibited by means of a straight inductor drawn across or rotated in a magnetic field, but these simple experiments must now be elaborated into more practical forms. In the first place, it is evident that the process of fig. 13, by which a straight inductor is drawn across a field in a straight line, is essentially finite; the motion is simple rectilinear motion in one and the same direction, and this cannot practically be maintained for any very great space, for the simple reason that the pole-pieces would have to be indefinitely long in order to produce an indefinitely long magnetic field. But our definition of the dynamo demands 'continuous relative motion,' and hence, when the inductor has been moved to one end of the magnetic field, we are under the necessity of reversing its direction of movement. We thus pass to oscillatory or reciprocating motion, first in one direction, and then back again in the opposite direction. Such a motion would be obtained by attaching the inductor directly to the piston of a steam engine; it would give an E.M.F. alternating in direction, and if the field were of uniform density, the value of the E.M.F. would vary as the speed of movement, reaching a maximum during the middle of the stroke, and reversing at either end (compare A B, fig. 24). But such an arrangement is not convenient, since it is not easy to secure a speed high enough to give an appreciable number of volts with any practicable length of inductor and strength of field. No such difficulty meets us when we have recourse to the rotatory motion of fig. 24, and hence in all practical dynamos, without exception, it is by rotation that the 'continuous relative motion' of field and conductor is secured, and an E.M.F. continuously generated. Reverting to the case of a straight inductor rotated in an air-gap formed in a single horseshoe magnetic circuit by a pair of opposing pole-pieces, it will be seen from fig. 39 that there are two distinct ways in which the cutting process can be carried out. In case A, as the inductor shown in section at *a* is rotated round the axis *o* to *a*′, it moves downwards across, or cuts, all the lines passing from the N. to the S. pole, and by further rotation from *a*′ to *a* it cuts the same lines again, but in the opposite direction, i.e. upwards; consequently the E.M.F. induced in it is, during the first half of a revolution, in one direction along its length (downwards into the plane of the paper), and then during the second half in the opposite

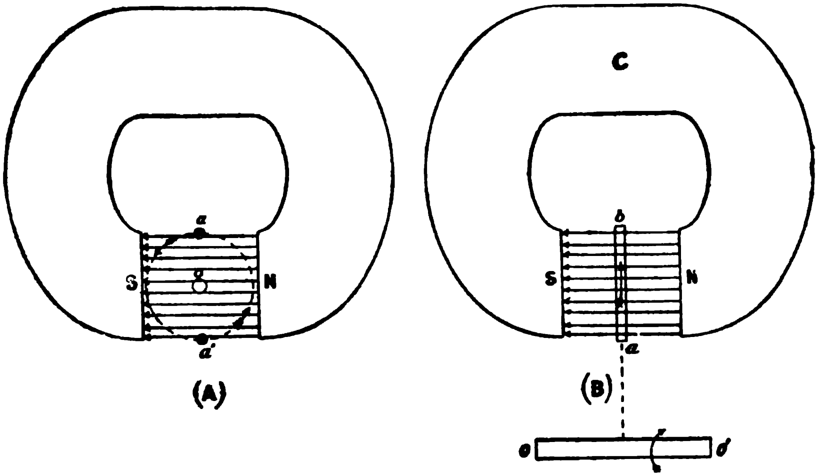


FIG. 39.

direction along its length (upwards out of the plane of the paper); in other words, it is the seat of an alternating E.M.F. whose direction undergoes reversal twice every revolution. But in case B, if the conductor is rigidly attached to the shaft *o o'*, and is thus continuously swung round, when an E.M.F. is induced in it by its passage past the poles, this E.M.F. is always in the same direction along its length (in the diagram from *a* to *b* or radially outwards).

Another most important distinction between the two cases of fig. 39 is connected with the position of the length of the inductor relatively to the axis round which it is rotated. In case A the length of the inductor is parallel to the axis of rotation; in case B it is at right angles to the axis of rotation, and from this difference of position great differences of structure arise.

Now the classification of dynamos is a matter of considerable difficulty, and several bases of division present themselves, each of which has its advantages in throwing light upon the affinity of one machine to another. It might be thought that the distinction between dynamos yielding an E.M.F. and current always in the same direction round the external circuit, and those yielding an alternating E.M.F. and current in the external circuit would be sufficiently sharp; yet on further examination it will be found that of two so-called 'continuous-current' dynamos giving a current unidirected in the external circuit, the one may be entirely distinct from the other in its whole nature and structure. As a matter of fact, dynamos which primarily and in themselves give a current continuously in the same direction are few and comparatively unimportant—at least up to the present time. Again, amid the bewildering variety of types of armatures and field magnets certain fundamental differences in structure may be seized upon, which help to characterise the machine of which they form a part; but any classification based on such characteristics alone will not suffice to exhaust the chief differences which distinguish one dynamo from another.

Two questions have, however, been suggested by fig. 39, which must be asked and answered when considering any form of dynamo machine; they are as follows :—

(1) Is the E.M.F. produced in each inductor as it rotates always in the same direction along its length, or does it alternate

by reason of its cutting the same lines twice over in each revolution in alternate directions?

(2) Is the length of the inductors parallel or at right angles to the axis of rotation?

If the above two criteria be followed up, it will be found that any dynamo under consideration can be placed in its proper position and rank. In the following pages it is therefore proposed to make use of three bases of division, applying each in turn so far as it is helpful to the due understanding of the different types of dynamos: the first of these is the question whether each inductor is the seat of an alternating E.M.F. or not; the second is the position of the length of the inductor, and the direction of the field with reference to the axis of rotation (for by this we may to a large extent determine other points, such as the systems on which the inductors must be connected together, the form of field-magnet required, and even the structure of the armature core), while third and last is the question whether the external E.M.F. and current are alternating or uni-directed. In any dynamo either the field or the inductors may be rotated, or both in opposite directions; it is merely a matter of convenience to be decided by the relative adaptability of field magnet or armature to withstand the centrifugal force and the various strains due to rotation.

At the outset, therefore, dynamos may be divided into Class I., those in which each inductor cuts the same lines twice over, in opposite directions, in each revolution, and by reason of its so cutting the lines has an alternating E.M.F. induced in it, henceforth called 'bi-' or 'multi-polar' dynamos; and Class II., those in which each inductor, when cutting lines, is always cutting them in the same direction, and therefore the E.M.F. induced in it is always in the same direction along its length, or 'unipolar' dynamos.

The above division so largely corresponds to the somewhat inaccurate and often misunderstood distinction between 'bi-' or 'multi-polar' dynamos on the one hand, and 'unipolar' dynamos on the other, that these terms are retained, but with the distinct and special signification above attached to them. In the ordinary two-pole dynamo of fig. 2 a definite group of lines existing between a pair of poles is cut twice by each inductor in each revolution, as in fig. 39 A: it is not merely that the same *number* of lines is twice cut, first in one direction, and then in the opposite direction, but the *same actual lines* are so cut. Similarly, in the multipolar dynamo properly so called, in which there are several magnetic circuits or groups of lines, the same fact holds good with respect to each pair of poles, viz. that each set of lines belonging to a distinct magnetic circuit is cut twice by each inductor in opposite directions in each revolution. In the so-called 'unipolar' dynamos there is a single field just as in the bipolar dynamo, and it must from the nature of lines of magnetic force exist between a pair of poles; but this single field is only cut once in each revolution, and the cutting always takes place in the same direction, so that there is no reversal of the direction of the E.M.F. induced along the length of the inductor: thus, the B arrangement of fig. 39 would form a unipolar dynamo. Now in Class I., even with a single field, as A, fig. 39, the action may be divided into two portions, separate in time, viz. a cutting of the lines as they emerge from the N. pole, and a second cutting of them, in an opposite direction, as they enter into the S. pole. The distinction here made between lines as they emerge from a N. pole, and the same lines as they enter into a S. pole, is of course unreal, but the action is dual and connected with the presence of a pair of poles by which the field is confined and concentrated within a definite space; consequently the dynamo is called 'bipolar.' In the unipolar machine, since the lines are always cut in the same direction, it might be said that they are cut as they emerge from a N. pole, or as they enter into a S. pole, and thus the necessary presence somewhere of the other pole might be left out of sight, the need of this pole not being at once apparent, except in theory, since it is not wanted to concentrate and collect the lines in order for them to be cut a second time in the same revolution. Such a view, however inaccurate, would be represented diagrammatically by fig. 40, where N is the N. pole of a bar magnet, the other pole of which is not in view (compare with B, fig. 39).

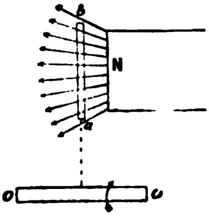


FIG. 40.—Conventional unipolar dynamo.

Having premised thus much, we pass to an examination of dynamos of Class I., and in the first place (A), those in which the inductor's length is parallel to the axis of rotation.

Through the middle of a rectangular gap formed in a simple horseshoe magnet by two pole-pieces having plane faces, let a shaft be placed which is capable of rotation, and at some distance radially from it let there be a wire, *a d*, supported at either end in a position parallel to the axis of rotation. Then, when the shaft is rotated, *a d* will cut the lines of the field and act as an inductor of an alternating E.M.F. (fig. 41). If the ends of the inductor are electrically connected with two collecting rings fixed on the shaft and insulated from it, against which springs, *b b*, are pressed so as to form a rubbing contact, we have an alternator, yielding an alternating difference of potential at the two stationary rubbers or brushes to which an external circuit, R_e, can be applied. Assuming for the present that the field swept through by the inductor is of uniform density, and that all the lines pass straight across from one pole to the other, the curve of instantaneous induced E.M.F., if plotted for one revolution forming a complete period, would be a sine curve, as in fig. 25.

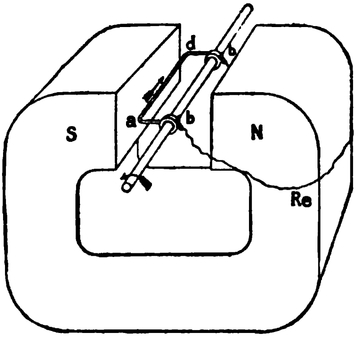


FIG. 41.

But the air-gap in which the magnetic field that is to be cut exists should be as short as possible in the direction of the lines, in order that the ampère-turns of exciting power required to produce the field may not be very large; for economy in the exciting power means economy either in the first cost of the machine or in its cost of working, or both. The first step would obviously be to shape the inner surface of the pole-pieces so that they follow the curve traced out by the rotating inductor, that is, instead of having pole-pieces with parallel faces, to bore out a cylindrical gap within which the inductor may rotate, allowing only such room or 'clearance' as is required for mechanical safety and durable working (fig. 42). But this will not entirely have the desired effect of strengthening the field cut by the inductor, inasmuch as it will still remain weak except at the edges, *a b*, *c d*, and there the field will be most dense outside the limits of the area swept through by the rotating inductor; lines will, in fact, leap across in considerable numbers from edge to edge where the distance is least, being lost as far as any useful purpose is concerned, and therefore to be reckoned as mere 'leakage.' A complete remedy is, however, found if, instead of removing the whole portion of the magnetic circuit which is bored out, an iron cylinder is retained on the surface of which the inductor can be placed (fig. 43). The exciting power required to produce a given number of useful lines within the sweep of the inductor will be immensely reduced, since the path of the lines is now more nearly a closed magnetic circuit of iron, like a horseshoe magnet, with its armature or 'keeper' between its poles; two short air-gaps are alone left, instead of one of great length, and the density under the pole-pieces will far exceed that of any straying lines between the pole-tips. Besides these two functions, viz. of reducing the exciting ampère-turns to a minimum for a given number of lines by shortening the length of the air-path, and of concentrating and guiding the lines in the required direction, the cylinder will also serve as a supporting structure on which the inductor and its connections can be secured. When thus used as a support for the inductors the iron 'core' needs to be rotated; although on any other ground it is entirely unnecessary. The problem of rotating the inductors round a stationary iron core has possessed great attractions for inventors; the difficulties inherent to it have, however, proved too great, and

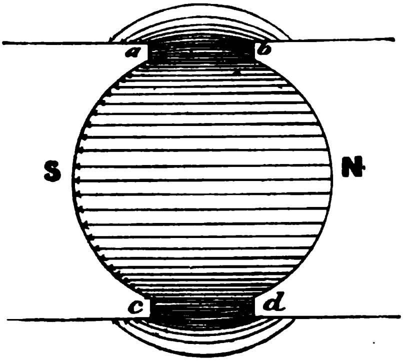


FIG. 42.

the mechanical simplicity of rotating both core and winding has led to its universal adoption. Apart from certain secondary and minor effects which need not here be considered, the rotating of the iron has no effect on the field of lines; these latter remain stationary, passing into and through the core just as if it were at rest. By the insertion of the iron core, the distribution of the lines in the field as cut by the inductor is essentially altered; they now enter the core much more radially, and are distributed over the arc embraced by the pole-piece almost uniformly, the result being a considerable modification in the shape of the curve of instantaneous E.M.F. If their distribution were perfectly uniform, and their direction at all points exactly radial, it would consist of a series of detached rectangles as in fig. 43 (contrast fig. 24); but in actual practice the corners are rounded off by the existence of a weak field or 'fringe' at the edge of the pole-piece, which is not truly radial, and the curve of induced E.M.F. for one period or complete revolution is intermediate between a pair of sine curves and a pair of rectangles (fig. 44). If B_g is the induction or density per square centimetre of C.G.S. lines passing radially through any part of the gap between the iron of the pole-piece

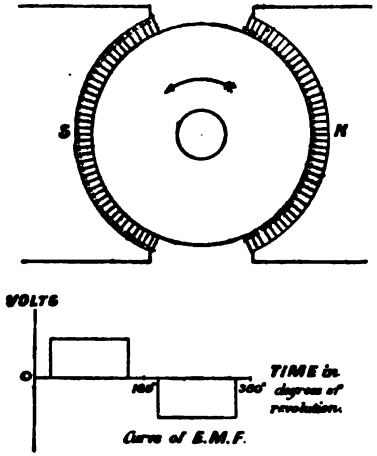


FIG. 43.

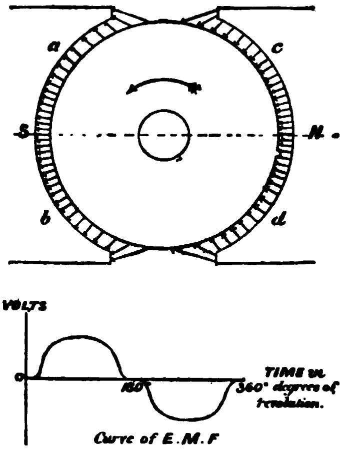


FIG. 44.

and the iron of the armature core, the E.M.F. induced in an inductor whose length is L centimetres, when traversing that part of the gap with a velocity of V centimetres per second, is, by our equation I*a* of p. 53—

$$\text{E (volts)} = \text{B}_g \, \text{L} \, \text{V} \times 10^{-8}.$$

But the E.M.F. generated by one single inductor of reasonable length thus rotating in a field as dense as it is possible to produce practically, and at as high a speed as is practicable, is very small, not amounting to more than a few volts at the most; hence it is usual in most commercial dynamos to have several such inductors and to add up their E.M.F.'s by connecting them in series.

Thus, in order to generate a considerable E.M.F., a number of inductors have to be rotated in an interferric space, which must be as small as is consistent with mechanical safety, and which should be approximately of uniform length at all points of a revolution. Evidently, therefore, in any dynamo, both the polar surfaces which the lines leave and enter, and the moving system of inductors must, as above, in some way form portions of an approximately cylindrical or spherical structure whose axis coincides with the axis of rotation; and according to the position of the length of the inductor relatively to this cylindrical structure the direction of the magnetic field with reference to the axis of rotation will be fixed. In our first type of dynamo, imagine a number of inductors arranged symmetrically round the same armature core (fig. 45) parallel to our original inductor, which was placed on the surface of a cylinder along its length, the direction of the lines being perpendicular to the axis of rotation. Then it will be seen that, with either direction of rotation, at any moment each of the inductors moving under the N. pole has an E.M.F. induced along its length, in the opposite direction (as viewed by an observer at either end) from the E.M.F. induced in each of the inductors moving under the S. pole; *e.g.*, with a counter-clockwise direction of rotation as shown, the rule of the hand tells us that the E.M.F. will be directed away from the

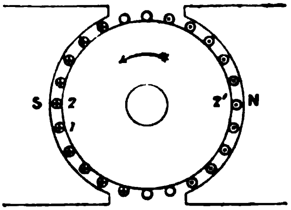


FIG. 45.—Inductors parallel to axis of rotation.

observer in all the inductors (crossed) under the pole-piece to the left of the neutral diameter, and towards the observer in the inductors (marked with a dot) under the pole-piece to the right. This fundamental fact must be the guide to any method by which the inductors can be connected together electrically in a useful manner.

Two methods of adding together the inductive action of a pair or more of inductors at once present themselves: by the first, inductor 1 of fig. 45 is connected in series with another inductor, 2, which is next to it on the surface of the core, and under the same pole-piece; by the second it is connected with another inductor, such as 2′, which is situated nearly diametrically opposite on the other side of the core.

(1) The first or 'Ring' method was employed by Pacinotti in 1865, but is also frequently called the 'Gramme' winding, from the French electrician Gramme, who reintroduced it in 1870. If the iron core, which served to guide the lines through the space between the pole-pieces, be supported at some distance radially from the shaft by means of an open hub or spider of some non-magnetic metal, such as gun-metal, a connecting piece of wire can be brought through the inside space between the core and the shaft, passing between the arms of the hub; by it the near end of inductor 1 (fig. 45) can be connected to the further end of 2, the inductor next to it on the surface of the core, and the whole arrangement is shown in the ring-wound armature of fig. 46, i. Without the iron core the lines of induction would go straight across the gap from pole to pole; the inner connector would therefore be cutting lines as well as, and in the same direction as, the outer inductor, and as the result an E.M.F. would be produced in each. These two E.M.F.'s thus acting round the loop would oppose and tend to neutralise each other, so that the net E.M.F. would be merely the difference between the two. But with the iron core interposed, the lines take a more or less curved path through the mass of the iron, as shown by dotted lines in the end view of the ring armature; and if the core be supported by a non-magnetic hub, very few lines will leak across the internal central space; consequently, there is little or no E.M.F induced in the inner wire, which thus serves purely as a conducting connector to sum up the E.M.F.'s produced in the two external inductors,

and does not itself cut the lines of the field. If any E.M.F. is induced in the inner wire, it is simply harmful, but with proper construction it becomes so very small that it may be neglected. It will be seen that this method really amounts to threading the same wire through the central hole, so as to form a 'loop,' and then winding a second loop of the same wire continuous and side by side with the first. We thus form a coil of two loops, containing two active inductors on the outside of the core, whose E.M.F.'s

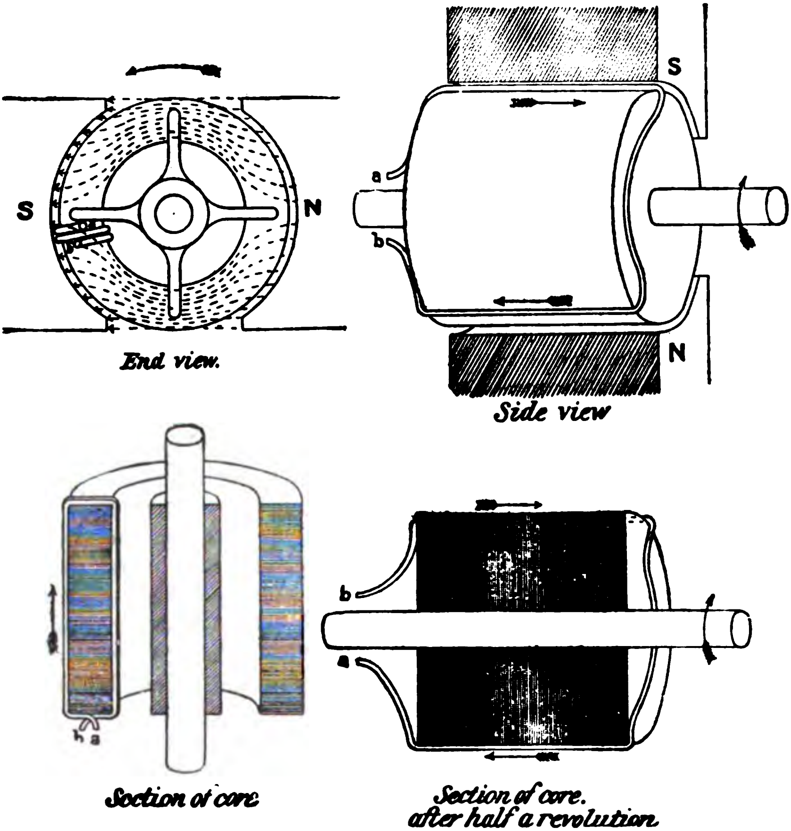


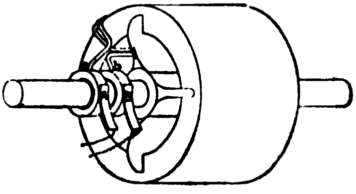
FIG. 46.—i. Ring-wound armature. ii. Drum-wound armature.
Two inductors in series.

are added together in series, and the core becomes a 'ring' overwound with a coil, whence the name for this winding arises. Further, it is evident that the process of winding the wire round the core may be continued so as to form more than two loops; for the present, however, we will confine ourselves to indicating the method by which two inductors can be joined in series by the formation of two loops on a ring core.

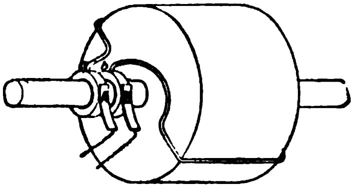
(2) The second or 'drum' method is identified with the name of 'Siemens.' In its complete form it was first introduced in 1873 by Von Hefner Alteneck as a modification of the original Siemens shuttle-wound armature. It is even simpler than the first method, and consists in joining the further end of one inductor by a connecting piece of wire passing across the end of the core to the further end of another inductor situated nearly diametrically opposite to the first inductor (fig. 46, ii). The E.M.F.'s induced in the two inductor wires will assist each other round the loop thus formed, being in opposite directions along their length as viewed from either end. The core now becomes, as it were, a cylindrical 'drum,' which may be solid right down to the shaft; as before, two inductors have been placed in series, and precisely the same amount of the surface of the armature core has been covered up and rendered useful; and, therefore, one loop of the drum winding is the exact equivalent of the two loops of the ring winding. To both the ring and the drum the iron core is essential, since without it almost all the lines would pass from the one pole to the adjacent edge of the other without being cut by the armature conductors at all; but it is doubly essential to the ring, in order that it may shield the internal conductors from any inductive action. By bringing the wire across the near end of the drum, i.e. by the addition of a second connector, the loops may be multiplied, the end of one loop forming the starting-point of the next.

If the curve of E.M.F. induced by either the ring or drum arrangement of fig. 46 be plotted, then with the same field and speed of rotation the ordinates will have twice the height that they have in fig. 44; and if the free ends of the loops *a b* be connected to two insulated collecting rings mounted on the shaft, we have the simple bipolar 'ring' and 'drum' alternators of fig. 47. This figure also shows the curve of E.M.F. acting during one revolution at the brushes A and B, which lead to the external circuit, the angle through which the loops have rotated being reckoned from a position midway between the pole-pieces as a starting-point.

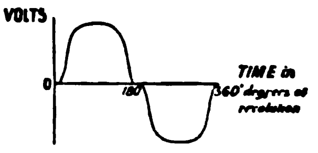
But for a variety of purposes an alternating E.M.F. is inconvenient or positively useless; yet, from the nature of the dynamos under Class I., the E.M.F. induced in each inductor is necessarily alternating, since it is produced by first cutting lines in one direction and then in the opposite, by continuous rotation, and, therefore, any current flowing under that E.M.F. must necessarily be alternating in the armature conductors themselves, whatever it may be in the external circuit. It remains to be inquired how the alternating E.M.F. of the armature loops can be 'commuted' or changed to an E.M.F. always acting in one direction in the external circuit. Let us consider the loop or loops of fig. 47: when in the course of rotation they arrive at a central position under the two pole-pieces, then the E.M.F. is a maximum, and collecting ring *a* is, say, positive; now, when the loops have turned through 90°, or have reached a position in the gap between the poles, no E.M.F. is being generated, since no lines are being cut, and they have reached the position of reversal. Immediately after passing this point, ring *a* will be negative, ring *b* positive. What is required, therefore, in order for the E.M.F. at the brushes to act always in the same direction round the external circuit is



Ring wound armature
Two loops



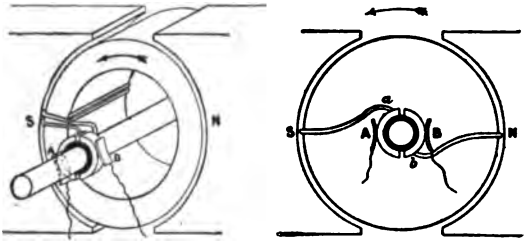
Drum wound armature
One loop



Curve of E.M.F

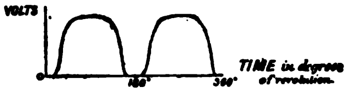
FIG. 47.—Alternator armatures with two inductors in series.

that the rings in contact with the springs or brushes should be automatically reversed at the instant of the reversal of the direction of the E.M.F. in the loop, so that *a* should now touch B, and *b* touch A. This is easily effected by making *a* and *b* each half of one and the same split ring, the two halves being separated from each other by air or other insulating material, and, further, by so setting the brushes that they pass over respectively from *a* to *b*, and from *b* to *a* at the instant of reversal (fig. 48). This device is the simplest form of 'commutator,' by which each brush always remains either positive or negative, as the case may be,



Ring-wound armature Side View

Drum wound armature. End View.



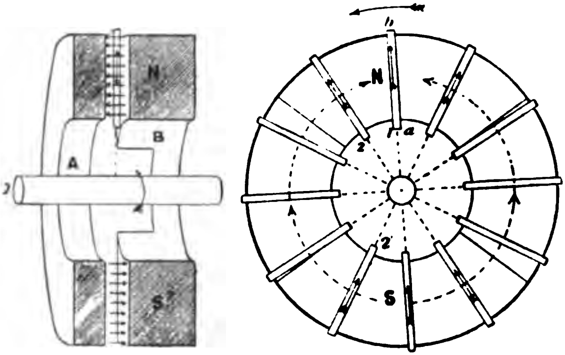
Curve of E.M.F.

FIG. 48.—Bipolar dynamos with two inductors in series giving a uni-directed E.M.F.

and therefore the current flowing in the external circuit is 'uni-directed,' although in the loops of the armature itself it is alternating in direction.

We have now obtained a ring and a drum uni-directed-current machine, and the curve of E.M.F. at the terminals or brushes will be entirely above the horizontal line (fig. 48). Given, therefore, two open ends of a loop or loops *a* and *b*, the same armature yields either an alternating or a uni-directed current, according as they are attached to a pair of collecting rings or to one split-ring commutator.

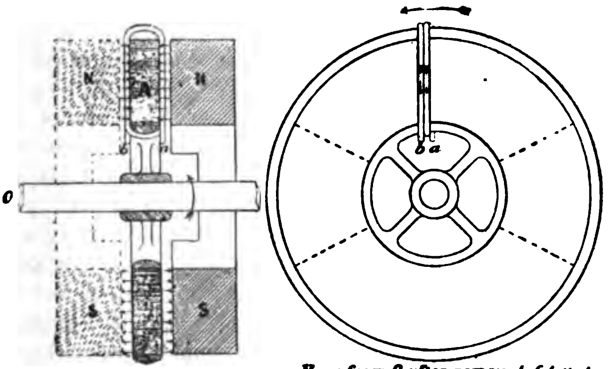
So far the rotating inductor has been one whose length is parallel to the axis of rotation ; we now pass to the second or (B) group of Class I., i.e. bipolar dynamos, in which the length of the inductors is at right angles to the axis of rotation, as in fig. 39, B. Now for a dynamo to belong to Class I. it is necessary that the inductor should cut the same lines twice in different directions, which cannot be done so long as the arrangement is that of fig. 39, B, and so long as reciprocating or oscillatory motion is impossible. Nothing, however, prevents us from breaking the magnetic circuit at another point, C (fig. 39), and bringing this second air-gap into the circular path of the inductor. In fig. 49 let B be a circular ring of iron with two pole-pieces, N and S, projecting from one side of it ; lines of induction entering at S will divide and pass round B in two groups by either branch of the magnet (as shown by dotted lines) and reunite in the N pole-piece, whence they emerge into the air-gap. Now, if we arrange a second iron ring, A, on the other side of the inductor, the lines will enter into it and pass in a similar manner round the two halves of A, and reunite to emerge opposite to the S pole-piece, thus completing their tour of the magnetic circuit. The inductor *a b* will now cut the same lines twice in each revolution in different directions at a different portion of their path, and the E.M.F. induced in it will be first radially outwards from the centre, and then inwards towards the centre, according as it cuts the upper or the lower group. With



View from O after removal of A.

FIG. 49.—Inductors at right angles to axis of rotation.

this arrangement it is easy to make the air-gap short by bringing the two portions of the magnetic circuit close together, leaving only sufficient room for the clear rotation of the inductor. But the same necessity exists, as before, of connecting two or more such inductors together in series in order to add up their E.M.F.'s. Since the direction of the magnetic field in the air-gap is parallel to the axis of rotation, the several inductors must be arranged somewhat as the spokes of a wheel radiating outwards from the centre, and the fields must be curved to a circular shape in order to follow the path of a line passing through the centre of the length of the inductors. In all the inductors under one pair of poles the E.M.F. will be directed radially outwards, in all those under the opposite pair, radially inwards, and this must be our guide in connecting them together. As in the case of Group A of Class I. two methods present themselves which are exactly analogous to the ring and drum windings: by the first, inductor 1 is joined in series with its next neighbour, 2; by the second it is joined to the inductor 2′, which is nearly opposite.



View from O after removal of dotted portion.

FIG. 50.—Discoidal-ring armature with two inductors in series.

(1) By the flat or discoidal-ring method a connecting piece of wire is passed round and through A, so as to join the outer end of *a b* with the inner end of the inductor by its side. The result is an arrangement (fig. 50) exactly analogous to the ring armature

of fig. 46, and the iron A, which now becomes an armature core, and is rotated with the inductors, serves exactly the same purposes, viz. guidance of the lines in the required path and mechanical support to the armature winding. The only change introduced has been the flattening out of the cylindrical ring core until its radial depth is greater than its length parallel to the axis of rotation, and it assumes a discoidal form ; the polar surfaces are then presented to the ends of the cylinder instead of to its periphery. But now a second magnet (shown dotted, fig. 50) can be presented to the other face of the flat-ring core, the poles of this second magnet being exactly opposite to those of the first and of the same sign. The lines will thus enter the core from both sides alike, and will pass in opposite directions round the armature, to leave it where the second set of poles is presented to its surface. By this means both sides of the winding become active, and the inactive connectors are reduced to the short lengths of wire at the top and bottom of the core. The whole arrangement may be evolved out of a ring armature, which, besides the external poles presented to its outer surface, has also a pair of internal poles arranged exactly opposite to the external poles, a N. facing a N., and a S. opposite to a S., so as to render the internal portion of the connecting wire active (fig. 51). The entire loop of the flat-ring machine, although containing two inductive portions, should therefore be regarded as equivalent to a single loop of the ring, the lines of one field being simply divided into two portions, one on either side of the core.

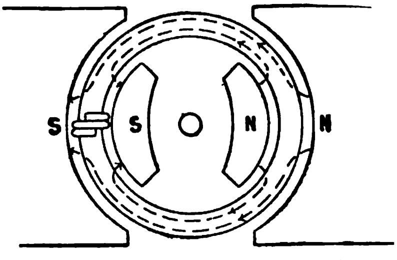


FIG. 51.—Ring armature with external and internal poles.

By bringing the ends of the winding to a pair of collecting rings, similar to those of fig. 47, the dynamo becomes a discoidal-ring alternator.

(2) The second, or true 'disc' method of connecting two radial inductors in series will be to join the outer end of one

inductor by a connecting piece passing round the periphery to the outer end of another inductor, situated nearly diametrically opposite, under the second pair of poles (fig. 52), the loop being completed, if required, by a second connector, joining the inner ends of two inductors, so as to render a series of continuous loops possible. The iron portion, A, of fig. 49 now forms no portion of the armature, and remains stationary : it simply serves to complete the magnetic circuit, which is cut at two points, and is exactly analogous to B. Opposite poles are of opposite sign, and the lines pass straight across each air-gap from one pole-piece to the other. In this, which may be called the true disc machine, it will be seen that the inductors are actually in the central plane of rotation, whereas in the previous flat or discoidal-ring machine they are arranged in a plane to the side of and parallel with the central plane of rotation. The peculiarity of the true disc winding is that it is indifferent whether the loop be wound on an iron core or not. A supporting core may be necessary, but it need not be of iron so as to guide the lines in the right direction through the air-gap, since the two opposite poles can be brought close together, only allowing sufficient room for the coils to pass between; and hence, if an iron core is present, it can only fulfil its first function of reducing to a minimum the exciting power required to produce a

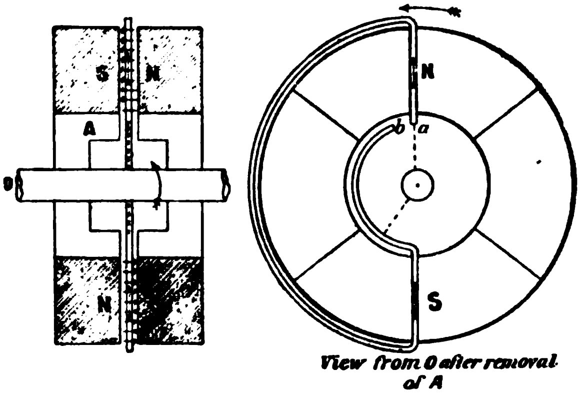


FIG. 52.—Disc armature with two inductors in series.

certain field. Just as the discoidal-ring armature can be evolved out of the simple-ring armature with two pairs of poles external and internal, so this second method is in reality drum winding adapted to a disc. If we replaced the core inside the drum winding with a pair of internal N. and S. poles, respectively facing the S. and N. poles of the field, and then imagine one end of the armature to be gradually enlarged, and the other reduced in size, it would take the form of a truncated cone, which would become flatter and flatter; finally, the winding would be turned into a flat disc, and the external and internal poles will be converted into two pairs of poles arranged on either side of the disc.

It would appear at first sight as if the disc bipolar machine had two fields, and therefore that it must of necessity have at least four poles, and so not be bipolar; but although this is to a certain extent true, yet there is no radical difference between the bipolar ring or drum and bipolar disc armatures, for in the former the opposite sides of the armature core are in reality S. and N. poles, since by them the lines enter and leave the core, and therefore, with equal truth, it may be said that there are two fields and four poles in the bipolar ring and drum dynamos. In reality they are all dynamos in which the inductors have E.M.F.'s generated in them, the direction of which along their length changes twice in each revolution, and by reason of that fact they may all be called 'bipolar.'

By the addition of a split-ring commutator, to which the ends of the loops of the discoidal and true disc winding are attached, we obtain continuous-current discoidal and disc machines.

The principles on which the dynamos of Class I. are based have now been diagrammatically exhibited in the case of bipolar dynamos; the further examination of their multipolar forms will be found to present no special difficulty, and will be postponed for subsequent treatment.

We now pass to dynamos of Class II., or those in which each inductor, when cutting lines of induction, is always, at any moment during the revolution, cutting them in the same direction, and further never cuts any line twice in one revolution; hence the E.M.F. generated in each inductor always has the same direction along its length, or is never reversed. As before, we may subdivide the class into the two groups of (A) those in which the inductors' active lengths are parallel to the axis of rotation, and (B) those in which they are at right angles to the axis of rotation.

Consider a single inductor rotated between two curved pole-pieces, as in fig. 53; the E.M.F. generated in these two cases will be always in the same direction along the inductor's length, and it will last so long as the inductor is moving through the field. Now if in both cases the pole-pieces are made tubular, there will be no break in the E.M.F., which will be maintained continuously at a constant value (fig. 54), and if the circuit be completed by joining the ends of the inductor by rubbing contacts to an external resistance, we have a unipolar dynamo giving a uni-directed E.M.F. and current, both externally and within the armature itself. In the second or B case, where the inductor's length is at right angles to the axis of rotation, it is simpler to mount the field magnet on the spindle as shown, thus making it the rotating portion, and allowing the armature to remain stationary; if the direction of rotation be the same as in previous cases, the E.M.F. in the inductor will be directed radially inwards.

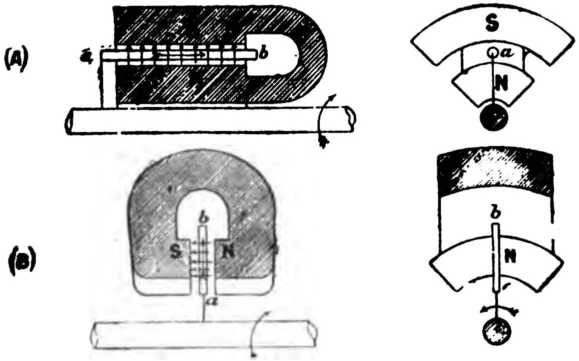
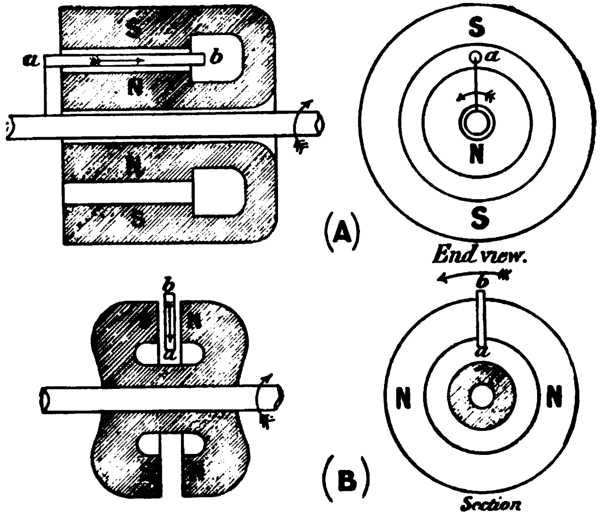


FIG. 53.—Unipolar dynamos.

Several such inductors can be arranged symmetrically round the axis of rotation, as in fig. 55, and these can be placed in parallel, so as to carry a larger current, but without any increase of E.M.F., until finally, when the whole available space is occupied, we arrive at a solid tubular inductor, or a solid circular disc of metal, which may be resolved mentally into a number of contiguous strips in electrical contact with each other; the action is the same as if there were only one inductor.

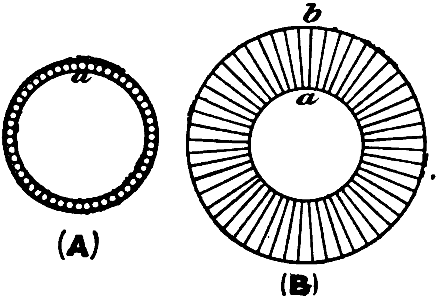
But, as stated before, the E.M.F. which can be thus produced is very small, and in order to increase this, two or more inductors must be connected in series. Let us consider the applicability of the previously-examined methods of the ring, drum, discoidal, and disc winding. In order to adapt the ring armature to this class of machine, it is evident that when an inductor on the external periphery of the core has cut all the lines as, *e.g.*, they enter the core on the right-hand side (fig. 56), the direction of E.M.F. will be reversed in it unless it avoids the lines as they issue from the core: this will be effected if the other pole on the opposite side of the core, instead of being external, as usual, be placed inside the ring, provided that the dimensions of the ring are such that the lines will not leak across inside in a direction unsuitable to our purpose. In fig. 56 the new internal position



Unipolar continuous-Current Dynamos.

Fig. 54.

of the second pole is shown, together with the new path of the lines of induction. The armature must now be supported from one end only, while the curved horse-shoe magnet, shown in section at M, bends round at the other end, one pole embracing the outside and the other projecting into the interior of the ring. But at once it is evident that the inner wires of the loops, which previously formed inactive connecting pieces coupling together the external inductors, will themselves become active inductors when the loops pass the inner pole-piece, and that they will be coupled together by the conductors on the outside of the ring. Hence the same conductors will serve first as inductors and then as connectors; the E.M.F. generated in each conductor will be always in the same direction along its length, but round the loops the E.M.F. will alternate. We have thus obtained a loop in which the E.M.F. alternates, and from it, by attaching the ends to a pair of collecting rings, a unipolar ring alternator.



(A) (B)

FIG. 55.—Unipolar continuous-current armatures.

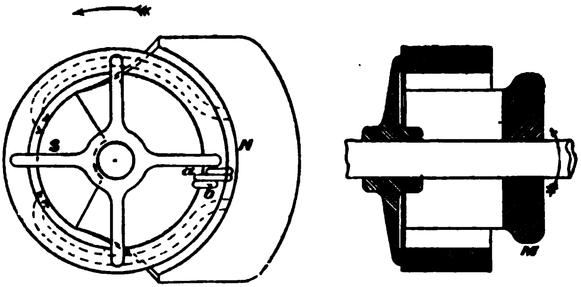


FIG. 56.—Ring unipolar dynamo with two inductors in series.

The drum winding is difficult of application to the present purpose, since it involves a moving joint in the magnetic circuit, the core forming itself one pole, and the lines issuing out of the end of the drum.

In the case of the discoidal-ring armature, with the loop wound round and through the core, the same idea can be applied as in the case of the simple ring armature ; for if the S. pole be shifted to the opposite side of the ring from the N. pole (fig. 57), opposite sides of the loop will alternately act as inductors, and the E.M.F. will alternate round the loop. The two poles in this arrangement must not overlap each other, and the lines must pass in a sloping direction across the core.

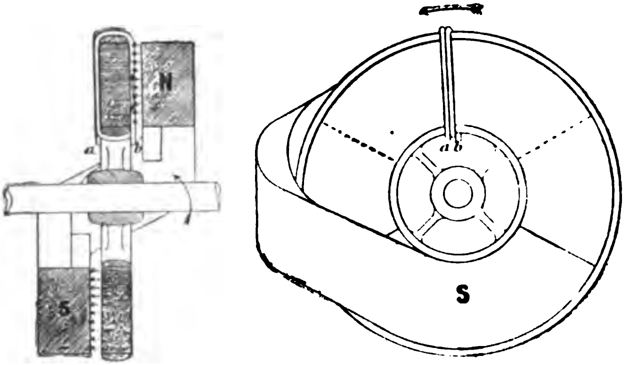


FIG. 57.—Discoidal-ring unipolar dynamo with two inductors in series.

In ring and discoidal unipolar alternators, therefore, the two poles are arranged either outside and inside the core, or one on either side of it ; hence we are not mechanically prevented from making each field extend over more than half the circumference of the mean path described by the rotating inductors ; but this is not electrically permissible any more than in the case of bipolar armatures, since were it done there would be no distinction always present between the wires which are inductors for the time being and those which are mere connectors. If this distinction is not maintained, and both sides of the loop are allowed to be moving under the pole-pieces simultaneously, the E.M.F.'s generated in the two sides act in opposition to each other, or, as it is termed, act 'differentially,' the net effective E.M.F. being simply the difference between the two. Any such differential action is therefore detrimental to the proper working of the machine.

The same holds good when we come to apply the disc method of forming a loop to unipolar armatures. If the field or fields be separated and clearly marked off by a neutral space or spaces, through which no lines of induction pass, then one side of each loop can act as inductor, and the other side as connector, provided that the width of the loop is at least equal to the width of the field, and also is not greater than the width of the neutral space. The three widths must be measured along the circular path traversed by the rotating magnet or loop, and therefore their actual dimensions vary according as the circle along the circumference of which they are measured be taken at the outside, middle, or inside of the inductor's length; it is, however, sufficient to consider only their widths as measured on the circle passing through the centre of the inductor's length, which gives the mean path traversed. The conditions stated above which govern the width of the loop are necessitated by the fact that the connector for the time being must be outside any field of lines in order to avoid differential action. For example, in fig. 58, which represents a unipolar disc alternator with rotating field magnet and stationary armature, while the leading edge moves from 1 to 3 no lines traverse *c d*, and therefore no E.M.F. is induced in it; but when by further rotation the leading edge moves from 3 to 1, *a b* does not in turn cut any lines itself, and consequently the E.M.F. is induced only in *c d*. Hence the E.M.F. is alternating in direction round the loop as one side or the other forms the active inductor, but in the conductor, which is for the time being cutting lines or acting as inductor, it is always in the same direction along its length. It is evident from the diagram that the field must not under any circumstances extend over more than half the circumference of the mean circle described by the inductor, the width of the loop being also equal to half the circumference, since otherwise the two sides of the loop will both be cutting lines in the same direction, and therefore acting differentially.

The distinctive feature of the ring, discoidal and disc loops, when applied to unipolar machines, is that they are a device for subsequently joining two active inductors in series ; they do not themselves do so. In this they resemble the loop of the ring bipolar armature, but may be distinguished by the fact that in the latter the connecting wires never become inductors, whereas in unipolar rings first one side of the loop and then the other is active. Thus, in the unipolar armature there is never more than one active inductor at any one time in each loop. Even in disc unipolar machines the loop does not itself join two inductors in series, though it forms a starting-point by which it can be done ; and hence the action of disc machines, however similar their winding in appearance, is entirely different according as they are bi- or uni-polar.

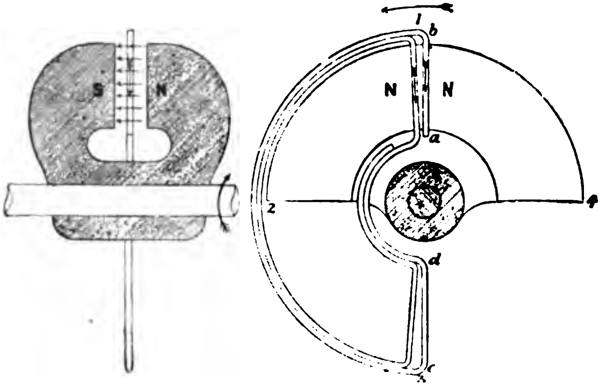


FIG. 58.—Disc unipolar dynamo with two inductors in series.

By connecting a split-ring commutator to the ends of the loops the unipolar machine can be reconverted into a dynamo giving a uni-directed E.M.F. and current in the external circuit.

CHAPTER VIII

BIPOLAR AND MULTIPOLAR ALTERNATORS

THE previous chapter has shown the methods by which, in the bipolar dynamo, two inductors are connected together in series, so as to form a pair of loops in the case of the ring and discoidal-ring armatures, or a single loop in the case of the drum and true disc armatures; the next step will be the formation of a 'coil,' or coils, of many such loops and their several modes of connection in order to still further increase the total E.M.F. of the machine.

From the nature of the loops described in Chapter VII. it is evident, as has been already pointed out, that they can be multiplied by simply winding more of the wire continuously on to the core after exactly the same fashion as the original loop, since the end of one loop will serve as the starting-point for the next. If these several loops are wound on the top of each other in the case of the ring or drum armatures, or by the side of each other in the case of the true disc, each loop will produce exactly the same E.M.F. as the first original loop, provided the strength of field be kept the same; but if the winding be thus piled up in two or more layers, the length of the air-gap is necessarily increased, and, therefore, with a given exciting power, the strength of field is greatly reduced. Hence it is advantageous to wind the loops side by side in ring and drum, or on the top of each other in disc armatures, so as to permit of the retention of the same air-gap as for a single loop.

Fig. 59 shows the loop thus wound for equivalent armatures of the ring, discoidal, drum, and disc types, in all of which four inductors are joined together in series. As before, the two free ends of the loops are connected to two collecting rings, one of which in the diagrams is shown of large diameter in order to be visible behind the other ; the whole then forms a bipolar alternator, from which current is taken by means of the brushes.

But when the loops are wound as shown in fig. 59, they have a certain width, and do not therefore all occupy identically the same position relatively to the polar surfaces at any point of a revolution ; consequently the E.M.F. induced in the different loops forming the coil need not at the same moment be always alike, either in direction or amount, and this renders it necessary to carefully consider the effect of the width of a coil.

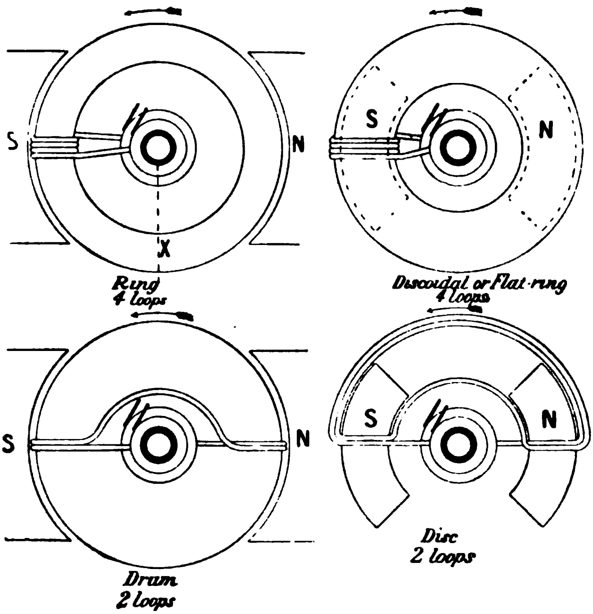


FIG. 59.—Bipolar armatures with four inductors in series.

Let us call a circle drawn round the external surface of the armature, and passing through the middle of the length of the inductors, the mean 'pitch-line,' the 'pitch' being the distance measured along this line between the centres of a pair of poles of opposite sign ; just as, in a toothed wheel, the pitch is the distance measured along the pitch-line from the centre of one tooth to the centre of the next. The pitch-line is thus the mean path traversed by the rotating inductors or fields, as explained on p. 129; *e.g.*, if the ring armature of fig. 59 be imagined to be cut across along the line marked X, and flattened out, as in fig. 60, A A is the pitch-line and *a b* the pitch.

In the following discussion the widths of each field of magnetic lines, of the neutral space between two fields, and of the coil, are all measured along the pitch-line, and are expressed in terms of the pitch as the unit of measurement.

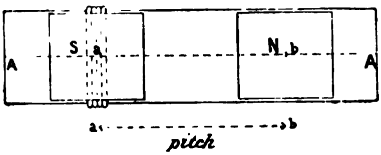


Fig. 60.—Development of armature showing pitch-line.

Take first the case of ring and discoidal-ring armatures. In all cases the E.M.F. generated in each inductor as it moves under a N. pole will be in the opposite direction along its length to that which is induced in it when, as its rotation is continued, it passes under a S. pole; and when loops are wound on to a ring, so as to form a continuous coil, and the E.M.F.'s generated in their active sides are to be added to each other, they must be in the same direction along the lengths of contiguous inductors. On this consideration, then, must be based the principle governing the width of a coil in ring and discoidal alternators, viz. that the loops of one and the same coil must never be moving at the same time under two poles of opposite sign, otherwise the E.M.F. generated in the portion of the coil which is moving under a N. pole will be in the opposite direction *round the coil* to that which is generated in the portion moving under a S. pole, and they will tend to neutralise each other; in other words, the two portions of the coil will act 'differentially,' the resultant E.M.F. being only the difference between the sums of the E.M.F.'s generated respectively in the two portions. The problem before us is the proper proportioning of the widths of field and coil, so as to make the best use of an armature of given dimensions running at a given speed.

First let us consider the effect of overwinding the entire core throughout with loops forming one continuous coil (fig. 61); in this extreme case it will be seen from the diagram that, whatever the width of the field, the single coil must always be under both poles at the same time, and consequently there will always be differential action between inductors on opposite sides of the core; further, the E.M.F. produced on either side being at any moment exactly equal, the resultant E.M.F. is nil. If, however, instead of forming one single coil, the winding be divided into two separate portions, as in the ring of fig. 62, where one of the two portions B is shown dotted, the action of B will be exactly similar to the action of the other portion, A, both in phase and intensity. It will suffice, therefore, in the first place, to consider the action of one half of the ring, A, apart from the other, B; having determined the proper width of coil A, we shall also have determined the proper width of coil B, and it will remain to be seen how the two coils, A and B, can afterwards be coupled together in order to produce a useful result.

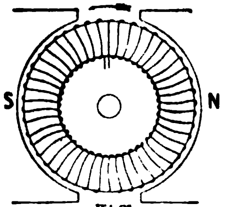
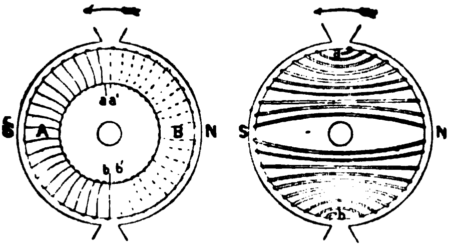


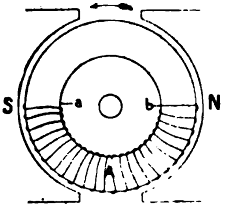
Fig.61



Ring Drum

Bipolar alternators with Maximum width of field & coils.

FIG. 62.



Position of greatest differential effect

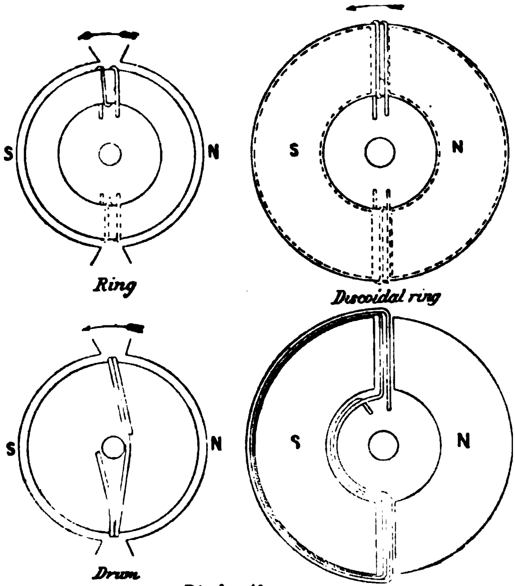
FIG. 63.

Starting, therefore, with coil A of width equal to the pitch, let us first make the width of each field equal to the pitch, as near as may be, neglecting the fact that there would be excessive leakage of the lines across from each edge of the poles into the opposite edges, where they come into close proximity with each other; this case is shown in fig. 62, and from the diagram it will be seen that there is no differential action between opposite ends of the coil, just at the moment when the coil is centrally situated under one pole. Immediately, however, that the coil A moves from the position shown in fig. 62 differential action is at once set up; the outside loops at end *b* will be cutting lines emerging from the N. pole, while those at end *a* are still cutting lines entering into the S. pole; and consequently the E.M.F. of the latter loops will be partly counterbalanced by the back E.M.F. set up in the former. When the coil is exactly midway between the poles, as in fig. 63, no E.M.F. at all will be obtained from the coil as a whole, since the outside loops at *b* are cutting lines issuing from the N. pole at the same rate as those at *a* are cutting lines entering into the S. pole, and therefore their E.M.F.'s are exactly equal and opposite. A little later, however, the loops at *b* will more than counterbalance those at *a*, until once again the coil is central under one pole, and the maximum E.M.F. is produced. Hence, when the coil is entering or leaving a field, all the loops are not equally active; and if the width of the field be equal to the pitch, there is only one moment when there is no differential action between the two ends of coil A (*cp.* fig. 185).

But an inspection of fig. 63 shows that the time during which there is differential action depends on the amount by which the width of the coil exceeds the width of the neutral gap between the fields, and will therefore be diminished if the coil be reduced to a width more nearly equal to the gap. By so doing, however, the number of inductors in the coil is necessarily reduced, until finally, with a width of field equal to the pitch, and no appreciable neutral gap, if the loops of the coil are never at any moment to be moving at the same time under two poles of opposite sign, the coil must have no appreciable width, but must, in fact, be concentrated approximately into a single line.

Fig. 64 shows a ring arrangement of this kind in which the width of the poles is nearly equal to the pitch, and differential action is entirely avoided by making the coil very narrow. When, however, this is so, there cannot at best be more than one or two loops, and therefore the total E.M.F. produced by the coil will be very low with any practicable density of field. Further, the greater portion of the available space round the armature core on which loops could be wound will not be utilised, although, when moving centrally under one or other pole, the coil might usefully be wider. The consequence, therefore, of adopting a maximum width of field is that a minimum width of coil is required in order to avoid any differential action.

The course followed so far has consisted in the reduction of the width of the coil, but, while still retaining a coil of width equal to the pitch, it is also open to us to reduce the width of the field. Again, the portion of a revolution during which there is differential action is diminished, but if there is to be no differential action the width of field must be reduced to a mere line, as indicated in fig. 65. Hence the retention of the maximum number of inductors necessitates a minimum width of field if differential action is to be avoided.



Ring *Discoidal ring*

Drum *Disc*

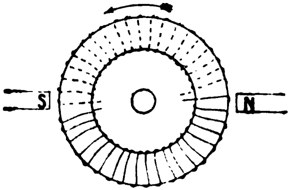
Bipolar Alternators with minimum width of coil & maximum width of field.

Fig. 64.

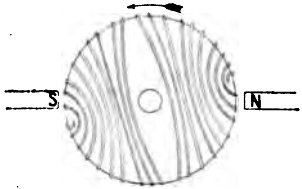
Both the above trains of argument, it will be seen, lead to the same conclusion, viz. that there will always be differential action until the width of the coil is reduced to not more than the width of the gap between the two fields measured along the pitch-line. We are thus led to a width of field less than the pitch, and a width of winding not greater than the gap between the two fields. How large the E.M.F. induced in the coil is with a given speed, strength of field, and number of turns will depend on the values given to the two ratios

$$\frac{\text{width of field}}{\text{pitch}} \text{ and } \frac{\text{width of winding}}{\text{pitch}}.$$

It will be observed that width of field has been spoken of throughout, and not width of pole, owing to the fact that there is a certain fringe of lines spreading out from the edges of the poles, which renders the width of field greater than the width of pole, and this difference must be taken into account in the theory and design of alternators. In modern alternators both field and coils are concentrated to different degrees; but the general rule to which most approximate may be considered to be width of winding equal to width of field, and both equal to half the pitch.



Ring



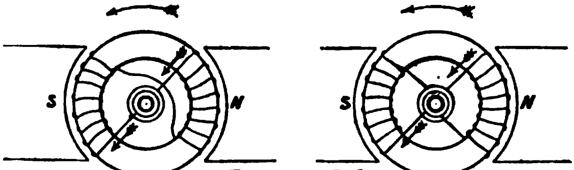
Drum

Bipolar alternators with maximum width of coil & minimum width of field..

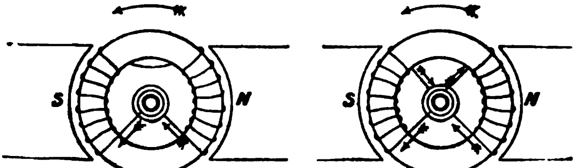
FIG. 65.

We have now, therefore, discussed the proportions which should exist between the pitch and a coil wound on one half of the armature ring in order that it may give its due E.M.F. But a second coil may be wound on the other half of the armature core, diametrically opposite to the first, since all that has been said about the first coil will hold good equally for the second; this latter will give a useful E.M.F. exactly similar to that of the first coil, both in phase and intensity; and, as has been hinted, it can be usefully connected with the first, either in series or in parallel. To do so, it must be remembered that if the E.M.F.'s generated in the inductors forming parts of the coil A (fig. 62) are up out of the plane of the paper, the E.M.F.'s generated in B will be down through the paper. Each coil may be wound right- or left-handedly;

just as a right-handed screw advances onwards when rotated right handedly, *i.e.* away from the end at which it is turned in a clockwise direction, so a right-handed loop or coil if imagined to be screwed right-handedly round its core would advance onwards. Thence it follows for ring-winding that if we follow any inductor above the horizontal diameter along the exterior of the core, *towards* our point of view, it will lead us on to another inductor further towards our *right* if the coil be *right*-handed or further to our left if the coil be left-handed ; *e.g.* in fig. 62 both coils are wound left-handedly. Hence, if the two coils are each composed



Series. *Left handed.* Parallel.



Right & left handed.
Series Parallel

FIG. 66. - Series and parallel connections for ring alternator.

of one or any uneven number of layers, and are to be joined in series, and (1) they are both wound in the same direction, the end of one coil must be taken across to the further end of the second coil (fig. 66), the other two ends being attached to the two collecting rings ; or (2) if they are wound in opposite directions the contiguous ends of the coils are joined together. If they are to be placed in parallel, the connections are exactly the reverse. Having regard to the rise of potential in each coil, and ease of connection, it will be seen, from fig. 66, that when the coils are in one layer, for series connection, a direction of winding alternately right- and left-handed is the more convenient arrangement; while for parallel connection a similar direction of winding in the two coils is the better. If the winding is to be disposed in two layers, it is most convenient to begin the winding of each coil at its centre by laying the middle part of the wire on the core, and thence winding up to either end of the first layer and back to the centre; thus both ends of the coil are brought to the centre and on the outside layer.

If both the coils are of width equal to half the pitch (the most usual proportion), as shown at the top of fig. 70, it will be seen that only half the total surface of the armature core is covered with winding, and this is grouped into two distinct coils. We can, in fact, already see the reason for the peculiar characteristic of the alternator which distinguishes it, not only in action, but even in appearance, from the dynamos to be described later, which give a uni-directed current. In the alternator the inductors are concentrated into coils sharply separated off from one another by intervening spaces of core on which there is no winding, and of these coils the bi- and multi-polar ring or discoidal alternators have as many as there are poles of opposite signs. Further, when entering or leaving a magnetic field, all the turns are not equally active, but only gradually come into play, the inductive effect of each inductor, as it enters the field, being added on to the E.M.F. produced in those inductors which are already cutting the lines of the pole under which they are moving. Owing to this action the E.M.F. of each coil as a whole rises gradually to a maximum when centrally situated under a pole, and then gradually falls to zero, to rise again to a maximum in the opposite direction. Consequently the curve of impressed E.M.F. for a whole coil during one complete revolution has an undulatory shape very similar to the sine curve of fig. 25; thus in fig. 67 each of the small curves represents the E.M.F. induced in one turn of the coil, and by the addition of these small curves the large curve is obtained which closely resembles a sine curve, although its component curves are much flatter and show no marked peak.

On passing to bipolar alternators whose armatures are wound on the drum or true disc method, the governing principle which determines the width of winding, viz. the avoidance as far as possible of differential action between portions of one and the same coil, is precisely the same as in the ring-wound armature; but the required conditions must be modified in order to suit the changed nature of the case; for in the drum and disc, as opposed to ring or discoidal, armatures, while the opposite sides of each loop must always be moving under poles of opposite sign, *one side* of a coil must never be moving at the same time under two poles of opposite sign, if there is to be no differential action. Starting with this rule, let us pursue the same course as in the case of ring armatures.

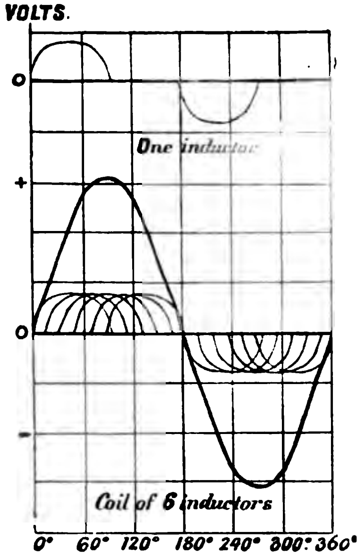
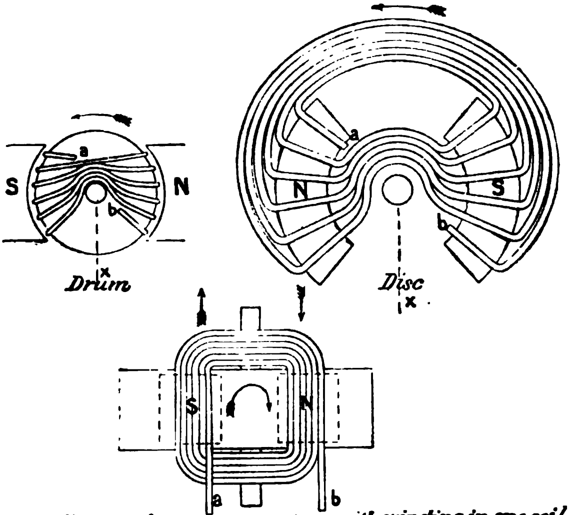


FIG. 67.—Combined E.M.F. of coil.

From the nature of the winding, the maximum width which a coil can have is twice the pitch, each side having a width equal to the pitch, as is shown in the diagram of a drum armature in fig. 62. Such an arrangement is the exact equivalent of the ring-wound armature shown at its side, wound with two coils each having a width equal to the pitch, and connected together in series; even when the width of field be equal to the pitch (as before neglecting the question of leakage) there is still a moment in each half-revolution, viz. that shown in fig. 62, when there is no differential action. At any other moment, however, there will be differential action, and the time during which such differential action lasts can only be reduced by reducing either the width of the winding on each side of the coil or the width of the field, or both. If there is never to be any differential action at all, and the width of field be retained at its maximum, the side of the coil must have no appreciable width, and the inner and outer loops of the coil must be taken off, until only one or two remain, and the width of the coil is equal to the pitch (fig. 64). If, on the other hand, the maximum number of inductors is still retained, the width of the field must be reduced to a mere line in order to avoid all differential action (fig. 65).

Again, therefore, we are led to a compromise between width of winding and width of field. Since opposite sides of each loop must never be moving under the same pole, the inner loop which has the minimum width must not be less in width than the field,



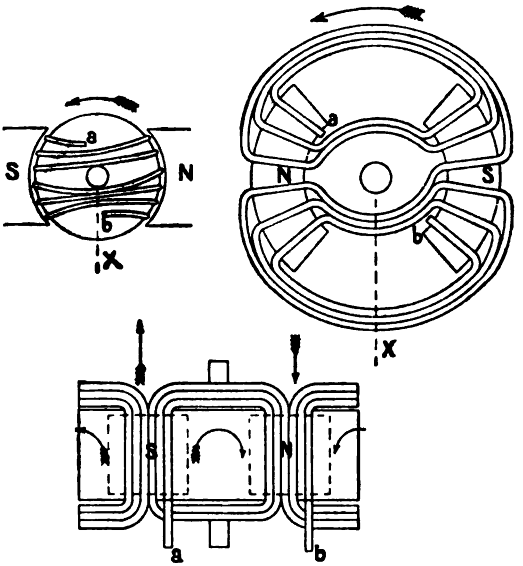
Bipolar alternator armatures with winding in one coil

FIG. 68.

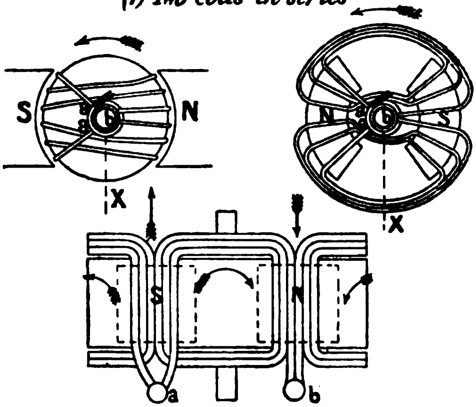
and, further, since one side of a coil must never be moving under two poles of opposite signs, the width of the winding in one side of a coil must not be more than the width of the gap between the two fields; only under these conditions, which are represented in the drum and disc armatures of fig. 68, will differential action be avoided. The second of the two conditions determines the permissible width of winding in the bipolar drum or disc alternator when wound with a single coil, as in fig. 68; but this single coil may equally well be regarded as divided into two separate coils, each giving an E.M.F. exactly similar in phase and intensity, and connected in series. If the drum or disc armature of fig. 68 be cut down to the spindle along the line x o and flattened out, then if the inactive end-connectors are entirely on one side of the spindle, as shown, the development of the flattened-out armature shows in plan one single coil containing two groups of inductors with E.M.F.'s in opposite directions, as viewed from either end (fig. 68). If, however, the single coil be divided into two halves by taking one half of the connectors round on either side of the shaft, as in fig. 69, the same two groups of inductors are divided into two separate coils in which the current circulates in opposite directions without in any way affecting the total E.M.F. When so divided the depth of the connecting pieces, which join together the inductors, is halved, and consequently space is economised, as is seen by comparing the bipolar drum and disc armatures of figs. 68 and 69; and this advantage becomes even more marked in the case of multipolar drum and disc alternators. Further, the two separate coils can now be connected in parallel equally as well as in series; when they are to be connected in parallel it is advantageous to wind them in opposite directions, as shown in fig. 69 (2), since then the adjacent wires of the two coils are always at the same potential.

When thus divided the width of each of the two coils is equal to the pitch; in other words, the outer loop spans the diameter of the armature, and the width of each side of each coil is equal to half the gap between the fields.

We thus arrive at two coils for a bipolar drum or disc, as for a ring, alternator, i.e. as many coils as there are poles in the case of drum, or as there are pairs of poles in the case of disc alternators; and the principles which govern their width are identical, the most general proportions being width of inner loop equal to width of field, and both equal to half the pitch, the outer loop having a width equal to the pitch. This leads to the same result as for the ring-wound armatures, one half of the armature core being covered with winding. Fig. 70 represents diagrammatically equivalent armatures of the four types, each with its coils in the position of reversal of direction of the E.M.F., and all entirely avoiding any differential action; since there are the same number



(1) Two coils in series

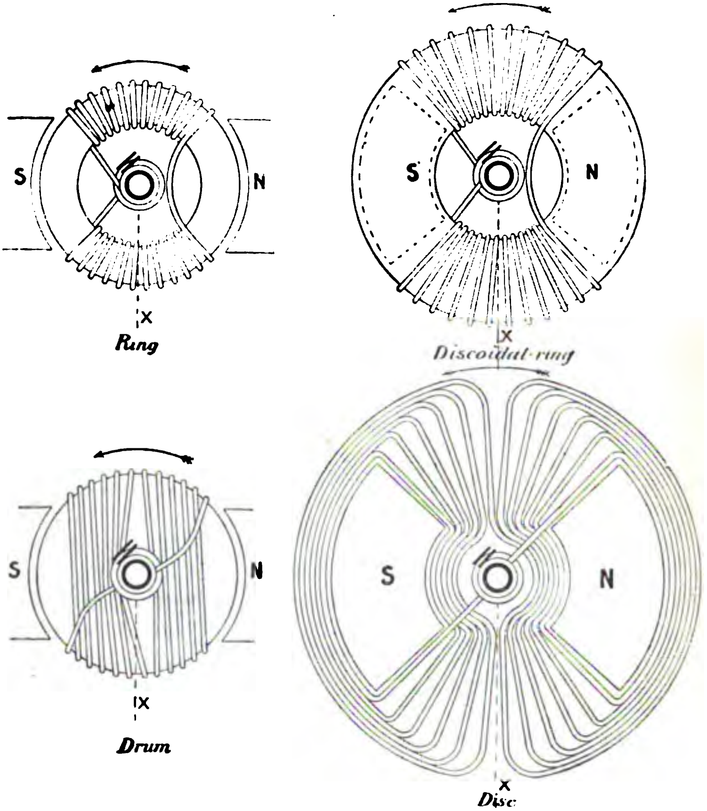


(2) Two coils in parallel

Fig. 69.—Series and parallel connections for bipolar drum and disc alternator armatures with winding divided into two coils.

of inductors similarly disposed in all the machines, their curves of impressed E.M.F. will be analogous, and in all will be approximately sinoidal in shape.

In most commercial alternators it is desired to produce a large



Ring

Discoidal-ring

Drum

Disc

Fig. 70.—Bipolar alternators with both width of winding and width of fields equal to half the pitch.

number of alternations per second, or a high 'periodicity,' ranging from 40 to 140 complete periods per second, and, further, a high E.M.F., usually of 1,000 or 2,000 volts. Each complete period or undulation of the E.M.F., as shown in fig. 67, is produced by the passage of a coil through two magnetic fields in opposite directions, and in a bipolar alternator the pair of fields is passed through once in each revolution; consequently its periodicity is equal to the number of revolutions per second. If, therefore, the speed at which an armature of sufficient size to give the required output can be safely or conveniently driven is not sufficiently high to give the desired periodicity, it is necessary while retaining the same width of winding on the armature relatively to the width of field to make the coils pass through more than one pair of fields in each revolution. In other words, the alternator must be multipolar. If we suppose the bipolar alternators of fig. 70 to be divided along the line O X by a cut passing through the armature core down to the spindle, and then to be opened out, so as to form a portion of a large cylinder with a flatter surface or a larger disc, we obtain the results shown by full lines in the left half of fig. 71. From this figure it is evident that the pair of fields to the left can be repeated indefinitely any number of times, if they are arranged symmetrically round the enlarged armature core, so as to form a complete circle, a N. pole and a S. pole succeeding each other alternately all round; and the dotted portion to the right-hand side of fig. 71 shows a pair of fields thus repeated. By thus multiplying the number of pairs of fields through which the coils must pass in each revolution, the number of complete undulations of the E.M.F. of the coils shown in full lines is increased correspondingly. If p=the number of pairs of fields, each pair is passed once every revolution, and therefore the number of periods in one revolution is p. Hence if N=the number of revolutions per minute, the periodicity of the multipolar alternator is $\frac{N}{60} \cdot p$.

In the case of ring and drum alternators, p is also the number of pairs of poles: but in the case of discoidal-ring and disc machines the number of pairs of fields is equal to the number of pairs of poles counted on one side only of the armature, the opposite poles forming a portion only of the same fields. If, therefore, P=the number of poles round a ring or drum armature, or on one side only of a discoidal-ring or disc armature, the formula for the periodicity may be put in the form $\frac{N}{120} \cdot P$.

Hence, by increasing the number of pairs of fields, the periodicity of the multipolar alternator for a given number of revolutions

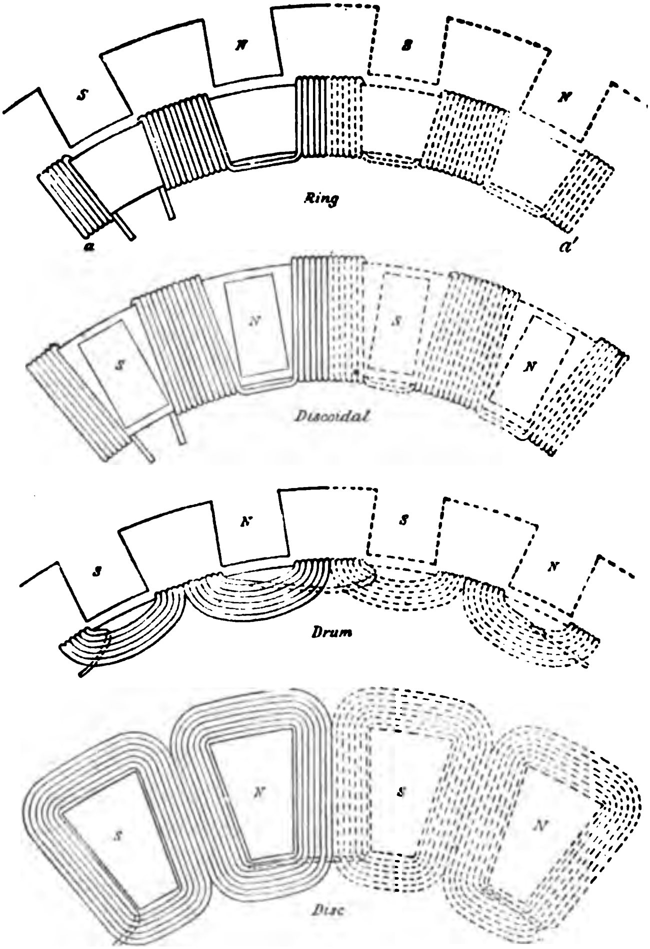


FIG. 71.—Portion of armatures of multipolar alternators.

per minute can be raised to any required figure. A very usual periodicity is 100, or, as it is symbolically expressed, 100 ~: the larger the output and size of the alternator, the slower the speed at which it is desirable to drive it; consequently the number of poles required to give a periodicity, or, as it is sometimes termed, a 'frequency' of 100, will range from 16 poles with 750 revolutions per minute to 20 poles with 600 or 24 poles with 500 revolutions per minute.

But it is not only the pairs of fields which can be repeated: the coils can also be repeated for every pair of fields, and fig. 71 shows by dotted lines a second pair thus repeated and connected in series with the original pair. We thus obtain in the multipolar alternator several pairs of fields, and as many coils as there are fields, both being arranged symmetrically in circles about a common centre or axis; all the coils will occupy at any moment exactly the same position relatively to a magnetic field, and therefore the phase and intensity of the alternating E.M.F. induced in each will be identical. Hence they can be coupled together in series, so as to form a continuous winding, or can be divided into two or more parallels, according as we desire a high E.M.F. or a large current; and the total impressed E.M.F. of the machine at any moment is equal to the E.M.F. induced by any one coil multiplied by the number of coils that are in series. Thus by increasing the number of coils that are in series, the E.M.F. of the alternator is increased, just as the periodicity is increased by increasing the number of pairs of fields. The presence, therefore, of several pairs of poles and coils introduces nothing new in the theory of the action, and the multipolar alternator may be simply regarded as made up of several bipolar alternators. The principle deciding relative widths of fields and of winding in each coil remains the same, and with the general proportion adopted, one-half of the armature core is covered with winding.

The methods of connecting up the coils in series or parallel are the same as those above described for bipolar alternators; if the entire winding be in series, the full difference of potential of the machine exists between the adjacent coils which form the beginning and the end, and it is therefore a frequent practice, especially in drum and disc machines, to divide the winding into two halves in parallel, so that each coil then carries only half the total armature current. When this is done, it is advantageous, as already explained, to retain the same direction of winding at the junction of the two halves in the case of ring and discoidal-ring armatures, but to reverse it in the case of drum or disc armatures. In all cases it is indifferent whether the armature or the field magnets rotate. In the former case the ends of the armature winding are brought to two collecting rings, which rotate with the armature spindle, and upon which rest the stationary brushes forming the terminals of the external circuit. In the latter case stationary terminals are attached to the armature, but collecting rings have to be applied to the system of rotating magnets, into and out of which the exciting current for the field winding flows by means of brushes or rubbing contacts.

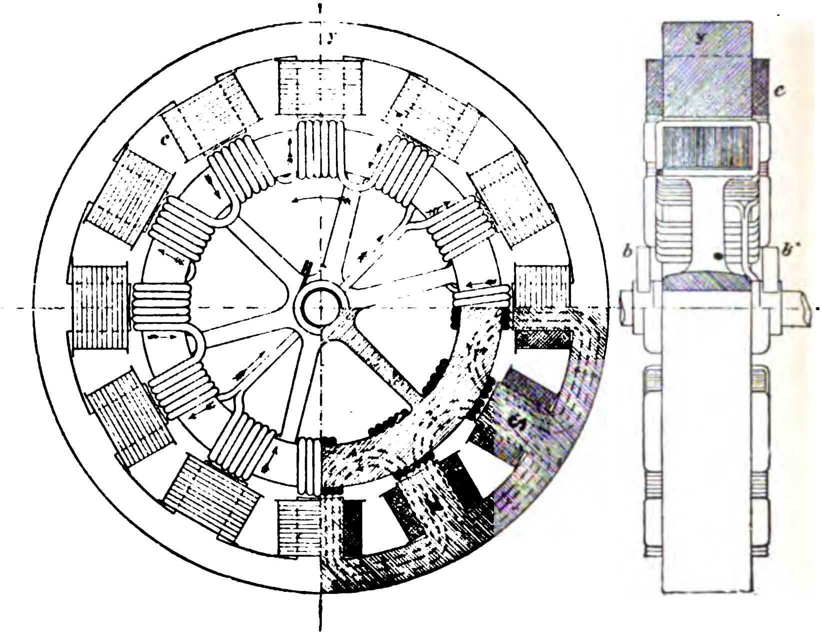


FIG. 72.—Diagram of ring multipolar alternator.

The form of the magnetic circuit and of the magnets which suits each of the different types of armatures remains to be considered. In the cases both of the ring and drum-wound armatures, the armature may be internal or external to the poles, these latter being in the one case united by a common circular yoke-ring external to the whole (fig. 72), or in the other by an internal ring, so as to form a kind of star, as in fig. 73. Fig. 72 shows diagrammatically a ring-wound armature, surrounded by twelve external poles, pointing radially inwards, and each wound with an exciting coil *e e*; one-half of the machine is shown in section in the end view, and one quarter in the side view, the paths of the magnetic induction being there marked by dotted lines. It will be seen that the total flux of lines forming one field passes entire through each magnet core, and bifurcates on reaching the armature core or the yoke-ring, *y*. In practice the system of magnets would be supported from the bed-plate, which is omitted in the diagram for the sake of clearness. The winding is divided into two halves in parallel; the inductors of each coil are shown as disposed in one layer only of six turns, although in most cases there would be a large number of turns, possibly in two layers. The current is collected by the brushes *b b'*, bearing on the two collecting rings, which are placed on opposite sides of the armature. Fig. 73 shows an equivalent drum-wound alternator with internal poles, which rotate while the armature remains stationary. Current is supplied to the exciting coils by the brushes and collecting rings, while the alternating E.M.F. and current is obtained from the stationary terminals of the armature *a d*. At the right hand is shown an internal view of one half of the armature winding after removal of the magnets; the coils which are in series are wound in the same direction, but the direction of winding is reversed at the points where the two halves of the armature are united in parallel. Since half the number of coils is an even number, this junction is effected by connecting together contiguous inside ends of one pair of coils (as at *a*) and the distant inside ends (as at *d*) of the coils at the opposite end of a diametric line. If half the number of coils had been an uneven number, contiguous inside ends and contiguous outside ends would have been joined together. The maximum difference of potential which exists between any two adjacent wires is equal to that generated by a pair of coils, and hence, even in a machine giving 2,000 volts of

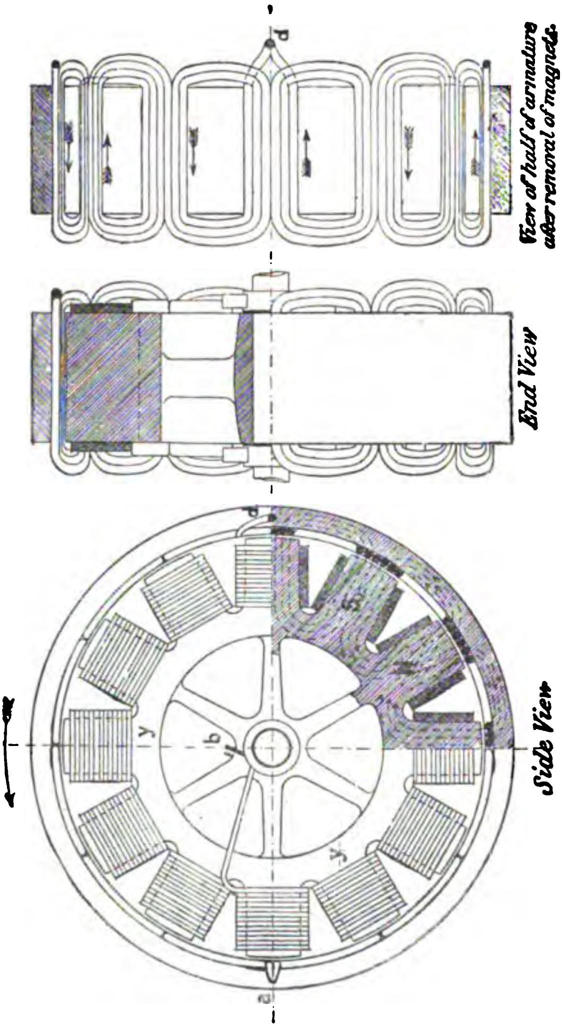
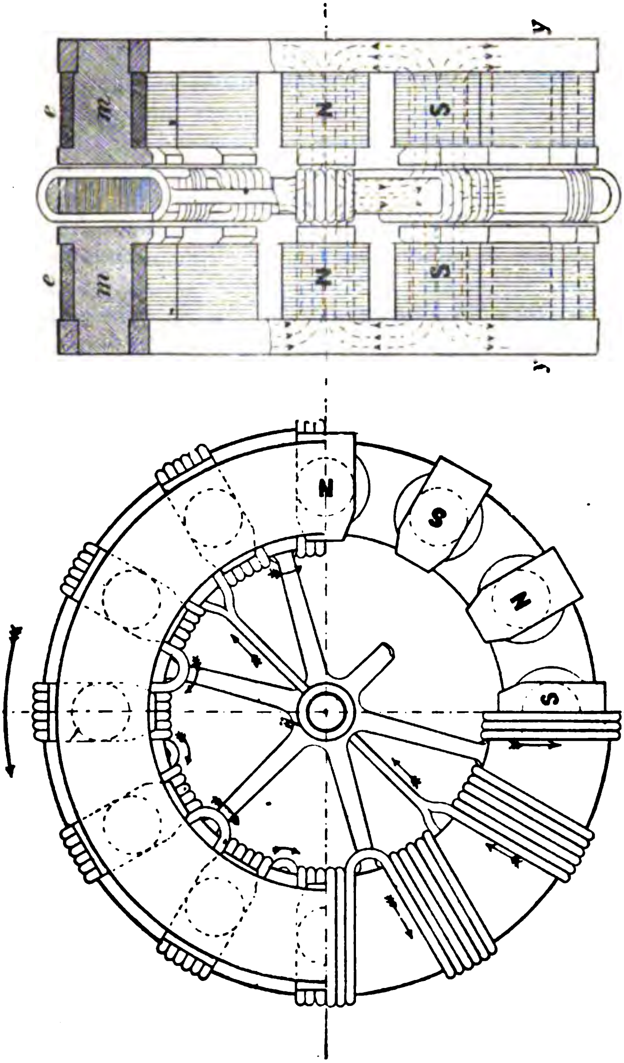


FIG. 73.—Diagram of drum multipolar alternator.



End View.

Side View.

Fig. 74.—Diagram of discoidal-ring multipolar alternator.

effective E.M.F., does not amount to more than about 950 volts at the moment of maximum induced E.M.F. Just as the ring armature can be adapted to suit a revolving internal magnet, so also the drum method of winding may equally be applied to the outside of a cylindrical iron core, which revolves within an external set of magnets. In the case of the discoidal-ring and disc machines (figs. 74 and 75) the magnets *m m* form two crowns, being united by two circular yoke-rings, *y y*, one on either side of the armature. In the former or flat-ring machine, poles of the same sign are presented exactly opposite to each other on either side of the armature core, and the lines which enter into it from a pair of N. poles flow in either direction through the armature, issuing forth where a neighbouring pair of S. poles is presented; in the disc alternator, on the contrary, opposite poles are of opposite sign, and the lines flow straight across the air-gap from one pole to another. In figs. 74 and 75 the left-hand lower quarter of the side view shows the armature with the nearer field magnets and yoke-ring removed, and the right-hand lower quarter shows the further system of poles, the armature itself being removed. In the discoidal-ring alternator of fig. 74 the winding is arranged in two parallels, and is exactly analogous to that of the simple ring alternator; the magnet cores on which the exciting coils are wound are cylindrical, while the pole-pieces have a rectangular shape, the width of field being equal to the width of winding in a coil when measured on the pitch-line or mean circumference of the armature.

In the disc alternator (fig. 75) the winding is analogous to that of the drum armature of fig. 73, except that twelve inductors are shown in each coil. The coils being arranged so as to form a complete circular disc, their shape, as shown in fig. 75, is more or less trapezoidal or pear-shaped, being broadest on the external periphery of the armature, and gradually tapering inwards along their radial depth. The polar surfaces are shown of a shape corresponding to the inside loop of a coil, the magnet cores, *m m*, having a similar trapezoidal section; as in the other alternator diagrams, the width of field is equal to the width of winding when measured on the pitch line, and both are equal to half the pitch.

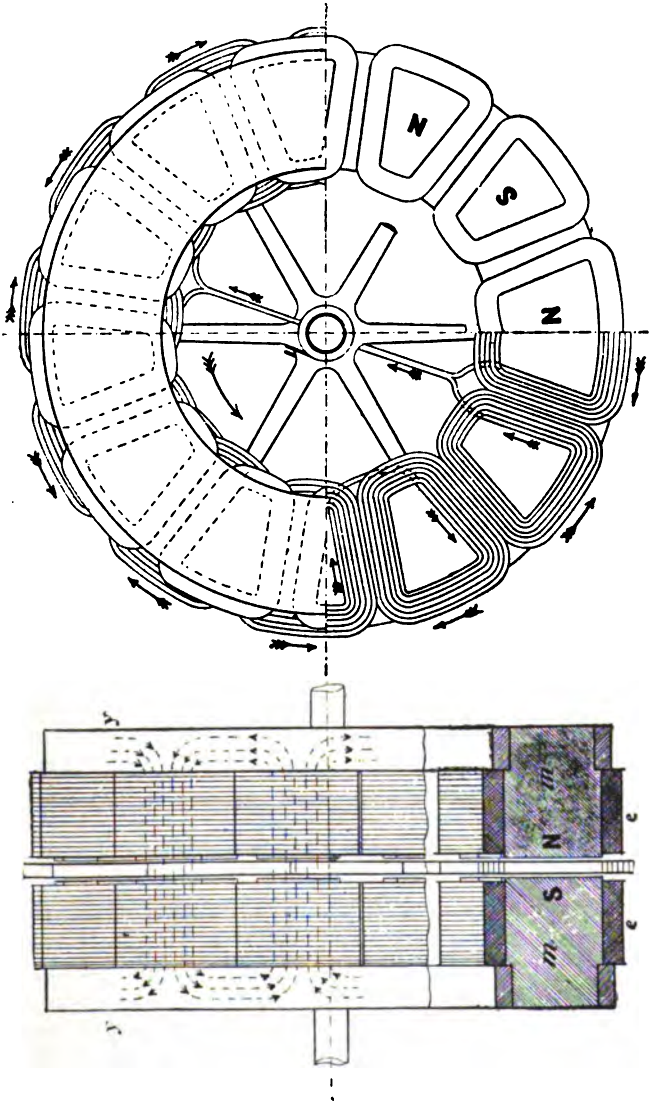


FIG. 75.—Diagram of disc multipolar alternator.

In both the discoidal and disc machines, if the neighbouring pole-pieces on one side of the armature come into close proximity to each other, a considerable number of the lines leak across from the neighbouring edges of each N. and S. pole without passing through the armature at all, and thus are entirely useless for the purpose of inducing E.M.F. The leakage of lines is therefore more marked in such machines than in ring and drum alternators, where the width of the gap between neighbouring magnets is not reduced to less than the width of the gap when measured on the pitch-line; it is especially marked in disc machines owing to the air-gap in them being usually longer than in discoidal alternators. In order to reduce this leakage, the pole-pieces of some discoidal machines have been slightly tapered at their inner ends, as shown in fig. 74, so that their shape is not strictly rectangular, and does not exactly correspond with the shape of the coils: these latter necessarily have parallel sides, since there are as many turns of wire on the inner periphery of the armature as there are on the outside.

Finally, it should be observed that if the pole-pieces have a rectangular shape in the case of discoidal rings, and a trapezoidal shape in the case of discs, the whole length of each inductor does not enter or leave the field at once and instantaneously; consequently it does not begin or cease to cut lines simultaneously throughout its entire active length, but the action is at first confined to one or other end (fig. 76). The effect of this is to render the rise and fall of impressed E.M.F. both in each inductor and in the coil, as a whole, more gradual than it would otherwise be, the curve of impressed E.M.F. being therefore rounded off into a gentle sweep throughout.

In general, the exact shape of the curves of E.M.F. and current which were discussed in Chapter VI. admits of very great variation with different dispositions of iron and copper in field magnet and armature. For an examination of some of these shapes the reader is referred to papers reprinted in the 'Electrical Engineer,' Aug. 16, 1889, Dec. 5, 1890, and 'Electrician,' Nov. 27, 1891.

In the alternators so far described, it has been stated that, roughly speaking, only one-half of the armature core is covered with winding; it is therefore possible to wind an entirely distinct set of coils forming a second armature circuit in the vacant spaces between the coils of the first circuit. The second set of coils may be used to feed an entirely separate external circuit, possibly at a different pressure. The phase of the impressed E.M.F.'s in the two circuits will differ by a quarter of a period, or 90°, the one reaching its maximum when the other is at zero. This process may be carried still further if the width of winding in each coil be reduced to less than half the pitch; *e.g.* if it be one-third of the pitch, three distinct armature circuits can be wound on the same core, giving curves of impressed E.M.F. differing in phase by 120°. Such alternators have lately (1891) come into prominence in connection with the use of alternating-current motors with rotating magnetic fields, and have been called 'polyphase' machines. It will suffice here to call attention to the possibility of winding alternator armatures with sets of coils so spaced that the E.M.F. induced in each set differs in phase from that induced in any other set by a certain fraction of a period.

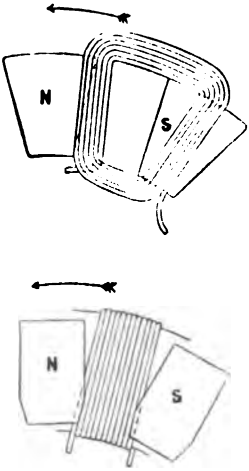


FIG. 76.

CHAPTER IX

UNIPOLAR ALTERNATORS

WHILE the bipolar dynamo may be said to be naturally an alternator, and only indirectly and artificially a continuous-current machine, it may with equal truth be said that the unipolar dynamo naturally gives a uni-directed E.M.F., and is artificially made to yield an alternating E.M.F. At first sight it might be supposed that a continuous-current machine which gives a perfectly constant and steady current without any need for continual starting, stopping, and reversing of it in the inductors, and which, therefore, requires no commutator, with all its attendant troubles of sparking at the brushes, must possess considerable advantages over the bipolar continuous-current machine; it was, too, the earliest form of dynamo and motor invented, since it dates from the years 1823 and 1831, when a disc unipolar motor and dynamo were respectively made by Barlow and Faraday. Yet practical experience shows that bi- and multi-polar machines have been and are in possession of the entire field of continuous-current work, and that there must be some inherent difficulties in the unipolar dynamo which have up to the present baffled all the ingenuity of inventors.

The chief of these is the small value of the E.M.F. which can be procured with the machine in its simplest form, even with a very high speed of rotation. It has, in fact, but one inductor, and therefore yields at best only from two to five volts. The remedy would be to connect several inductors in series, so as to add up their E.M.F.; yet if the reader refers back to Chapter VII. it will be seen that the methods of connecting inductors in series as there described only result in forming loops, round which the E.M.F. and current alternate, the characteristic feature of the dynamo being thereby entirely lost. If we take the rotating armatures of fig. 55, with a number of insulated inductors arranged either in a tube or in a disc, and join the end of one inductor on to the opposite end of another by means of a connecting wire fixed to the two inductors and rotating with them, it will be found that these connecting pieces, when so fixed, must necessarily cut the lines of induction at exactly the same rate as the inductors proper, and in the same direction; they thus become inductors themselves, inducing an E.M.F. which is in the reverse direction round the loop to that given by the inductors proper, and the total E.M.F. of the combination is nil. Hence, if the armature rotates, as we have supposed, the connecting pieces must be stationary, so that they can be taken through the iron of the field-magnet without cutting its lines, and the current must pass into and out of them by some kind of rubbing or sliding contact. There is, in fact, no method of *winding*, properly so called, which is applicable to the unipolar armature, so long as it has to give a uni-directed E.M.F. All that can be done is to join one inductor on to another through the agency of rubbing contacts, and the multiplication of these at once introduces such difficulties as to render the whole arrangement unworkable. In practice, the utmost that has been done may more properly be described as the placing of two distinct machines in series, the two being mounted on the same spindle; yet even then the twofold collection of a very large current by means of brushes or mercury troughs presents great difficulties.

It will suffice, therefore, to pass immediately to unipolar alternators, in which one or other of the definite methods of unipolar winding described in pp. 127–130 has been adopted. As in the case of bipolar armatures, it will be necessary, when we deal with coils instead of simple loops, to determine the best widths of winding and field, and precisely the same principle, viz. the avoidance as far as possible of differential action, must be our guide. The pitch is, as before, the distance between the centres of a pair of poles of opposite sign, this distance being measured along the circumference of a circle passing midway through the active lengths of the inductors; the only difference is that now such two poles are either outside and inside of the armature core, in the case of ring armatures, or on opposite sides of the core, in the case of discoidal and disc armatures, instead of being either both outside or both inside, or both on the same side of the core.

In the case of ring and discoidal-ring armatures, as shown on p. 128, Chap. VII., the maximum permissible width of field, even with only one loop, is not more than the pitch, since otherwise differential action reduces the time during which any resultant E.M.F. is produced, and when there are many loops the rule determining the proper width of fields and winding takes exactly the same form and leads to the same results as were previously found for the corresponding bi- or multi-polar alternators. The outer portions at the sides or edges of one and the same coil must never be moving at the same time past two poles of opposite sign, whether these are outside and inside, as in the ring form, or on each side of the armature, as in the discoidal form; if opposite sides of the same coil are under inductive action at the same time, the wires which for the time being are connectors will also act as inductors of a back E.M.F., and so neutralise the effect of the inductors proper. Hence, the width of the coil must be not more than the width of the neutral space between two fields, where no lines enter or leave the armature core; and the law to which the winding must approximately accommodate itself is width of field equal to width of coil, and both equal to half the pitch. Thus half the armature core is covered with winding exactly as in the bipolar alternator.

If the armature cores of figs. 56 and 57 be thus wound with a pair of coils, each of width equal to half the pitch, and the fields be correspondingly reduced in width, we obtain a dynamo which, when cut through to the spindle and flattened out, is shown diagrammatically in the full-line portion of the upper diagram in fig. 77. The path of the lines is shown dotted, with the poles outside and inside the core, as they would be in the case of the ring form. If the armature were discoidal, it will be understood that the poles would then face opposite sides of the core, and the lines would pass aslant across the armature. As in the case of bipolar alternators, the whole can be repeated a second or any number of times, as shown by the dotted half of fig. 77, and thus is obtained a unipolar alternator with two or more pairs of poles, the coils being similarly multiplied, and in practice there are usually more than one pair of poles and pair of coils. The several poles and coils are symmetrically placed round the armature, as shown in figs. 78, 79, there being always as many coils as there are fields, where the lines enter or leave the armature core, i.e. as there are poles. If p = the number of pairs of fields, the periodicity is the same as in the case of multipolar alternators, viz. $\frac{N}{60}p$, or $\frac{N}{120}$ P. In the case, however, of the discoidal-ring unipolar alternator, since half the poles of the corresponding multipolar alternator are absent, the number of pairs of fields is equal to the number of pairs of poles counted on both sides of the armature.

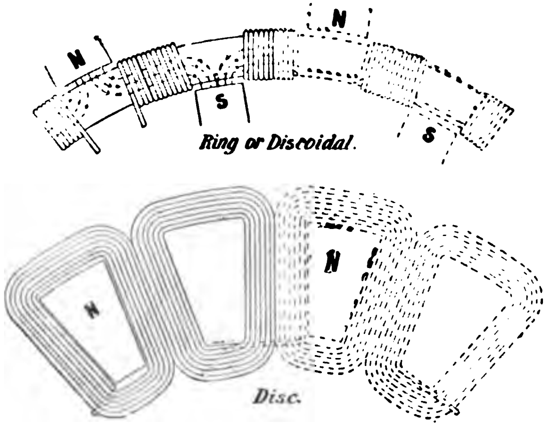


FIG. 77.—Armatures of unipolar alternators.

A chief feature of the above alternators is that all the poles form branches of one and the same magnetic circuit, and therefore only require one exciting coil wound on that portion of the circuit where all the lines unite to flow in a common stream; each inductor thus cuts the lines from the numerous subdivisions of one common pole, as the term 'unipolar' emphasises when applied to an alternator with many pairs of poles. Although not confined to unipolar alternators, the single exciting coil is always employed in them.

Each type has its own appropriate form of magnetic circuit. In the ring unipolar alternator all the external poles are of the same sign, and so also all the internal poles; hence they are formed by branches out of a magnet-core common to them all,

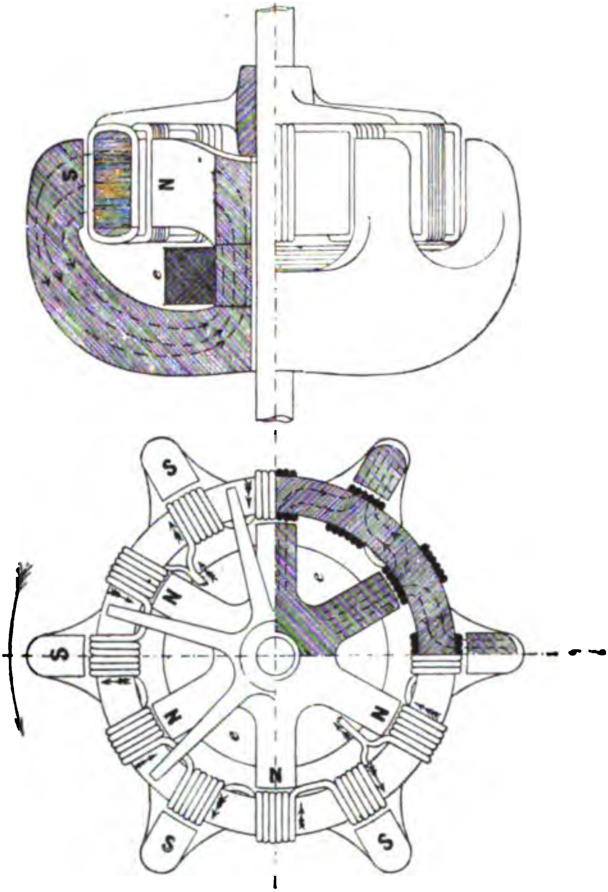


Fig. 78.—Diagram of ring unipolar alternator.

and placed on one or other side of the armature (fig. 78): upon the magnet-core is wound the single exciting coil shown at *e*.

In the discoidal form all the poles on the one side are N., and all those on the other are S., and their common yoke may be either external or internal to the armature. If the latter rotates, it is most convenient to place the yoke on the outside, as was

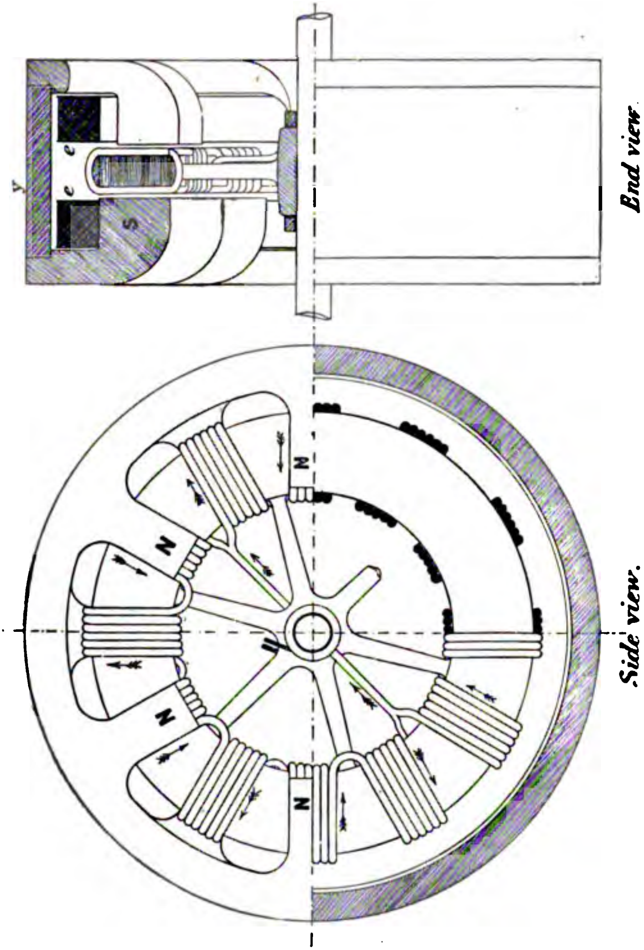


Fig. 79.—Diagram of a unipolar discoidal-ring alternator.

shown in fig. 57, and the exciting coil, *e*, is either suspended from the external yoke-ring, or is divided into two halves, one on either side of the armature, and supported by the poles. Fig. 79 shows an external yoke-ring, *y*; in the upper half of the side view the exciting coil is supposed to be removed in order to show the curved poles on the opposite side.

When we pass to the disc unipolar alternator, our definition of the pitch leads to the conclusion that for a single-field dynamo with only two poles, as shown in fig. 58, the pitch is the entire circumference of the circle, which gives the mean path traversed by the rotating inductors, and it has been shown in Chapter VII. that the maximum width of field permissible is equal to half the pitch, for the width of a single loop alone must not be less than the width of field, nor greater than the width of the neutral space; hence the latter must be at least equal to the width of the field. Now, when there are several loops wound on the top of each other, so that each side of the flat coil has appreciable width, there will be differential action when such a coil is rotated, unless the width of field be reduced to equality with the inner loop, and the width of the outer loop be not greater than the width of the gap. Hence differential action is entirely avoided if the inner loop have a width equal to that of the field, and the outer have a width equal to the pitch less the width of the field; *e.g.* if the width of field and of inner loop be equal to one-third of the pitch, and the width of the outer loop be equal to the gap, or two-thirds of the pitch, the width of the winding on each side of the coil being one-sixth of the pitch. If the width of field be increased, the width of winding must be decreased, and *vice versâ*, so that, as before, various proportions are possible, all of which avoid differential action. By the same train of reasoning as has been applied before to bipolar alternators, we arrive at the most favourable result when the widths of the field and inner loop are each equal to one-quarter of the pitch, and the width of the outer loop is equal to three-quarters of the pitch, in which case half of the armature core is covered with winding, and the result is identical with the disc alternator of fig. 68, if one or other of the pair of poles be entirely omitted. But such a single coil may be conveniently divided into two, so that the circumferential connecting portions have less depth, exactly as was done in the case of bipolar disc alternators, the connections of the winding being identical; and this is shown in fig. 77, which further shows how the whole may be repeated with a second field. We thus arrive at *twice* as many coils as there are fields, the coils being virtually arranged in pairs, each pair corresponding to one field, instead of

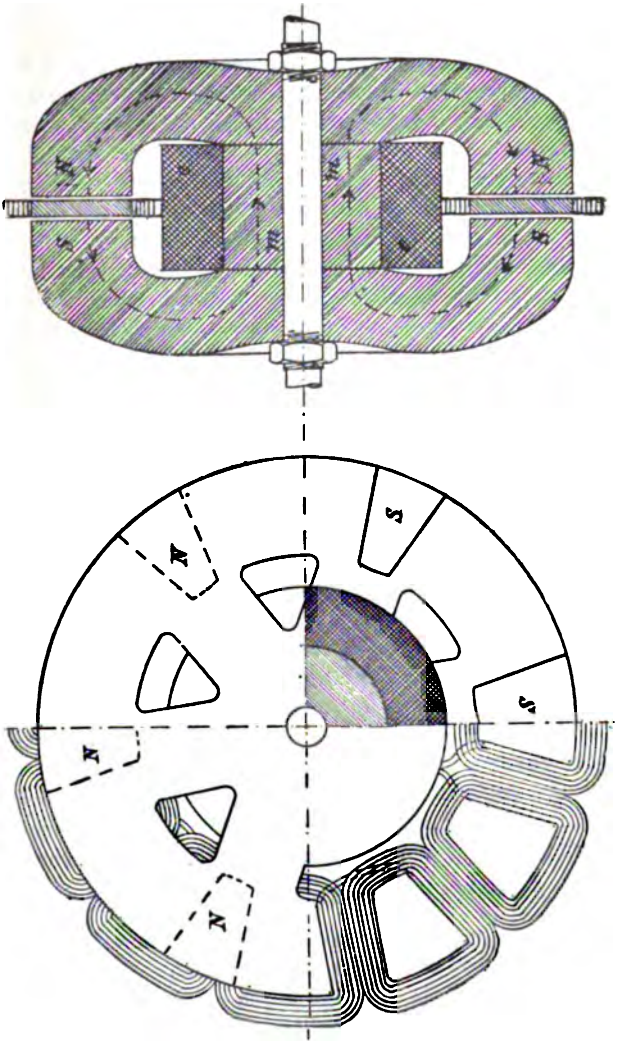


FIG. 80.—Diagram of a unipolar disc alternator.

two fields, as in the corresponding bipolar dynamo; and since half the poles of the multipolar dynamo are absent, the width of the outer loop of each of the two coils is now half the pitch, instead of being, as before, equal to the pitch.

Fig. 80 shows a disc unipolar dynamo evolved out of fig. 58 with revolving magnet and stationary armature; the several fields are again formed by dividing one magnetic circuit into a number of curved branches, each terminating in a pole-face. All the poles on one side of the armature are N., and all those on the other side S.; those on either side of the armature are shown united on the outside by a thin web of iron, pierced with holes and with deep recesses intervening between the projecting poles. The lines of induction thus form separate groups, between which intervene neutral spaces, clearly marking off the several fields; the field is never reversed in direction, but may be described as intermittent. The single exciting coil which is shown at *e* need not of necessity rotate with the magnet, but might be attached to the armature, in which case no collecting rings for the exciting current would be required; it is, however, more advantageous for other reasons to rotate it with the magnet. When the armature is rotated, the yoke or undivided portion of the magnetic circuit may be made external to the armature in the shape of a ring, similar to that shown in fig. 79, but with the opposite poles directly facing each other. Since the poles on each side are of the same sign, there is no tendency for lines to leak across from one pole to a neighbouring pole on the same side of the armature, as in the case of a multipolar disc armature (p. 154), and the corresponding economy in the production of the fields to a large extent compensates for the fact that at any one time only half of the winding is active.

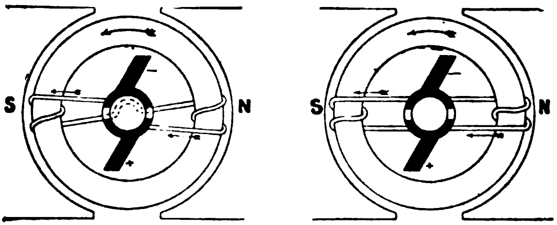
Since the passage of one field past a pair of coils gives one complete double wave of E.M.F., the number of complete periods per second or the 'periodicity' of a unipolar disc alternator is $\frac{N}{60}p$, where p is either the number of pairs of coils or the number of poles on one side of the armature. It is therefore the same as the periodicity of a multipolar disc alternator with an equal number of armature coils, although in the latter there are twice as many poles as in the unipolar machine.

CHAPTER X

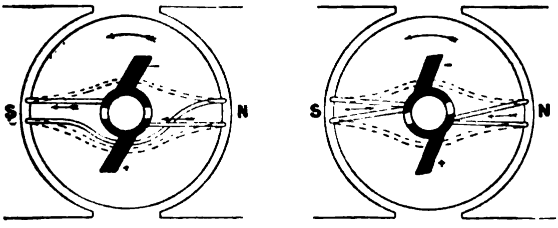
OPEN-COIL ARMATURES

In the two preceding chapters the subject of alternators has been discussed; in all of them, whether bi-, multi-, or uni-polar, the current due to the induced E.M.F. is alternating in direction in the external circuit, just as is the current in each armature loop and in each armature coil formed of loops. Returning to our first group of dynamos, viz. those which are bi- or multi-polar, we now enter upon a consideration of the second division, consisting of those dynamos which yield a uni-directed, or, as it is often called, a direct current in the external circuit, and in which, therefore, since their terminal E.M.F. must be uni-directed, some form of commutator developed out of the split ring of fig. 48 must be present. This second division is, however, itself divisible into two sub-classes, viz. those with 'open-coil' armatures and those with 'closed-coil' armatures; in neither does the E.M.F. at the terminals alternate, but in the former it still pulsates or fluctuates in value during each revolution, while in the latter it has a practically constant value. 'Open-coil' armatures therefore occupy an intermediate position between alternators and 'closed-coil' machines. Their use is almost entirely confined to arc lighting, for which they have been found peculiarly suited; the pulsations which still remain in their E.M.F. curve, and which cause the current in the external circuit to fluctuate, although to a still smaller extent, are not disadvantageous to arc lamps, while their method of construction lends itself to the generation of a high E.M.F. and constant current, as required by arc lamps when run in series.

The previous discussion of alternators will have familiarised the reader with the principle that, in the case of ring or discoidal armatures, with one coil wound as in fig. 48 on one half of the ring, and placed in a bipolar field, a second coil can be wound on the portion of the ring diametrically opposite, and in this second coil an E.M.F. will be induced exactly similar to the E.M.F. of the first coil; it can therefore be connected either in series or in parallel with the first, the same split-ring being made use of without any modification (fig. 81). Similarly, in the case of drums, a second coil can be wound on by the side of the first and connected with it in series or in parallel, exactly the same proportion of the entire surface of the armature core being covered as in the ring. So, too, in the case of the disc armature, a second coil symmetrical with the first, but on the opposite half of the disc, introduces no fresh element to our theory. When the two coils are connected in series they virtually form one continuous coil: this is most evident in the case of the drum (fig. 81), but in reality the two opposite coils of the ring and disc may equally



Series. *Ring* *Parallel.*



Series. *Drum* *Parallel*

FIG. 81.

be regarded as one coil divided for practical reasons into two portions.

But whether in series or parallel, the two coils thus symmetrically placed do not alter the character of the curve of E.M.F. as given in fig. 48, and do not, therefore, assist us in the work of reducing the pulsation of the E.M.F. to a comparatively small amount. An entirely new method must be tried, viz. the arrangement of coils on the armature in such positions that they are not diametrically opposite to each other, and so are not symmetrical in every respect.

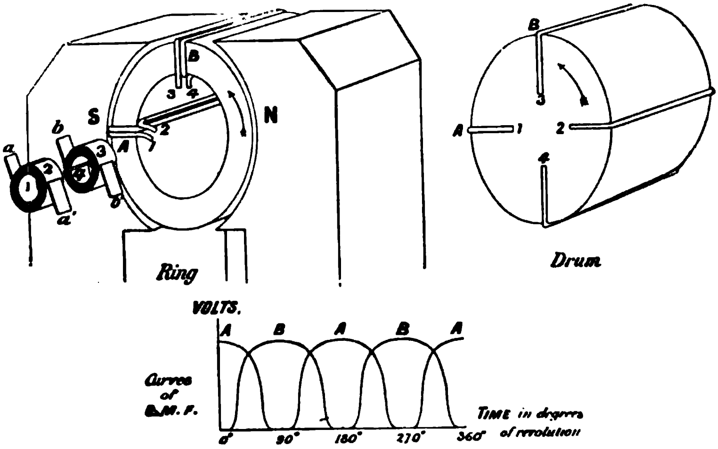


FIG. 82.

Reverting to the simple coil and split-ring of fig. 48, let us arrange a second coil on the armature at right angles to the first, and attach its ends to a second split-ring, the angular position of which relatively to the first differs by a corresponding interval of 90° (fig. 82). The armature shaft on which both split-rings would be mounted and the connecting wires which join the ends of the coils to the commutator segments are omitted for the sake of clearness, the connections being simply indicated by numerals ; at the right hand is shown the corresponding drum armature, wound with two loops 90° apart. Let us suppose the insulating gaps between the two halves of each split-ring to be very small, and let the rubbing surface of the two pairs of brushes or contact springs, *a a′*, *b b′*, be very narrow—in fact, hardly more than a mere line. Now, since the second coil B is 90° behind the other, it will come into inductive action when the first coil is going out of action, and will be in the position of greatest action when the other is entirely inactive. If, therefore, we plot along the same horizontal axis the curves of the terminal E.M.F.'s on the two pairs of brushes for one revolution, the maximum E.M.F. or crest of the E.M.F. wave due to one coil will coincide with the hollow of the wave of the other; thus, in fig. 82, curve A A gives the value at any moment of the E.M.F. at the brushes *a a′*, while B B is the corresponding curve for coil B. It is evident, therefore, that if the crests of the second wave were used to fill up the hollows of the first, the fluctuations of the joint curve of E.M.F. would be very largely decreased. Further, if we arrange three coils round the armature at angles of 120° from each other (fig. 83) we again see that if their curves were in some way amalgamated the pulsations might be still further decreased. In the class of machine now before us, therefore, we require a comparatively small number of coils, so arranged relatively to one another that when one ceases to be active another takes up the action, this change being continually and successively repeated at the proper moments in each revolution. Further, since in practice the edges of the pole-pieces are not brought into very close proximity to each other on account of the excessive leakage of lines which would ensue, the portion of each half of a revolution in which a comparatively narrow coil cuts few lines or none, and therefore yields no E.M.F., is of appreciable length. Hence, whenever a coil is inactive, it may be of advantage to cut it entirely out of circuit, since if it formed part of the circuit a portion of the E.M.F. of the coil in action will be spent in driving the current through its useless resistance. Or, again, it may, while comparatively inactive, be thrown into parallel with another

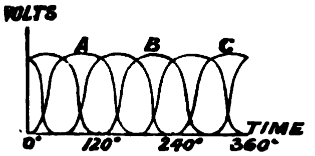


FIG. 83.

coil, also comparatively inactive. An incidental advantage so gained will be that the electrical resistance of the armature is during that time reduced, but the main object of such parallel connection will be to prepare a coil for the rapid changes which it undergoes from action to inaction, and *vice versâ*.

Any armature in which a coil is either left open-ended, or has its connections with the other coils rearranged during any portion of a revolution, is an 'open-coil' armature, the exact meaning of which term will be more apparent later, when it can be contrasted with 'closed-coil' armatures.

Starting with two coils wound on a bipolar armature at right angles, as in fig. 82, let us increase the width of the gaps in the split-rings until each commutator segment has an angular width of something less than 90°, while the angular width of contact of each brush is still kept small; then in the position shown in fig. 84, since the brushes *a a′* are in the middle of their segments, they are yielding an E.M.F. while coil B is not in contact with its brushes, but shortly after there will be an E.M.F. at *b b′*, and none at *a a′*. Hence, one pair of brushes of greater length along the axis of rotation might be substituted, which should press on both split-rings alike, as *a b*, *a′ b′*, in fig. 84, and the whole is represented in fig. 85 by the four commutator segments, each having an angular width of nearly 90°, and connected to one end of a coil. The pair of coils are shown diagrammatically by the coiled lines A A′, B B′, each of which may represent either a simple coil on a ring armature, or a pair of such coils wound on opposite sides of

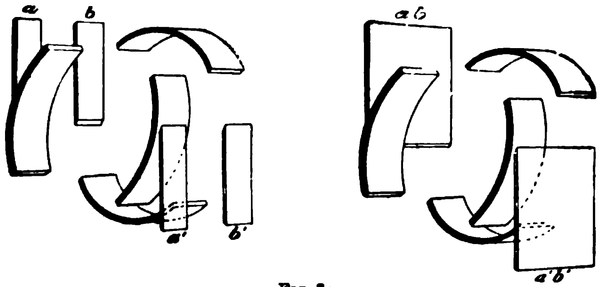


FIG. 84.

the ring, or the corresponding forms which these take in the drum, discoidal, and disc machines, as shown in fig. 81, and explained on p. 166.

Let *m m* be the line of maximum field, where the lines of induction are most dense, and *n n* be the neutral line, where there

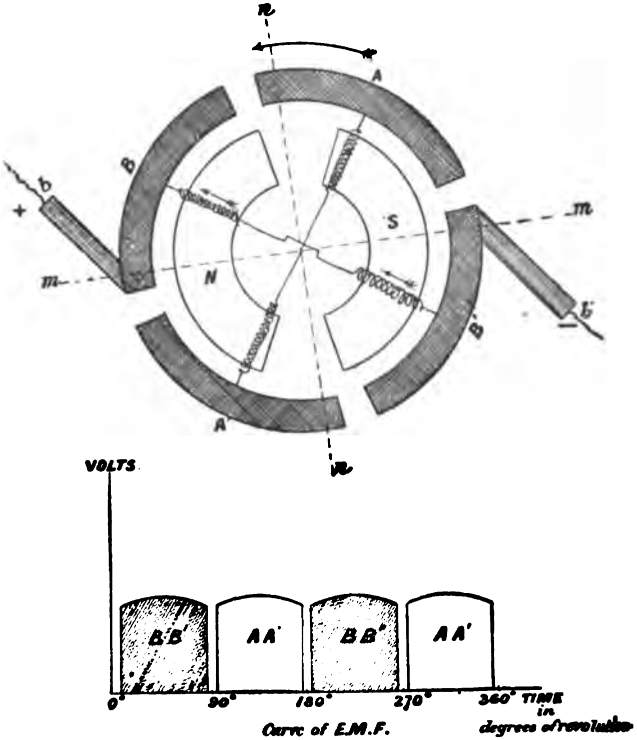
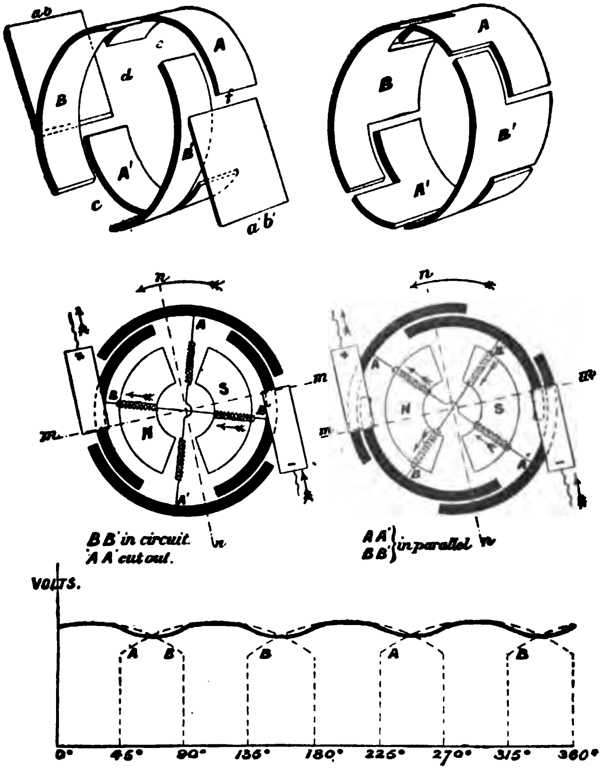


FIG. 85.

is no field at all; the reason why these are not drawn exactly horizontal and vertical will be explained in a subsequent chapter. It is important to observe that the brushes are now placed near to or on the line of maximum field, and, as shown, coil B B′ has just come into action, and is supplying E.M.F. and current to the external circuit, while coil A A′, which is just passing into the position of least action, is cut out of the circuit. The width of each commutator segment being nearly 90°, each of the two coils will in turn be in contact with the brushes for nearly a quarter of a revolution, but as the gaps of insulating material are not bridged over by the brushes, the circuit will be broken four times in each revolution. If now the curve of E.M.F. acting at the brushes on the external circuit be plotted for a revolution, starting from a point where the brushes are on the gap, just in front of the position shown in the diagram, it will be entirely above the horizontal line, and the fluctuations in its value are very much reduced ; yet it will be discontinuous, being divided by four breaks corresponding to the passage of the insulating gaps under the brushes. At these moments the brushes cease to press on one pair of segments, and have not yet begun to press on the other pair. The arrangement is therefore totally unworkable, owing to the fact that the circuit of the current is continually being broken, which would produce destructive sparking on the commutators. And, further, when a coil is just cut out of circuit, as A A′ in fig. 85, it is only half-way between the line of maximum field and the neutral line, and is, therefore, still generating some E.M.F. which might be utilised. To remedy these defects it is only necessary to retain the commutators as shown in fig. 82, that is, with the neighbouring segments overlapping each other and with one pair of broad brushes *a b*, *a′ b′*, pressing on both commutators ; each brush before leaving one segment will now touch another belonging to the other split-ring (fig. 86). Below the commutator there shown is a diagrammatic representation of two positions of the armature, the brushes being supposed to touch along the line *m m* ; it will be seen that the time during which a coil is entirely cut out of circuit is lessened, and during the time so gained the two coils are placed in parallel, as shown at the right hand. If each pair of segments overlap the other by an angle of 45°, and each revolution be divided into eight equal parts, then for the first eighth the two coils are in parallel, for the next only one is supplying E.M.F., and the other is open-circuited ; for the third eighth the two coils are again in parallel, after which for the next eighth the second coil is alone supplying E.M.F., the whole being repeated during the second half of the revolution in the same order. The curve of E.M.F. is now no longer discontinuous, but is a continuous undulatory line without any breaks; it is obtained by plotting the E.M.F. of each coil, when acting separately, and the ordinate,



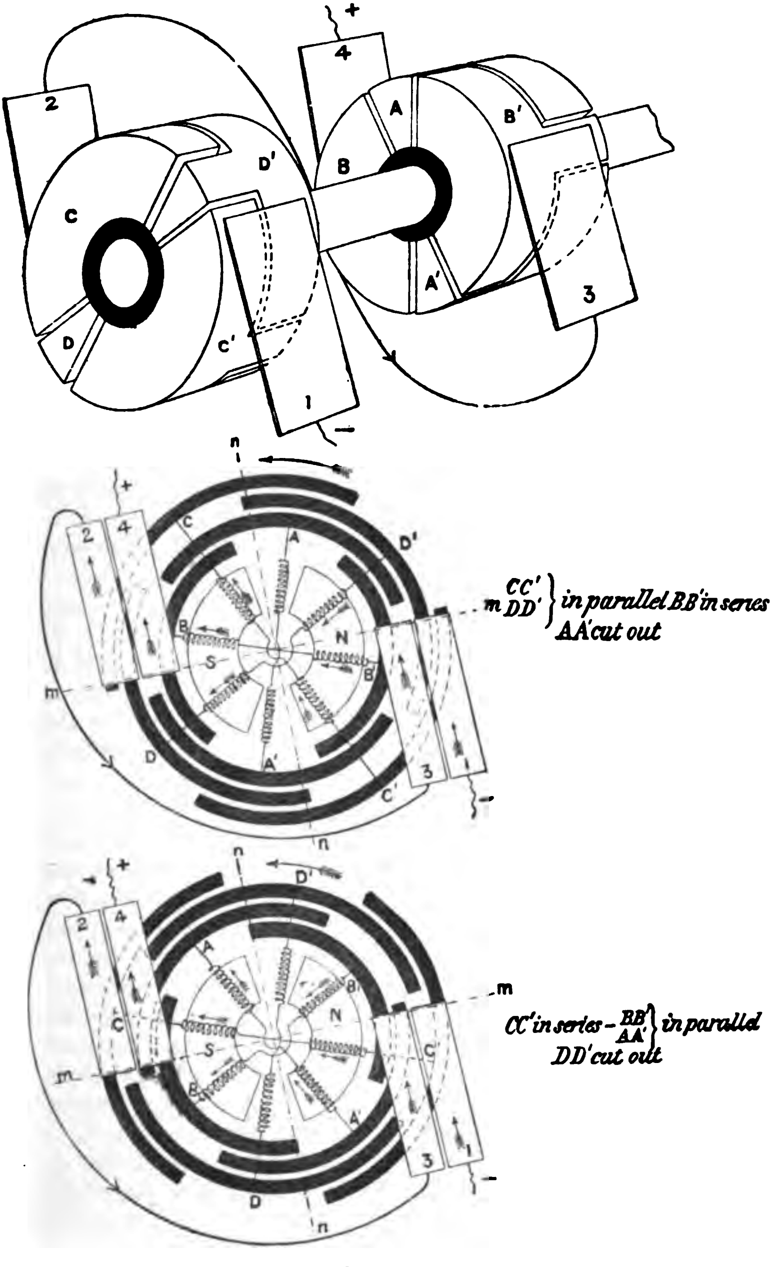
Curve of E.M.F'

FIG. 86.

which is common to the two separate dotted curves (fig. 86); the intervening periods of 45° are then filled up approximately by curves representing intermediate values between the E.M.F.'s of the coils when in parallel: these will fall and rise rather more suddenly than the curve of the coil which has the higher E.M.F. Four times in each revolution the two coils are in parallel, and during these periods the E.M.F. in one is increasing as it approaches the position of best action, and in the other is decreasing as it recedes from that position; the two are theoretically equally active only for an instant when they are equidistant from the line *m m*. It might be thought that this inequality between the E.M.F.'s of the two coils at the moment when they are put in parallel, and just before they part company, would cause large local currents and waste of power, since the higher E.M.F. of the one coil naturally tends to drive a back current through the other of which the E.M.F. is lower. Were the coils free from self-induction, this would be the case, and the net E.M.F. acting at the brushes during the periods of parallelism would be simply that of the coil which at the moment was less active, the excess E.M.F. of the more active coil being expended in producing an internal and useless back current. Owing, however, to the self-induction of the two coils, the current in them cannot be suddenly reversed, increased, or started; and consequently the curve of E.M.F. at the brushes falls and rises gradually, taking intermediate values between the E.M.F.'s of the stronger and weaker coils. Thus when first thrown into parallel, the resultant E.M.F. of the weaker coil is higher than its induced E.M.F. owing to the self-induced E.M.F. called into existence by the current which is started in it, while the resultant E.M.F. of the stronger coil is less than its induced E.M.F. by reason of the increase of current through it. Further, when a coil is about to be cut out of circuit, the inequality of E.M.F. is of advantage, since it stops the current flowing in it much more quickly than would otherwise be the case; for the greater E.M.F. of the other coil tends to drive a current in the reverse direction through the leaving coil, and so prepares it to be cut out of circuit without an excessive amount of sparking.

Each coil has its own commutator, and the winding of the armature does not form a closed circuit in itself, as will be found to be the case in 'closed-coil' armatures. The two overlapping commutators may be conveniently united, so as to form a single mechanical structure, by making the neighbouring segments of one split-ring project into the gaps left between the two segments of the other split-ring while still being insulated from them. Thus in fig. 86, if A projects into the gap *e* between B and B′, and A′ into the other gap *c*, while B and B′ similarly project into the corresponding gaps *d* and *f* between A and A′, we get the joint commutator shown at the right-hand side. Owing to the high voltage which this class of machine usually gives, air forms the most suitable insulator between the adjacent segments, since there is always a certain amount of sparking present, which would otherwise damage the surface of the commutator; hence the two commutators, though projecting into each other, are still divided by comparatively narrow air-gaps. The whole is mounted on the shaft of the armature, and held in place by a thick ring and washers of insulating material.

The method above described may be pushed a step further by winding on the armature more pairs of coils intermediate between the first pair; the pulsations of the E.M.F. are thus still further reduced, while more of the available space round the armature is rendered useful. The first pair of coils were 90° apart, and in every case a pair of coils at right angles to each other must be taken and connected to a joint commutator exactly similar to the joint commutator of the first pair. Thus fig. 87 shows diagrammatically an armature of four pairs of coils, two pairs at right angles to each other being connected to each of the two joint commutators; one joint commutator is set an eighth of a revolution, or 45° in advance of the other, just as any two neighbouring coils are separated by an angle of 45°; the arrangement is really equivalent to two separate machines in series with each other, the two pairs of brushes being connected in series by joining together one brush of one commutator with that brush of the other commutator which is on the opposite side, the other two brushes leading to the terminals of the machine. The perspective view of the two commutators corresponds with the first diagrammatic representation of the armature and coils. In the inner commutator, in the position there shown, coil A A′ is cut out of circuit, being in the position of no action, and coil B B′ is alone supplying E.M.F. to the pair of brushes 3, 4; in the outer commutator both coils, C C′, D D′, are in parallel, and supplying E.M.F.



CC'
DD' } in parallel BB' in series
AA' cut out

CC' in series − $\frac{BB'}{AA'}$ } in parallel
DD' cut out

FIG. 87.

to the other pair of brushes 1, 2; brush 2 is connected with brush 3, so that the current flows through coils C C′ and D D′ in parallel, and thence through coil B B′. The lower diagram represents the same after the armature has revolved through an angle of 45°, when D D′ is cut out, B B′ and A A′ are in parallel, and C C′ in series. This arrangement of one coil cut out, two in parallel, and the third in series with the two, is continually repeated as each coil arrives at the positions of neutral field, intermediate action, and best action.

Similarly, if three pairs of coils are used, there must be three joint commutators, each arranged one-twelfth of a revolution, or 30°, in advance of or behind its next neighbour.

We have now to return to the case of three coils arranged at angles of 120° with each other, represented diagrammatically in fig. 88, and in this case one end of each coil must be brought to a common junction, as shown at the centre. The other free ends are joined to three commutator segments, each having an angular width of rather less than 120° and insulated from each other. Evidently when in the position shown in fig. 88, since coil C is in the neutral line, no E.M.F. is being generated in it, while in coils A and B the E.M.F. is respectively radially outwards and radially inwards; if, therefore, the brushes are set on the line of maximum action a current will flow through A and B; but shortly after, when the insulating gap comes under brush *b*′, the circuit will be broken, and a similar break of the circuit will occur six times in each revolution—thrice at brush *b* and thrice at brush *b*′. In this case, therefore, while keeping the insulating gaps small, we require to widen the arc of contact of the brushes until each spans or bridges over the insulating gaps (fig. 89). The effect of this is that at the instant when a coil is about to be cut out, it is placed in

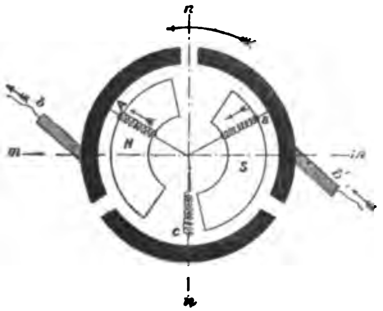


FIG. 88.

parallel with a second coil, since the two come under the same brush; the current, therefore, at this moment flows through the two in parallel, and so through the third which is in series with them. The series of changes which take place in a half-revolution is as follows: Starting from the position shown in fig. 89, in which B and C are in parallel, and A in series with them, B is cut out, leaving A and C in series; then A and B in parallel, C in series with them; A cut out, B and C in series; A and C in parallel, B in series with them; lastly C cut out, leaving A and B in series. There is now no break in the circuit, and the E.M.F. curve will be continuous with only slight fluctuations as the various changes are passed through. These are best followed by cutting out a small disc of paper on which are represented the three coils and commutator segments, and rotating it round a pin on another sheet of paper on which are drawn the two stationary brushes. Each coil is first in parallel, then cut out, again in parallel, then in series, and lastly in parallel again before it is again cut out. The time during which two coils are in parallel depends on the relative angular widths of the air-gaps in the commutator and of the brushes: if they are nearly equal the two coils will only be in parallel for a short time, but if, while keeping the width of the air-gap small, the width of the brushes be increased, this time will likewise be increased. The effect of different widths of the brush surface requires to be carefully considered.

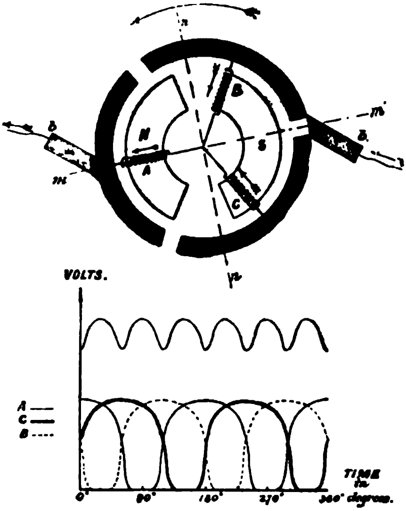


Fig. 89.

If the brush surface be only just wider than the gaps, as in fig. 89, the time when the two coils are in parallel will be a mere instant; and, further, the two coils are then at equal angles from the line of maximum action, *m m′*, so that their E.M.F.'s are precisely alike. Hence we may practically disregard the moments of parallelism and say that during the whole revolution the E.M.F. of the machine is that due to two coils in series, the one approaching and the other receding from the line of maximum action; each coil is under one brush for nearly a third of a revolution at a time, and is cut out of circuit for a sixth of a revolution. The joint curve obtained by compounding together the curves of the three coils in the required manner is shown in fig. 89, and is composed of six crests and hollows. But now let the width of the brushes be greater, and the time of parallelism longer; then the E.M.F.'s of the two coils in parallel are not always equal, and there is a tendency for the one of higher E.M.F. to drive a back current through the one of lower E.M.F. by means of the path of the brush surface. The effect is exactly the same as was previously described under the first type of open-coil machine; when first thrown into parallel, the weaker coil by reason of its self-induction does not admit of a large back current being instantly passed through it, while when about to be cut out of parallel, the inequality of E.M.F. is positively of advantage, since it serves to stop or even reverse the current at the moment when the circuit of the coil is to be broken. Next, let the width of the brush be just 60° (fig. 90), i.e. half a commutator segment plus half an air-gap. This entirely alters the series of changes, for now the coils are open-ended for a moment only, and except for these brief moments, there are *always* two coils in parallel and in series with the third. Each coil is under one brush for half a revolution, and this time is divisible into three equal parts: for

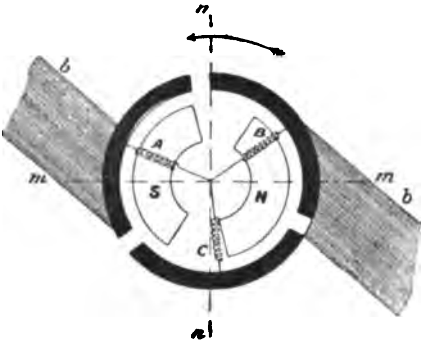
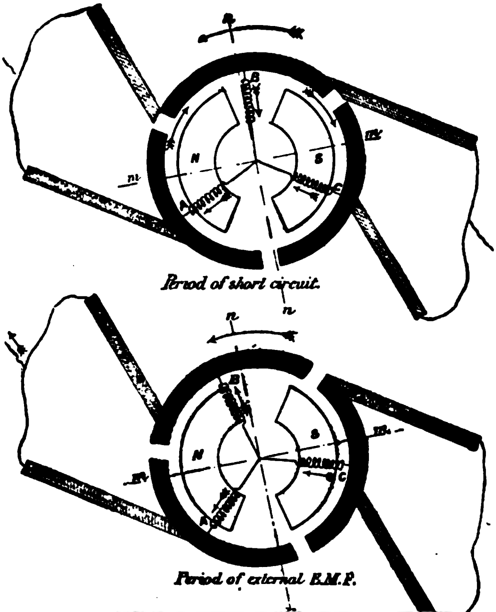


FIG. 90.

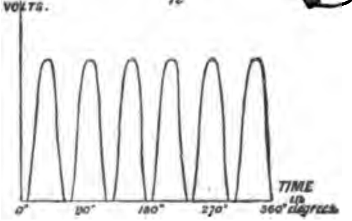
a third of it, it is in parallel with the coil in front of it; for the next third, it is in series with the other two which are in parallel; and for the last third it is in parallel with the coil behind it. It will be seen that the most active coil is in series only for a sixth of a revolution at a time, and that when it is only 30° from the line of maximum action, it is thrown into parallel with a less active coil, which is itself practically on the line of no action. But this latter coil has itself had a reverse current started in it by the higher E.M.F. of the coil from which it has just parted company; for a coil is only withdrawn from parallelism when its partner is 30° from maximum action, and it is itself on the neutral line, and therefore it is already prepared to be put in parallel with the third coil, its self-induction sufficing to keep up the flow of current until its E.M.F. is equal to that of its new partner. Shortly after, its E.M.F. becomes higher than that of the coil with which it has just been put in parallel; consequently in its turn it prepares this latter coil to receive a reverse current prior to its leaving one brush and coming under the other. By such mutual action and reaction, the total E.M.F. of the machine is kept fairly constant, and there are not the violent fluctuations which might perhaps be expected from so small a number of coils so continually rearranged as to their connections. The crests and hollows of the E.M.F. curve are still the same as when the brushes were only slightly wider than the gap. Yet the average E.M.F. throughout a revolution is not quite so great as that which is obtained when the width of brush is less, and the difference of E.M.F. of the two coils which are placed in parallel is smaller.

Next let the width of the brush be more than 60° (fig. 91); then the two brushes can both rest on the same segment at once, and the whole machine is for the time being short-circuited on itself; the segment which is under both brushes at the same time affords a path of almost negligible resistance to a local current, due to the combined E.M.F.'s of the two coils attached to the other two segments, neither of them being far from its position of maximum action; consequently the E.M.F. of the machine is for the time virtually abstracted from the external circuit. Such short-circuiting will occur six times in each revolution, and the actual length of time during which each short-circuit lasts will depend on how much wider the brush surface is than 60°. Further, the time during which each coil acts separately before it is thrown into parallel with a coil of weaker E.M.F. is proportionately reduced ;



Period of short circuit.

Period of external E.M.F.



Curve of E.M.F. at the brushes.

Fig. 91.

for example, if the width of brush were as much as 80°, and the air-gaps are 10°, as in fig. 91, each coil would be in series for only 25° on either side of the line of maximum action, or for only about one-seventh of a revolution at a time. The total result is, therefore, that the average E.M.F. which the machine supplies to its terminals throughout a revolution is very largely reduced ; the actual height of the crests is reduced, but chiefly owing to the six periods of short-circuit, each lasting for 10°, the total area of the curve of E.M.F. acting at the brushes is much diminished. The local short-circuit current does not produce the heating that might at first be expected; in fact, in a machine running at 800 revolutions per minute it has to be started 4,800 times per minute, and each time the self-induction of the coils prevents it from growing to any great strength.

Evidently, therefore, if an arc of 60° were chosen as the normal width of brush contact, and an easy means were provided for altering this width, the voltage of the machine could be regulated within certain limits by increasing or decreasing it accordingly as we require a lower or greater E.M.F. The use of this device in connection with the Thomson-Houston arc-light dynamo will be described later; suffice it here to say that it is carried out in practice by dividing each brush into two, set at a certain angle apart (fig. 91), which angle can be varied according to the requirements of the moment; the two brushes are electrically connected together, so as to form in reality one brush spanning a considerable arc of commutator surface.

Two forms of 'open-coil' armatures have now been described, and beyond these no other 'open-coil' arrangement has ever been successfully introduced and embodied in a practical machine.

CHAPTER XI

CLOSED-COIL ARMATURES

THE third and last class of bi- and multi-polar dynamos still remains for consideration. Like the machines of the last chapter, they give a uni-directed E.M.F., and therefore presuppose a commutator; but they differ in that this E.M.F. is for a uniform speed of rotation constant in value during the whole period of a revolution; the current, therefore, which such a constant E.M.F. causes to flow through a circuit of fixed resistance is itself also constant and steady, without fluctuations or pulsations. Hence, dynamos of this type are also called 'continuous-current' machines, the E.M.F. and current which they yield being not only continuous in direction, but continuous also in value.

The purposes to which such machines can be applied are so numerous, and so important, that they are used much more extensively than any other class of dynamos; whether it be incandescent lighting, charging accumulators, motive purposes and transmission of power over long distances, or electro-deposition of metals, they are equally suitable for all alike, while the voltages for which they are made vary from 5 to 3,000 volts. Their characteristic features will be found to be intimately connected with the fact expressed by their other name of 'closed-coil' dynamos; they are so called because the winding of their armatures forms an endless coil closed upon itself, and, in contrast to the previous sub-class, no portion of the armature-winding is ever open-circuited, or has its connections with the rest rearranged.

Since the E.M.F. of machines belonging to the present group is to be steady and free from fluctuations, it is reasonable to recur to and extend the method which was found so effective in the case of open-coil machines, viz. that of placing two or more coils on the armature in positions which are not symmetrically opposite to each other. We have, therefore, first to determine how the E.M.F.'s of such coils can be combined together, their connection with the external circuit being still reversed at the right moment when the direction of the E.M.F. induced in them changes. Further, and most important, this must be done, not only without opening or breaking the entire circuit, but also without opening or leaving open-ended any coil. An entirely new device must therefore be adopted, by which, whenever a coil is in the position of reversal of E.M.F. between two adjacent poles, it is 'short-circuited' on itself. To effect this, it is only necessary to make each of the two brushes of fig. 48 touch both segments of the split-ring simultaneously at the moment of reversal; the coil is then closed upon itself through the interposition of the brushes, and this allows the current previously flowing round the coil in the one direction to die away, and a current in the reverse direction round it to be started by the reversed E.M.F. as it comes under the other pole. The brushes must be so set that the short-circuiting takes place approximately at the position of zero E.M.F., and the curve of E.M.F. of the loop, as given in fig. 48, is thus practically unaltered.

Starting, therefore, with a pair of coils at right angles to each other, and connected with a pair of split-rings, as in fig. 82, instead of widening the insulating gaps (as was done in the case of the first sub-class of direct-current machines, viz. those with open-coil armatures), we must keep the gaps small, give the brushes a sufficiently wide contact surface, and set them in such a position and at such an angle to the split-rings that they bridge over the insulating material between the segments whenever a coil is in the position of reversal. Having done this for both pairs of brushes, let us connect one brush of one split-ring with the brush on the opposite side of the other ring; thus fig. 82 will take the form shown in fig. 92, which represents a two-coil ring, and by its side the corresponding drum-wound armature; as before, for the sake of clearness, the actual connections of the coils to the segments are omitted, and merely indicated by corresponding numerals. Brush *a'* is joined to brush *b*, and the remaining pair of brushes form the terminals to which the external circuit is applied; as shown in the figure, coil B is in the position of reversal, and, therefore, is short-circuited through its brushes; the current flows into *a*, through coil A, which is alone supplying E.M.F., and out by *a'* into *b* and *b'*. But a moment later, when the armature has moved further round, both coils will be in action, and neither will be short-circuited by the brushes; the current due to the E.M.F induced in both will now flow through coil A, and thence through coil B, leaving the armature by brush *b'*. As

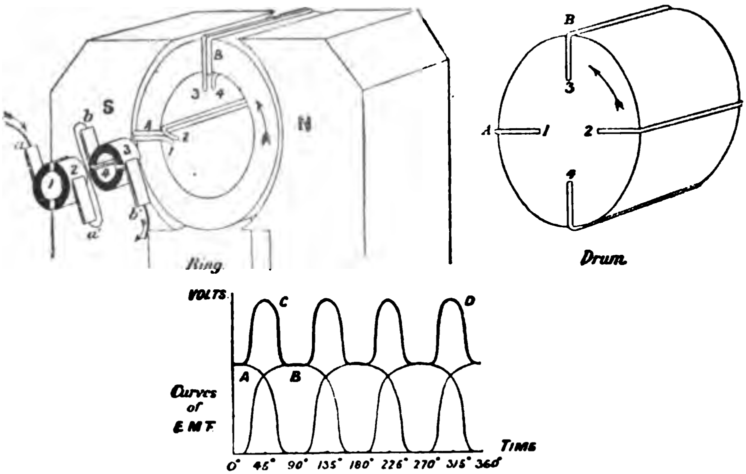


FIG. 92.

the armature continues to rotate, coil A will in turn be short-circuited by its brushes when it reaches the second position of reversal, and coil B will alone supply E.M.F. to the external circuit; finally, both coils will come again into action, until B is again short-circuited at the end of half a revolution.

The armature circuit is thus never broken, and no coil is ever open-circuited; each gives a curve of E.M.F. as shown in fig. 82, but the important result has been obtained that the two coils are now in series, and the curve of the total E.M.F. acting at the brushes *a* and *b'* will be given by adding together simultaneous ordinates of the two separate curves and plotting their sums as a third curve. This has been done in the lower part of fig. 92, the curve C D showing the effect of adding together in series the E.M.F.'s generated at each instant in the two coils. At once it will be seen that the fluctuations in the third curve of joint E.M.F., though still marked, are very much reduced in value from what they are when each coil acts separately. Expressed as a percentage, the fluctuation, which is roughly 100 per cent. on either side of a mean value in the case of a single coil, is reduced to about 30 per cent. in the case of two coils in series.

The curve C D should be contrasted with the open-coil curve of fig. 86. The general effect of both open-coil and closed-coil arrangement in reducing the fluctuation is alike, but there is a difference in the two methods of obtaining it, and consequently in the shapes of the two curves. In that of fig. 86 a coil was first open-circuited, next thrown into parallel, and then acted separately. Now, the two coils are either in series or one of them is short-circuited; as the E.M.F. induced in one coil is diminishing in value, that of the other coil is rising, so that at 45°, 135°, 225°, and 315° the two are equal, and the total E.M.F. then reaches a maximum, while it never falls below the value of the maximum E.M.F. given by one coil when in the position of best action. A further step towards greater constancy of E.M.F. would be made by arranging still more coils at successive small angles in front of each other, so as to come into and out of action successively, and fig. 93 shows the effect of superadding the E.M.F.'s of four coils arranged with angles of 45° between neighbouring pairs, there being only eight small undulations in a revolution. The fluctuation is now reduced to only 10 per cent. on either side of the average value throughout a revolution.

If such third and fourth coils be added, their split-rings must be so arranged that the angles between the relative positions of the diametric line through the insulating gaps of each commutator correspond with the relative angular position of the coils on the armature (fig. 93); and, as before, brushes on opposite sides of the commutators must be joined together in order to sum up the E.M.F.'s of the coils. Evidently the levelling process may be continued until the total E.M.F. is practically continuous and uniform, provided that there be a sufficient number of small separate coils regularly spaced round the armature. More coils must therefore be wound on, and similarly joined together, until finally there is a number of coils and split-rings, all arranged with a regular angular advance on one another until an angle of 180° has been passed through. There will then always be a sensibly constant number of inductors actively cutting the lines of the field, and placed in series so that the average E.M.F. due to them as a whole will be constant. Not only does the continued sequence of one coil after another level the curve, but from the fact that each coil now contains few inductors, and is one of a large number, the effect upon the whole, as each is short-circuited, is enormously reduced. Such a set of symmetrically-arranged coils might be made to cover the whole of the armature core; if composed of drum-wound loops they would embrace the whole of the core, and if of ring-wound loops, then for each coil on the one half of the ring a corresponding coil may be wound diametrically

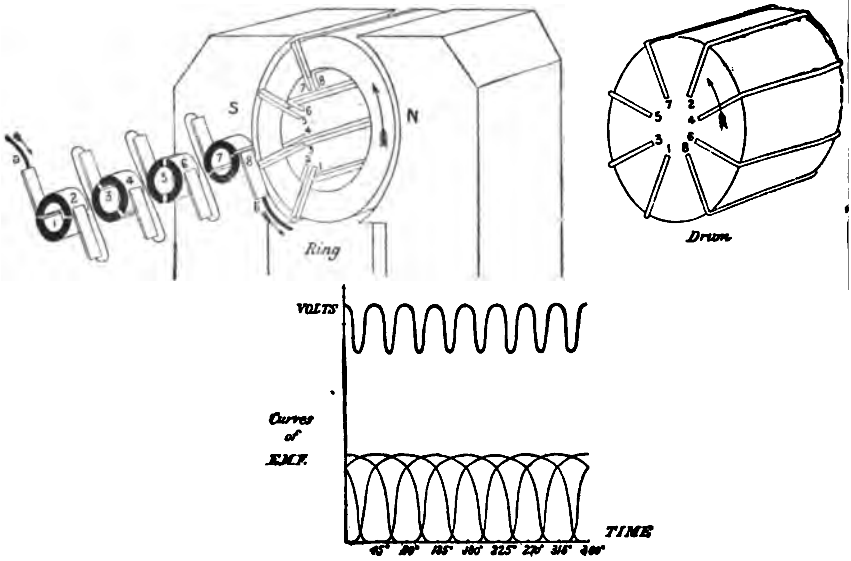


Fig. 93.

opposite on the other half of the core, which will give an E.M.F. similar in phase and intensity, and which may therefore be joined either in series or in parallel with the opposite loop, as in fig. 81. Thus the whole of the armature surface would be utilised for winding, and, with a large number of coils, the fluctuations of the E.M.F. would be reduced to an inconsiderable amount. But this would necessitate a correspondingly large number of split-rings and brushes in a row, which would be at once cumbrous and inconvenient. Hence a simple device is adopted, which results in the division of the armature winding into two equal and similar halves in parallel. In fig. 92 the effect of joining brush *a′* across to *b* is virtually to connect one end of coil A with the beginning of the next coil, B. Similarly, in fig. 93, the end of one coil is electrically connected to the beginning of the next, as 2 to 3, 4 to 5, 6 to 7. If, therefore, the four split-rings of fig. 93 be concentrated into a portion of a tube or cylinder divided into four segmental pieces insulated from each other, upon which one pair of brushes rests, the effect at the moment chosen for illustration will be unaltered. An end view of the new arrangement is shown by the full-line portion of fig. 94. Pairs of segments in fig. 93 are replaced by single segments as indicated by the numerals, and the coils are diagrammatically represented by coiled lines external to the commutator. But immediately after the moment illustrated, as the rotation is continued, brush *b′* will leave the segments, unless the whole arrangement is repeated with other coils so as to make the joint commutator symmetrical, as shown in dotted lines. If this be done, the coils as they pass from between the brushes on the one side are continually replaced by other coils from the other side ; it will, however, be seen that the possibility of having a second set of coils joined to the dotted segments must now be considered.

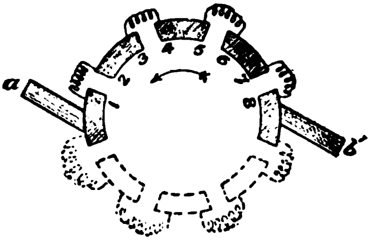


FIG. 94.

First let us repeat the arrangement of fig. 92, so as to obtain four coils symmetrically placed round the armature, and each with its ends attached to its own split-ring commutator (fig. 95); as before, the commutators are shown in perspective, and the connections of the coils are marked by the numerals 1, 2, 3, &c. As in fig. 92, a current can be collected from coils A and B by applying the external circuit to brushes *a* and *b*; similarly, a current can be collected from coils A′ and B′ by applying the external circuit to *a*′ and *b*′. If, therefore, we join all the brushes consecutively on opposite sides of the commutators, and finally join brush *b*′ to brush *a*, both currents can be collected by applying the external circuit to brushes *a* and *b* as before. We now have a dynamo giving an E.M.F. curve identical with that of fig. 92, but carrying twice the current; the two halves of the armature winding are in parallel, and the current entering at *a* divides within the armature into two equal portions, one flowing through coils A and B, and the other through B′ and A′, the two reuniting to leave at *b*. If we now replace the separate brushes and separate commutators by one cylindrical structure divided into four segments insulated from each other, and upon which one pair of brushes rests, the effect is to convert the whole winding of the armature into a closed spirally-wound coil, joined at intervals to commutator segments. The end of one coil is consecutively joined to the

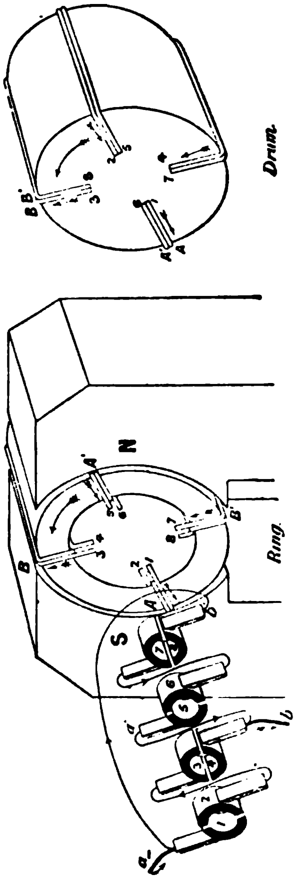


FIG. 95.

beginning of the next, as the numerals show, until the end of the last is joined to the beginning of the first. Fig. 96 shows the joint commutator in perspective, and below are shown the connections of the coils; at the right of the diagram the brushes are shown in the act of short-circuiting coils, A and A′. The whole is now symmetrical on both sides of the brushes, and rotation can be indefinitely continued without, at any time, breaking the circuit or leaving any coil open-circuited. As soon as any coil passes away from one pole-piece, it is short-circuited under a brush, and passes over into the other half of the winding. The complete closed-coil armature is now seen to justify its name, not only because a coil is never opened, but also because it is itself a closed helix, formed of a number of continuously wound loops; starting from any point, the winding can be traced right round the armature, without any breaks, until the starting-point is again reached. In sharp contrast to open-coil machines, the brushes must be set at the opposite ends of a diameter corresponding, not with the line of maximum field, but with the neutral line of zero field.

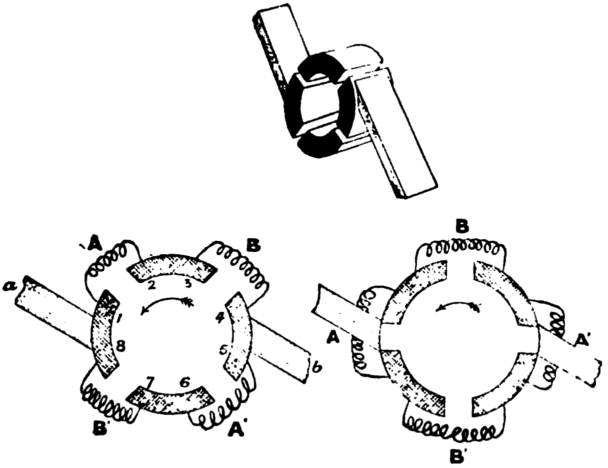


FIG. 96.

Having once arrived at an armature winding divisible into two halves in parallel, it is easy for us to pass from two coils in each half to three or four, or any larger number, in each half; all that is necessary is to introduce more segments into the commutator, each forming a connection between the end of one coil and the beginning of another.

Thus, with eight coils, we get the complete commutator of fig. 94 with eight segments. In fig. 97 is shown a ring-wound armature with an eight-part commutator, and also the analogous drum-wound armature, each with the same number of inductors, occupying the same amount of available space on the core; in the latter each loop is indicated by the same numerals, as 1, 1′, the end connections at the further end of the drum being shown dotted. Both armatures are wound left-handedly, and the current enters and leaves by corresponding brushes; further, the direction of the current along the length of the inductors is shown by marking those in which it flows towards the commutator end with a dot, those in which it flows away from that end being crossed,[1] while the short-circuited inductors are left white. Such armatures with eight coils will give a curve of E.M.F. similar to that given by the four coils of fig. 93, but will again carry twice the current.

To still further decrease the fluctuations of the E.M.F., more coils or 'sections' can similarly be wound on to the armature, with a correspondingly increased number of commutator segments, until with any number of segments more than thirty-six the E.M.F. at the brushes may be regarded as practically constant. With a large number of commutator parts, the simple tubing divided into segments, as shown in fig. 96, has to be replaced by a built-up structure, consisting of a number of wedge-shaped bars of hard-drawn copper, gun-metal, or phosphor-bronze placed side by side, but completely insulated from each other by intervening strips of mica, so as to form a smooth cylinder upon which the brushes rest, the whole being insulated from the sleeve and washers by which the segments are held tightly together.

The fundamental principle which formed the starting-point to a discussion of the armature winding of bipolar dynamos was that

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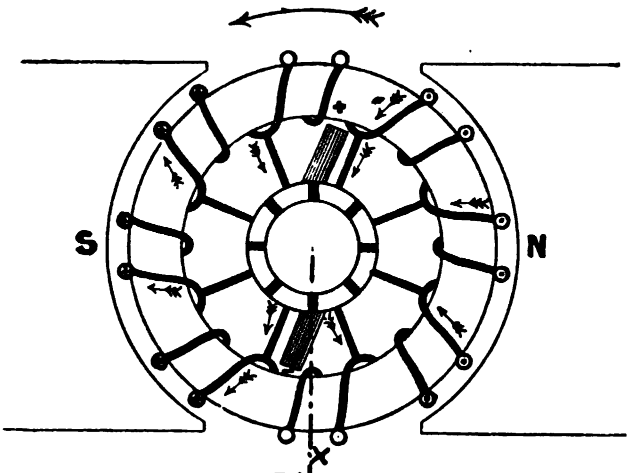
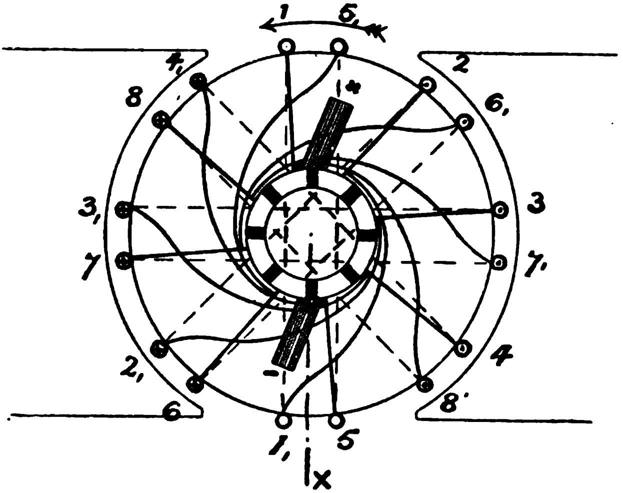
[1] A useful convention, employed by Prof. S. P. Thompson, and easy to recall if we consider the dot as representing the head of an advancing arrow and the cross as the end view of the feathers of a retreating arrow.



Ring



Drum.

Fig. 97.—Eight-coil bipolar armatures.

in all the inductors under, say, the N. pole the E.M.F.'s induced along their lengths were in the opposite direction to the E.M.F.'s in the inductors under the S. pole; an inspection of fig. 97 shows that, by both methods of winding, the small E.M.F.'s induced simultaneously in each inductor are added together in each half of the winding, so that it is their sum which causes the current to flow through the circuit, entering and leaving at the brushes. Further, since the total E.M.F. is practically constant throughout the whole revolution, the total E.M.F. induced at any moment coincides with the average E.M.F.; hence it is sufficient, in order to determine its value, to find the total number of lines cut in any given time by all the inductors which are in series, and divide that number by the time taken to cut them. Every bipolar closed-coil armature overwound with inductors may be divided diametrically into two halves by a line perpendicular to the line of magnetisation of the field, or in other words by a line at right angles to the general direction of the lines of induction as they cross from the one pole-piece to the other; along the outside of the half which is on the one side of the dividing line, say under the N. pole, there is a sheet of current flowing in all the inductors in the one direction, as viewed by an observer at either end; while along the outside of the other half, the sheet of current flows in the opposite direction. The brushes remaining stationary on the revolving commutator form the contact surfaces, by means of which the current is collected from both halves of the winding, and passed into the external circuit, while further, by short-circuiting the coils which are in the position of zero E.M.F. they serve also to 'commute' the current, or enable a coil to pass from the one half of the armature over to the other, where the current round it is in the reverse direction.

The commutator segments which are at any moment situated between the brushes serve merely as junctions between the end of one loop or coil of many loops and the beginning of the next, but their true function is called out as rotation continues, and they pass successively under the brushes. As at any moment they are situated relatively to the brushes, their potential rises by successive and approximately equal increments as we pass from the negative to the positive brush round either side of the commutator. Each commutator segment has its potential raised over that of the segment behind it by the amount of the E.M.F. induced in one coil alone. In ordinary incandescent-light dynamos the volts induced in one coil, and therefore the difference of potential between two contiguous commutator segments, may be taken as varying between two and five volts; thus in a bipolar dynamo with 100 inductors, giving 100 volts, and a 50-part commutator, the two halves are in parallel, so that each inductor gives about two volts; and since there are two inductors to each 'section,' we have roughly four volts generated per commutator bar. It should be observed that with the drum-wound armature the minimum number of inductors in a 'section' is two, whereas

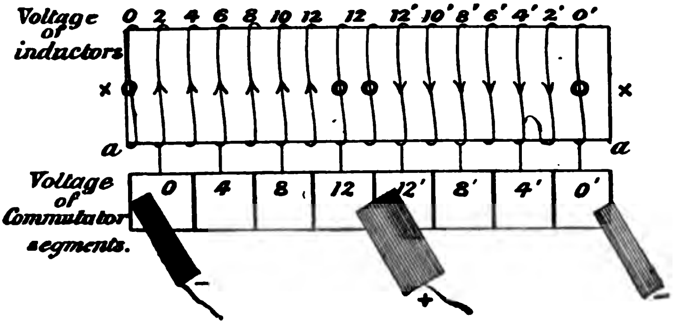


Fig. 98.—Development of ring armature.

in a ring armature there may be only one loop to a commutator bar, and therefore only one inductor; in other words, in the Gramme-wound ring armature the number of loops and the number of inductors are identical, whereas in the Siemens-wound drum the number of loops is always half the number of inductors.

In the ring-wound armature the rise of potential round the sections of the commutator simply reproduces the more gradual rise of potential in the inductors as we pass from the negative to the positive brush round either half of the armature winding. In fig. 98 the ring of fig. 97 is supposed to have been cut along its length at the point marked ×, and to have been opened out flat. Arrow-heads on the lines representing the external inductors indicate the direction of the current in each inductor, those which are short-circuited by the brushes being marked by open circles; and the rise of potential in the inductors and in the commutator is shown on the supposition of two volts being generated per inductor. Between adjacent coils there is no great difference of potential, but each coil adds on its E.M.F. until their sum acting from brush to brush becomes considerable. Even in 'high-tension' machines, giving over 1,000 volts, the number of volts per coil or section is not very great; and, therefore, the difference of potential between any two neighbouring wires is correspondingly small. In this lies a distinguishing feature of the ring armature which separates it from the drum, wherein the full difference of potential of the machine exists between two pairs of neighbouring inductors, as is in fact shown by the drum of fig. 97; for there it is seen that the two short-circuited loops, one of which is under the positive brush and the other under the negative, are contiguous to each other, i.e. loops 1, 1′, and 5, 5′, while further between the short-circuited loops and the neighbouring loops on either side the full pressure of the whole E.M.F. of the machine exists.

The winding of the drum armature requires further consideration, for there is more than one method by which the inductors can be joined together into loops, so as to fulfil the requisite conditions of an armature circuit of continuous winding closed upon itself, through which the current divides into two halves, flowing in one direction along one half of the circumference, and in the opposite direction along the other half. In the bipolar dynamo each loop of a drum armature must approximately embrace half the circumference of the armature, or in effect be wound nearly diametrically across the ends of the drum; but it is not necessary that the width of the loop should be exactly equal to the pitch, so long as it be greater than the width of the field, and therefore has but little E.M.F. induced in it when in the position of commutation. Hence if we distribute a number of inductors equally spaced round an armature core, as in fig. 99, and start a loop from the commutator segment marked 1, we can form the second side of the loop by means of any of the three inductors marked 8, 9, 10, and, further, the loop so formed may be ended at either of two commutator segments, viz. 2 or 8, the one so chosen forming the starting-point for a second loop. In the case of an even number of commutator parts, as shown in fig. 99, to join 1 with 9, the diametrically opposite inductor, gives an irregular and unsymmetrical winding, which is now seldom if ever used. Hence one inductor is connected to another which falls just short of being diametrically opposite. Having decided on which side of the diametric line passing through the first inductor the loop is to be completed, i.e. whether by inductor 10 or inductor 8, two distinct methods present themselves, according as the commutator segment which forms the junction of the first and second loops is on the opposite or the same side of the diametric line as the first loop. Further, each of these two methods may be right- or left-handed; and, lastly, the connecting wires or strips at the ends of the drum while joining any two given inductors may pass round on either side of the shaft.

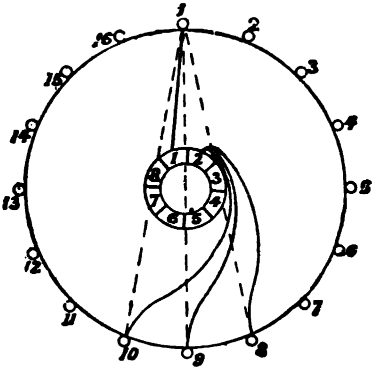


Fig. 99.

First let the commutator segment, which ends the first loop and begins the second, be on the opposite side of the diametric line to the first loop. The left-handed form of this first method joins 1 with 10 at the further end of the drum, and thence proceeds to the commutator segment numbered 2; it has already been shown in fig. 97, but is repeated in fig. 100, where the inductors are numbered successively all round the armature instead of by their loops; further, the end connections of the armature instead of being shown passing straight across from side to side at the far end are curved round the shaft, as in practice they must necessarily be.

In fig. 100 the far-end connection of 1 and 10 is shown to the left of the shaft, as in practice is most advantageous, since if the connector passes round to the right of the shaft its length is greater than it need be; and the same would hold good for all the connectors at the further end of the armature. Consequently

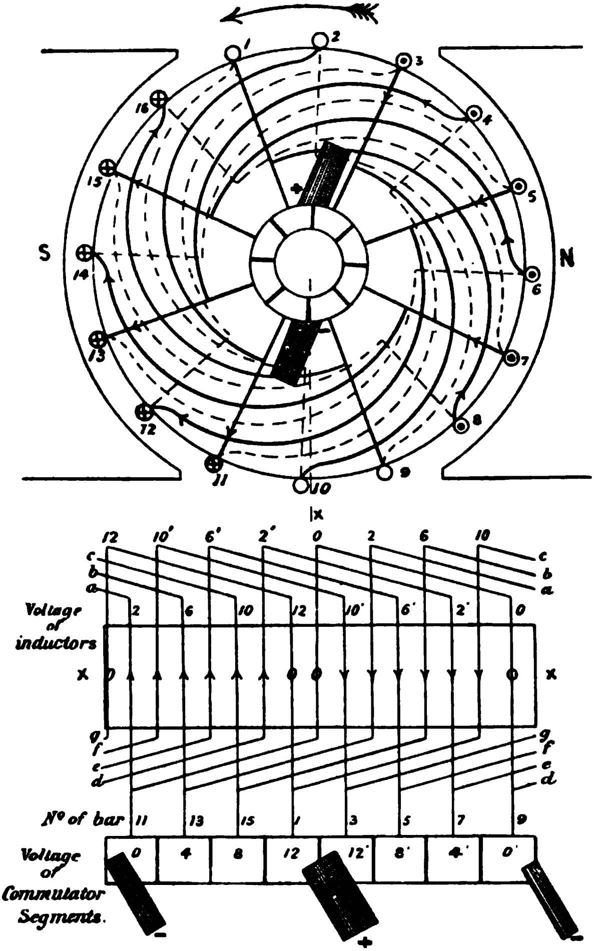


FIG. 100.—Left-handed drum winding on method 1 (wave winding).

with this method the connectors at the two ends of any bar should be arranged so that they pass round the armature shaft on opposite sides. At the bottom of fig. 100 is shown a development of the winding, the armature being supposed to be cut at the point × and flattened out after the same manner as the ring armature in fig. 98; the inductors, while of the same length, are supposed to be arranged so that at each end of the drum every other bar projects beyond the intermediate ones; the end connections are shown by lines all equally inclined to a centre line round the armature, and joining the long end of one bar to the short end of another, while the letters *a*, *b*, *c*, *d*, &c., mark corresponding portions of the end connectors, where they would join if the armature were again bent up into a cylinder. If the winding thus viewed in plan be followed out, it is seen to work continuously forwards in zigzag fashion, and this corresponds with the fact that the slope of the connectors at either end of the drum is in opposite directions, as viewed from the commutator end. This method is known as 'wave winding.'

The whole is perfectly regular and symmetrical, each loop being of exactly the same total length as every other, and spanning the same extent of the circumference of the armature; if $\tau =$ the total number of inductors, each connector spans $\frac{\tau}{2} - 2$ inductors. The rise of volts round the two divisions of the armature is marked at the top on the supposition of two volts being generated per bar, and also on the commutator segments; it should be contrasted with that of the ring armature in fig. 98.

The corresponding right-handed form of the first method joins inductor 1 in fig. 99 with inductor 8, and ends the first loop at the commutator segment marked 8, which, relatively to the diametric line passing through inductor 1, is on the opposite side to the first loop; fig. 101 shows the winding complete. It will be seen that it is exactly analogous to the left-handed winding of fig. 100, but now with the same arrangement of bars and the same field the current enters at the top brush and leaves at the bottom.

The second entirely distinct method ends the first loop and begins the second at a segment on the same side of the diametric line as the first loop. The left-handed form joins inductors 1 and 8 and ends the loop at segment 2, and is shown in detail in fig. 102. It will be seen that the current enters and leaves by the same brushes as in the left-handed form of the first method. The winding is again perfectly regular and symmetrical, but by this second method it works backwards and forwards round the armature; hence it is known as 'lap winding,' since the series of

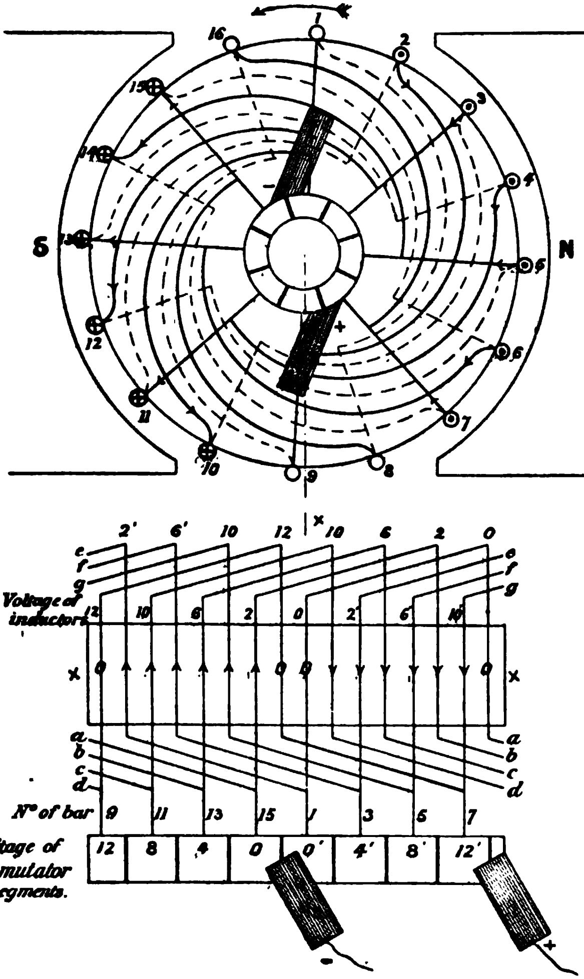


FIG. 101.—Right-handed drum winding on method 1 (wave winding).

loops overlap each other, the first inductor of every following loop lying within the arc embraced by the preceding loop. With the same arrangement of bars as before, the slope of the connectors at either end of the drum is in the same direction as

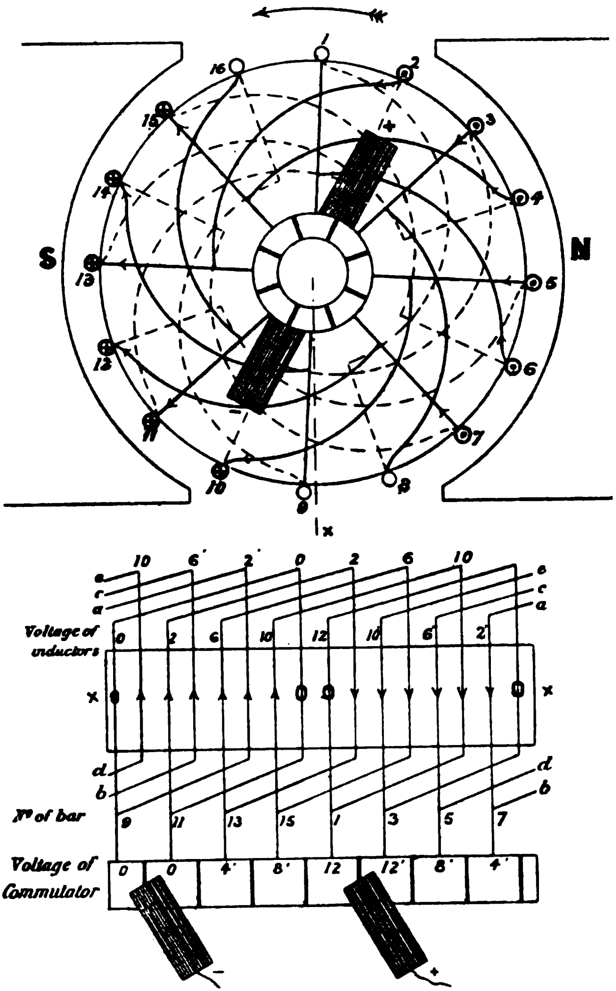


FIG. 102.—Left-handed drum winding on method 2 (lap winding).

viewed from the commutator. Each connector at the far end spans $\frac{T}{2} - 2$ inductors, while those at the commutator end are shorter and only span $\frac{T}{2} - 4$ inductors.[1]

So far the number of commutator segments has been assumed to be even; there remains for consideration the question of an uneven number. In this case each loop is made perfectly diametric, so that the two previously described methods may be regarded as coalescing, and the only variation possible in the winding is that it may be either right- or left-handed. Fig. 103 shows a left-handed armature with eighteen inductors and nine commutator segments; the winding is regular and symmetrical in so far as each loop is of the same length, and the number of active inductors in each half of the armature winding is identical; but only one section is short-circuited at a time. Thus, in the diagram the lower brush is in the act of short-circuiting loop 9–18, but when the armature has rotated through a small angle, loop 1–10 will be short-circuited by the upper brush.

Owing to the fact that the two halves of the armature winding of any bipolar continuous-current machine are in parallel, they may be likened to two similar batteries joined in parallel so as to supply current in equal degree to one and the same external circuit (fig. 104). For this to be the case it is necessary that the E.M.F.'s and resistance of the two batteries should be alike, but it is even more important in the case of the dynamo that the E.M.F.'s induced in each half of the winding and their resistance should be exactly equal to each other. For consider the consequences if in the one half of the winding there is generated an E.M.F. greater by one volt than that which is generated in the other half. First suppose that the machine is running 'on open circuit,' that is, without any brushes to collect the current; then since the winding, whether ring or drum, forms an endless helix closed on itself, we should virtually have a constant E.M.F. of one volt driving a

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[1] It may be remarked that for both the above methods of winding it is not necessary that the inductors should be of the same length and arranged as shown in the diagrams; they may also be alternately long and short, the effect being simply to alter the slope of the connectors at one or other end, rendering it the same at both ends in method 1 and opposite in method 2.

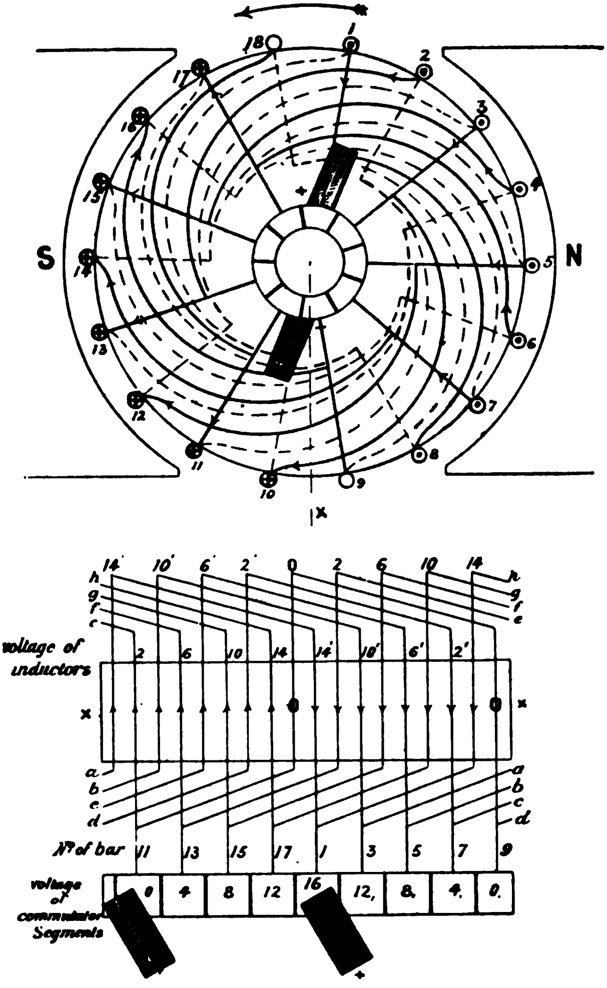


FIG. 103. Left-handed drum winding with uneven number of segments.

useless current round the armature through the resistance of the entire winding in series. Such a local current must absorb a corresponding amount of power, and this evil may assume considerable proportions if we remember that the resistance of all the armature coils in series may not be more than the hundredth part of an ohm, in which case a current of 100 ampères is produced, and power is wasted at the rate of 100 watts. Next let the brushes be 'down' in contact with the commutator, and suppose that current is being supplied to an external circuit (fig. 104); then if we reckon from the negative brush as at zero potential, the voltage of the commutator segments upon which the positive brush rests must be equal to the internal E.M.F. of that half of the armature which has the highest E.M.F. minus the loss of volts corresponding to the passage of the current through the resistance of that half. *E.g.* let E_1 be the internal E.M.F. induced in the one half of the winding, and c_1 be the current flowing through its resistance r_1; then so long as $E_1 - c_1 r_1$ is greater than E_2, the second half of the winding will not supply any current to the external circuit, and, further, will have a wasted current driven in a reverse direction through it; while the first half of the winding will carry both the useful and the useless currents, these together making up c_1. But if the resistance of the external circuit be decreased, and so C_e, the external current, be increased, the loss of volts over r_1 will be increased, until after a certain point is reached $E_1 - c_1 r_1 = E_2$; then $c_1 = C_e$, the second half of the winding being entirely useless. If

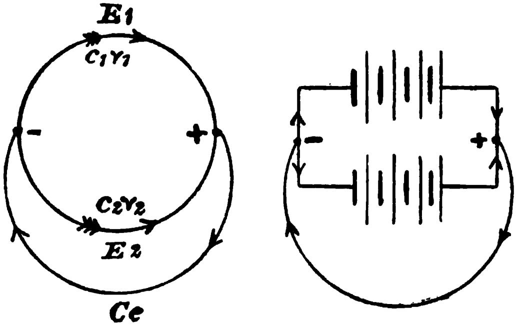


FIG. 104.

the external useful current be further increased, $E_1 - c_1 r_1$ will become less than E_2, and the useful current will now divide through the two halves in such proportions that $E_1 - c_1 r_1 = E_2 - c_2 r_2$, either value being the potential of the commutator segment E_b on which the positive brush rests. Hence, if $r_1 = r_2$,

$$c_1 - c_2 = \frac{E_1 - E_2}{r_1};$$

in other words, the current divides in unequal proportions through the two halves of the winding, the current through r_1 being greater than that through r_2 by the fixed amount necessary to lower E_1 to a terminal voltage equal to E_2. Hence if the inequality of E.M.F. be great, an excessive current will be continually flowing through one half of the winding, while the other is carrying less than that which it is designed to pass without overheating.

The above is the result of inequality in the E.M.F.'s induced in two halves, which have the same resistance. Next let their internal E.M.F.'s be equal, but not their resistances, r_2 being greater than r_1. Then on open circuit there is no loss; but when the external circuit is closed, since the terminal voltage E_b of either half must be the same,

$$E_b = E - c_1 r_1 = E - c_2 r_2;$$

whence

$$\frac{c_1}{c_2} = \frac{r_2}{r_1};$$

and therefore the current divides unequally in the inverse ratio of the two resistances. As the armature rotates, this division of the current will adjust itself to the changing pair of resistances, and hence the current in each half will fluctuate, which is again undesirable.

It is therefore essential with either the ring- or drum-winding, or any development of them, that both the internal E.M.F.'s and the resistances of the portions of the armature which are at any time in parallel should be very approximately equal.

In the case of a drum armature overwound with loops, since each loop passes simultaneously through the fields on both sides of the armature, it is unimportant, so far as equality of E.M.F. of the two halves is concerned, whether as many lines are cut by each inductor when moving under the one pole as when moving under the other; and therefore one field might, owing to a lateral leakage of lines from the core, be weaker than the other without prejudicial effect. But in the case of the ring armature the number of lines entering it on the one face must be the same as the number leaving it on the other, if two similar halves of the winding are to produce equal E.M.F.'s. In practice, however, this condition is always fulfilled, and need not, therefore, be further considered. But given fields of equal strength and similar distribution on either side of the armature, it is evident that, whether it be ring- or drum-wound, since it rotates, the actual set of coils which are in parallel at any moment is continually and progressively changing. Hence in order to secure equality of internal E.M.F. and of internal resistance in the two halves each loop should be exactly similar to every other loop in all respects, or at least all the sections of the winding as attached to the commutator segments must be alike. They must therefore consist of the same number of turns or loops, with the same resistance and wound similarly; they must all be symmetrical relatively to the axis radially and circumferentially. If wound in more than one layer, the length of wire in each coil must be the same in order that all the coils may have the same resistance. In ring-wound armatures this is easily secured, since each coil, whether in one or many layers, is wound exactly similarly to every other; but in drum-wound armatures, in which there are two or more layers of winding, precautions must be taken to ensure an equal length and resistance in each and every coil.

In our previous diagrams of drum armatures the winding has been disposed in one layer only, and the loops of one half of the winding occupy intermediate spaces between the loops of the other half; now, in the case of small drum armatures, in which the winding of each coil of many turns is composed of an unbroken length of wire wound along the surface and over the ends of the core without any joints, the above arrangement must be reproduced; that is to say, intermediate spaces must be left between each successive coil of the first half of the winding, so that the remaining coils forming the second half of the winding may be wound into the spaces so left vacant, the whole surface of the armature being thus eventually covered with one uniform layer of winding. In practice, after the first coil has been wound the core would be turned round through an angle of 180°, and the coil belonging to the diametrically opposite commutator segment would then be wound side by side with the first coil, after which the core would again be turned round, and another coil wound, the beginning of which would be afterwards joined to the same commutator segment as the end of the first; thus, in fig. 97 coils 1, 1′ and 5, 5′ would be wound first, and then coil 2, 2′. The same process would again be repeated, until the whole number of coils was wound, and each would thus have the same length of wire and be similar to every other. The total number of loops to be wound may, however, be so large that with wire of suitable diameter they will require to be disposed in two or more layers. When this is the case the simplest plan would be to entirely cover the surface of the core with half of the total number of coils, and then wind the second half in a second layer on the top of the first. But this simple method is open to the great objection that the coils of the second half of the winding are longer than those of the first half. Two courses are therefore open to us, by either of which this evil is obviated: either each coil must be built up in two or more layers, so as to exactly reproduce the arrangement described above for a single layer, as is shown in fig. 105, where the black circles indicate the loops of a coil belonging to a diametrically opposite commutator segment; or each complete coil must be split into two equal portions, and the second half wound on the top of the first half of the diametrically opposite coil. Fig. 106 shows a drum-wound armature with each coil divided into two halves: these lie alternately over and under the corresponding halves of the other coil which belongs to the diametrically opposite commutator segment; consequently one half of each coil is wound

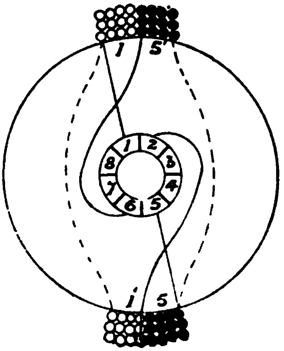


Fig. 105.

first, and the second half can only be completed when the first half of the neighbouring coil has been wound. In the diagrams the beginning and end connections of each coil are shown, the intermediate connections between the two halves of each coil, which are omitted for the sake of clearness, being easily traceable.

Next, in ring or drum the brushes must be set at the opposite ends of a diameter, so as to divide the commutator segments, and therefore the armature coils, into two equal halves.

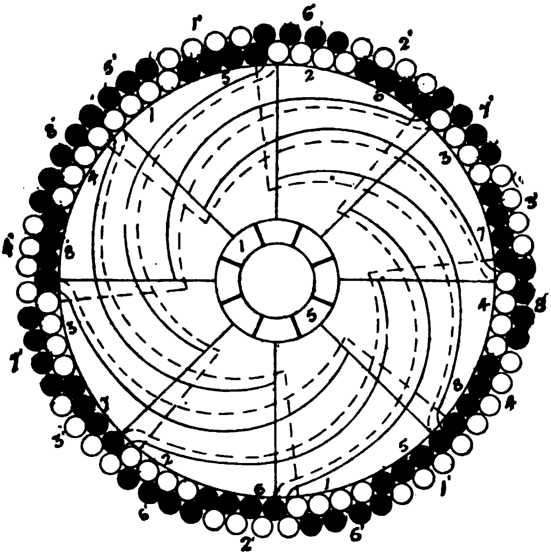


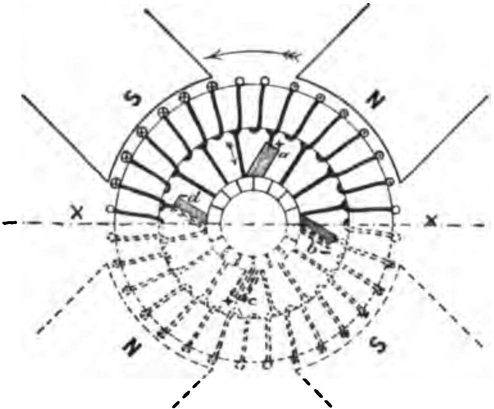
FIG. 106.

We now have an armature which is electrically symmetrical, and this is sufficient in the case of drum-wound armatures to secure equality of the E.M.F.'s of the two halves and an equal division of the current between them. But in the case of rings a further condition must be fulfilled, and this is that the field should be symmetrical. We have so far only assumed that the number of lines cut by the inductors on either side of the armature is the same; but, more than that, if, as in fig. 44, we divide the armature into two halves by the dotted line S N, passing through the centre of the fields on either side of the armature, the distribution of the field in the upper half must be similar to the distribution in the lower half. If the density of the field, *e.g.* at the lower corners, be greater than the density at the upper corners, the exact position in which a coil ought to be short-circuited by the brush will not be the same at the top as it is at the bottom; hence if both brushes are set so as to short-circuit the coils at their proper moments the armature will no longer be electrically balanced, since the portion of the winding between the brushes on the one side will not be similar to the portion between them on the other side, and therefore their resistances, if not their E.M.F.'s, will be unequal. Hence, for ring armatures, not only must the winding be electrically symmetrical, but the field must also be magnetically symmetrical.

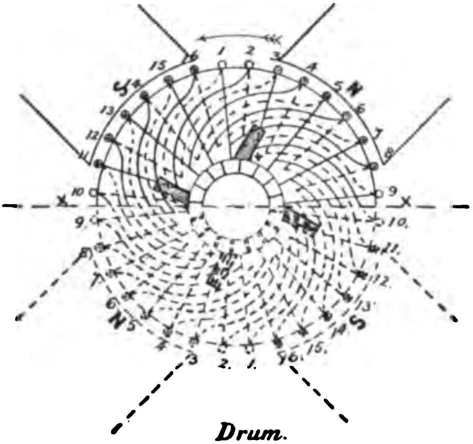
To sum up,—the condition of a successful continuous-current armature with a commutator, of which one portion is in parallel with another, is perfect symmetry, by reason of which the two portions are as nearly as possible similar.

It will now be clear that the bipolar continuous-current machine is in reality two dynamos in parallel, each giving the same E.M.F., but together capable of carrying twice the current that either could carry by itself without overheating.

From this point it is easy to pass to multipolar continuous-current machines whose armatures are divisible into as many parallels as there are poles. By the same method as was applied to alternator armatures (p. 145, figs. 70, 71) let us cut the ring or drum of fig. 97 radially along the line ×, and open them out; then the whole can be repeated a second or any number of times: in fig. 107 the dotted lines show a repetition of the bipolar armatures of fig. 97, the full and dotted lines together forming a 4-pole dynamo. The winding still forms a closed spiral, the only changes being that there are as many points of collection of the current as there are poles, and that the loops of the drum span a smaller proportion of the circumference, the extent depending on the number of poles. In a 4-pole field the loops of a drum, instead of spanning approximately half the circumference, as in the 2-pole field, will now span approximately a quarter of the circumference. Similarly, in a 6-pole field the end connections join together inductors separated approximately by 60°, or a sixth of the circumference. Fig. 108 shows a development of the winding of the 4-pole drum armature of fig. 107. A 4-pole dynamo is, in fact, two bipolar



Ring



Drum.

FIG. 107.—Parallel-wound multipolar armatures.

machines with a common armature core, and therefore four dynamos in parallel, together capable of carrying four times the current that each quarter of the winding could carry separately.

Just as in the 2-pole dynamo there are two neutral points where the coils are commuted, and two corresponding points on the commutator where the current is collected by the brushes, so for the parallel-wound armature placed in a 4-pole field there are four such points. To collect the current, opposite brushes *a* and *c*, *b* and *d*, are joined together, or, as this is virtually equivalent to connecting opposite commutator bars, the same end can be attained by means of a set of cross-connections, which join together such bars as are at the same potential. When thus cross connected there are only two external points of collection and two sets of brushes; in a 4-pole machine each commutator bar must be connected to that which is diametrically opposite, and the two positions for the brushes are at right angles; in a 6-pole field each cross-connection must connect three segments situated 120° apart, the brushes being placed 60° or 180° apart; in an 8-pole field each cross-connection must join four segments 90° apart, and the brushes are 45° or 135° apart (cp. p. 254).

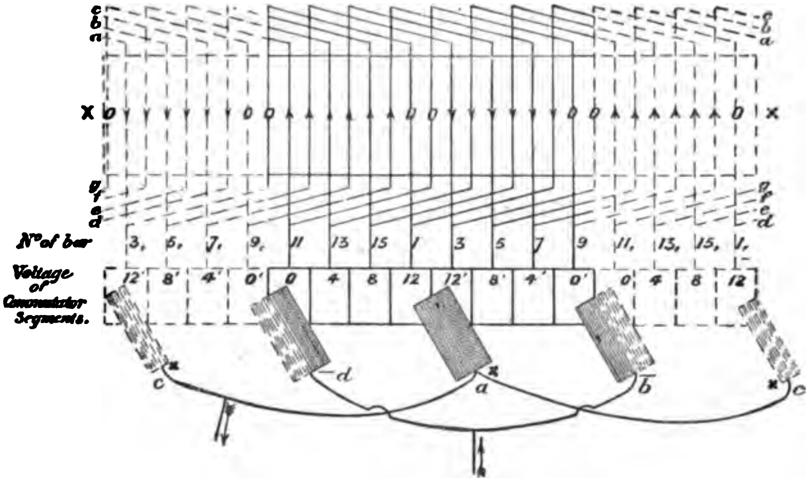


FIG. 108.—Development of parallel-wound multipolar drum.

There still remains the multipolar drum wound armature, in which the winding is not divisible into as many parallels as there are poles, but is 'series-wound,' as it is termed; by this method the E.M.F.'s produced by half the number of inductors are summed up in series, just as in the bipolar continuous-current

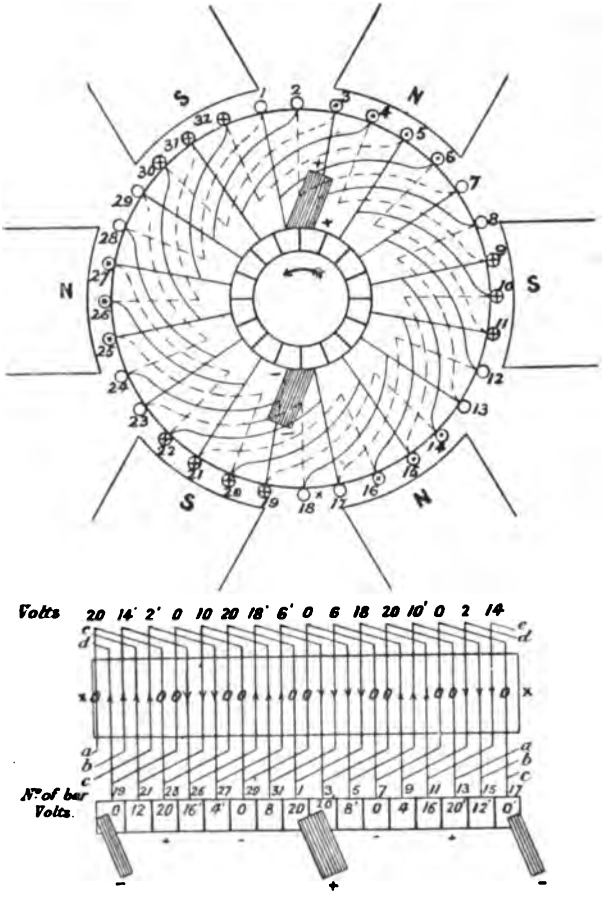


FIG. 109.—Series-wound multipolar drum.

dynamo, and there are therefore only the two halves of the armature in parallel, whatever the number of poles. Starting from any one inductor, the 'wave' winding works continuously forwards in a zigzag round the entire armature, and, after traversing as many inductors as there are poles, returns to the second inductor in front of or behind the starting-point. Thus, in a 4-pole machine, starting along an inductor under, say, a N. pole, the far-end connection joins it to an inductor nearly but not quite 90° ahead of it and under a S. pole; thence the circuit passes on in succession to a third and fourth inductor, nearly 180° and 270° ahead of the starting-point; and finally to a fifth inductor, next but one to the starting-point, a little more or less than 360° having thus been traversed. A similar tour of the armature is again made, and is repeated until the winding closes on itself. In order that the winding should so close, it is necessary that the total number of inductors should be equal to $\text{P} \times n \pm 2$, where P is the number of poles and $n =$ any odd number. Fig. 109 shows a 6-pole series-wound drum armature, the total number of inductors being $32 = 6 \times 5 + 2$. Whatever the number of poles, only two sets of brushes are needed. If the number of pairs of poles be odd, the brushes must be placed diametrically opposite to each other, as in the diagram for three pairs of poles; but if the number of pairs is even, the angular distance between the brushes corresponds to that between poles of opposite sign, being 90° for four poles and 45° or 135° for eight poles.

The formula which determines the value of the internal E.M.F. induced in any continuous-current closed-coil armature is easily obtained, owing to the fact that with any fairly large number of sections in the commutator the E.M.F. has practically a constant value, and therefore it is sufficient to determine the average E.M.F. during any period of time, such as one revolution. In any armature, whether bi- or multi-polar, the number of lines cut in one revolution by each inductor must be $\text{P} . z_a$, where z_a is the number of lines that enter into or leave the armature core by any one pole, and P is the number of poles. In a bipolar machine, where $\text{P} = 2$, the lines passing into and through the armature are cut twice by each inductor in each revolution, once as they enter the core from a N. pole, and once where they leave it to pass into a S. pole, thus making the total number cut in one revolution $2z_a$; similarly, in a 4-pole machine, the number cut by each inductor in one revolution is $4z_a$. In the case of flat-ring machines, in which poles are presented to both sides of the armature, it is most convenient to consider any pair of similar poles facing opposite sides of the armature as forming one polar surface, and z_a as the number of lines entering into the armature from any such pair of similar poles, P being then reckoned only on one side of the armature. Now the time in seconds taken to perform one complete revolution is $t = \frac{1}{\frac{N}{60}} = \frac{60}{N}$, where N is the number of revolutions per minute. The average E.M.F. in volts induced in each inductor in one revolution is proportional to the total number of lines cut divided by the time taken to cut them, and is therefore

$$e_a = \frac{P \cdot Z_a}{\frac{60}{N}} \times 10^{-8} = P \cdot Z_a \times \frac{N}{60} \times 10^{-8}.$$

The total internal E.M.F. induced in the armature is the sum of the E.M.F.'s induced in the inductors which are in series, and it only remains, therefore, to consider how many of the total number of inductors are in series, or, conversely, into how many parallel circuits the armature winding is divided. Let $\tau =$ the total number of active inductors counted all round the surface of the armature; then in bipolar machines half of them are at any one time in series, and consequently the E.M.F. induced in the machine as a whole is

$$E_a = \frac{\tau}{2} \cdot e_a = \frac{\tau}{2} \cdot P \cdot Z_a \times \frac{N}{60} \times 10^{-8}.$$

But P also $= 2$; whence

$$E_a = \tau \cdot Z_a \cdot \frac{N}{60} \times 10^{-8}. \quad \ldots\ldots \quad (11)$$

Similarly, in the case of multipolar machines with parallel-wound armatures, there are as many parallels as there are poles, or the number of inductors in series is $\frac{\tau}{P}$. Hence we arrive at exactly the same expression as for bipolar machines, viz.

$$E_a = \frac{\tau}{P} \cdot P \cdot Z_a \cdot \frac{N}{60} \times 10^{-8}$$

$$= \tau \cdot Z_a \cdot \frac{N}{60} \times 10^{-8}. \quad \ldots\ldots \quad (12)$$

If, however, the multipolar dynamo has a series-wound armature, and $p =$ the number of pairs of poles, or $\frac{P}{2}$, then, since half of the total number of inductors are in series, we have

$$E_a = \frac{\tau}{2} \cdot P \cdot Z_a \cdot \frac{N}{60} \cdot 10^{-8}$$

$$= p \cdot \tau \cdot Z_a \cdot \frac{N}{60} \cdot 10^{-8}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

CHAPTER XII

THE MAGNETISATION OF IRON

THE magnetisation of iron has already been alluded to in Chapter III., but it is now necessary to consider the subject more fully, and at the outset to recur to the experiment of wrapping a number of insulated turns of copper wire uniformly round a ring (fig. 11), and passing a current through the wire. The number of lines of induction that flow through the interior of the magnetised helix is then measured by means of an exploring or test coil and the deflections which it produces in a ballistic galvanometer.[1] Attention has already been called to the primarily important fact that with a given magnetising current the number of lines flowing through the magnetised ring is very much greater if it enclose an iron core than if its coils encircle merely air, wood, or other non-magnetic material. But, further, if we make the same experiment upon two rings of iron of the same dimensions but of different quality, *e.g.* one of soft annealed wrought iron, and the other of cast iron, it will be found that the same magnetising current will not produce nearly so many lines in the cast iron as in the wrought iron, from which it follows that the permeability of the latter is greatly superior to that of cast iron. The physical and chemical qualities of the substance do, in fact, exert a powerful effect upon its magnetic properties, as is shown by the very great differences which exist between the permeabilities of different classes of iron and steel, *e.g.* if steel be alloyed with 12 per cent. of manganese, it becomes almost non-magnetic.

Next, if we take a ring of any magnetic material and vary its magnetising current, the number of lines flowing through it will also vary, but not by any means in proportion to the strength of the current—a very striking fact which sharply divides off iron from air; for the permeability of air is strictly constant, and the number of lines through a ring of air is simply proportional to the magnetising ampère-turns. Roughly speaking, the number of lines induced in iron will for a small number of ampère-turns be almost proportional to varying strengths of current, but after a certain magnetisation is reached each successive increase in the strength of the magnetising current will produce less and less effect; the permeability of the iron, therefore, falls off, or, as it is popularly termed, the iron becomes 'saturated,' and presents a greatly increased reluctance to further induction of lines.

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[1] For a description of the exact method the reader is referred to Ewing's *Magnetic Induction in Iron and other Metals*, chapter iii.; Vignoles 'Researches in Electro-magnetic Induction' in the *Electrician*, May 15 and 22, 1891; Hopkinson, 'Magnetisation of Iron,' *Phil. Trans.* 1885.

Lastly, if we raise the magnetising current to a certain value, and then gradually reduce it to zero, the lines produced in a ring of air also vanish entirely; if, however, there be an iron core to the ring there are still some lines passing through it, even when the magnetising current has been reduced to nothing; in other words, the iron 'retains' a certain amount of magnetism, so that zero magnetising current does not necessarily imply zero lines, and we are confronted with the problem of a flux of lines without any corresponding magneto-motive force. In more general terms, if the current be raised to some high value, and is then gradually reduced to a smaller value, the number of lines flowing through the iron is not the same as when the current had the same smaller value, but was being increased in strength; it is, in fact, greater when the current is being decreased than when it is being increased, although the actual value of the current-strength may be the same. Again we find that no such phenomenon takes place in the case of air or other non-magnetic substance.

It is evident, therefore, that the permeability, μ, of any material depends, not merely upon whether it is magnetic or non-magnetic, but if it be the former, upon three conditions. These may be summed up as follows: (1) Its physical and chemical state, *e.g.* whether it be wrought iron or cast iron, annealed or hardened; or whether it be alloyed with other substances, and what is the percentage in which these are present. But the permeability even of a definite chemical substance in a definite physical state is not a constant quality; it also depends on (2) the value of the induction or extent to which the iron or other magnetic material is at the time magnetised in any given case. Regarded from this point of view, the permeability is a function of B, the number of lines passing per square centimetre. Herein it differs decisively from the analogous property of electrical conductivity, for in the case of the electric circuit the specific resistivity of the material composing it is not a function of the current, but has a value independent of the amount of current flowing through it, save in so far as this indirectly affects the temperature of the conductor. Although, therefore, the analogy between magnetic permeability and electric conductivity is perfect in the case of air and non-magnetic bodies, it is far from holding in the case of iron. While the permeability of iron may be 3,000, or even more, with an induction $B=5,000$, yet when the induction has been raised by special methods to the enormously high value of 45,350, the value of μ is reduced to less than 2. From this it will be seen that although air, gun-metal, zinc, &c. may be regarded as magnetic insulators relatively to iron when the latter is weakly magnetised, yet when the iron is 'saturated' the difference in their permeabilities becomes enormously reduced; for values of the induction in wrought and cast iron such as are ordinarily reached in practice, their permeabilities may be said to stand to that of air in the ratio of about 200 or 100 to 1, but for higher values it continually decreases, until finally for intense saturation it is reduced to about double that of air. Lastly, the permeability depends upon (3) the previous magnetic history of the metal, *i.e.* upon whether it has been previously subjected to a larger or a smaller induction. The iron has as it were a magnetic memory by reason of which the number of lines passing through it differs in the two cases of an ascending or a descending magnetisation. Such being the complex nature of the permeability $\mu=\frac{B}{H}$, it is necessary to examine carefully the whole process of the magnetisation of iron, and represent it graphically by 'curves of magnetisation,' connecting together corresponding values of B and H.

From the known number of turns, T, and ampères, A, flowing in our magnetising helix, we can obtain $\text{H}=\frac{0{\cdot}4\pi\text{AT}}{l}$, i.e. the total magneto-motive force acting round the ring of fig. 11 divided by the length in centimetres over which that force is distributed, or 1·256 times the ampère-turns per centimetre length. This quantity H in C.G.S. units is usually known in the present connection as 'the magnetising force' to which the iron is subjected; but it may also be expressed as the fall of magnetic potential which takes place over a centimetre length of the substance forming the interior of the helix when B lines flow through each square centimetre of its cross-section. By varying A and measuring the corresponding $z=\text{B}a$ (a being the area of the iron in square centimetres) a number of corresponding values of H and B are obtained, and these when plotted as abscissæ and ordinates respectively are joined to form a curve of magnetisation. Let us first take a soft annealed wrought-iron ring; let this be previously unmagnetised, and therefore in an entirely neutral state, or else let it be carefully demagnetised by means of a gradually decreasing alternate current passed through the magnetising helix; let us then determine its *ascending* curve of magnetisation, the current being increased in strength step by step, and the induction being separately determined for each step. At the outset, for very small magnetising forces, the value of B rises at a certain slow rate almost proportionately to the increase of magnetising force; when, however, the force has reached a value of about 1 or 2 C.G.S. units the rate of rise of the induction changes very rapidly to a much increased value; at this new rate the induction again continues to rise almost proportionately to the increase of the magnetising force, but when the latter is raised to a value of from 5 to 10, although the induction B continues to increase, its rate of rise falls off, and becomes gradually less and less rapid. The curve thus obtained for B and H is that marked O E in fig. 110; since the iron was at the outset unmagnetised, it starts from the origin, and rises with the increasing magnetising force, as shown by the ascending arrow. It is divisible approximately into three different portions, marked *a*, *b*, *c*; for a short distance from the origin it is but slightly inclined to the horizontal axis, along which the magnetising force is reckoned; it then passes by a rapid bend into a long part, *b*, of great steepness, and approximately straight; gradually, however, the curve bends over, forming a rounded corner, or 'knee,'

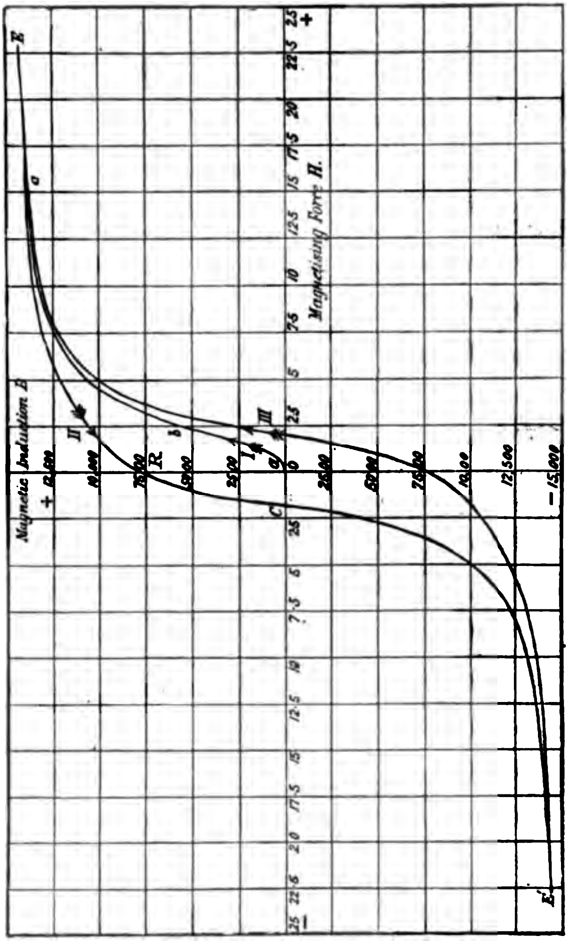

FIG. 110.

and finally continues to rise in a slightly curved line, *c*, inclined at a small angle to the horizontal axis.

After having reached the point E on the *ascending* curve of magnetisation, let us gradually reduce the strength of the magnetising current, and so of the magnetising force. Again measuring values of the induction at different stages, we are thence enabled to trace the curve of *descending* magnetisation, and this, as has been already stated, is by no means identical with the ascending curve. It is marked II. with a downward arrow in fig. 110, and is considerably higher than the ascending curve, so that when the magnetising force is zero it cuts the vertical axis at some point, R, considerably above the horizontal axis. The height O R measures the number of lines still passing per square centimetre of the iron when the magnetising force is gradually reduced to zero, or the 'residual magnetic induction' of the iron; its exact amount depends upon the value to which the induction in the iron has been previously raised, and from which, as a starting-point, the reduction began, while the proportion which it bears to the maximum induction reached is an indication of the 'retentiveness' of the iron. After reaching point R on the downward curve a negative magnetising force is required, or, in other words, the direction of the magnetising force must be actually reversed in order to reduce B to zero. The amount of this negative force, which has to be applied in order to reduce B exactly to zero after it has been previously raised to a high value, is called the 'coercive force' of the iron; it is measured by the length of the line O C in fig. 110.

If, after reaching the point E on the ascending curve, the magnetising force, instead of being reduced, had been still further increased past the value corresponding to that point, the curve would become flatter and flatter, but it never becomes truly horizontal; although the iron may now be said to be 'saturated,' the induction B never ceases to increase when the magnetising force is increased, so that there is no definite limiting value of B beyond which it cannot be raised; the permeability of the iron is enormously reduced, but even when the peculiar action of the iron in increasing the magnetic flux is almost imperceptible, more lines can always be propelled through the helix, just as if it were a simple solenoid enclosing merely air and without any core of iron. 'Saturation' is, in fact, a relative term, and no particular numerical value can be attached to it. The iron is usually said to be 'saturated' when the curve of magnetisation has begun to bend over at the rounded knee between *b* and *c*, but the exact appearance of the curve and position of the bend largely depend on the respec-

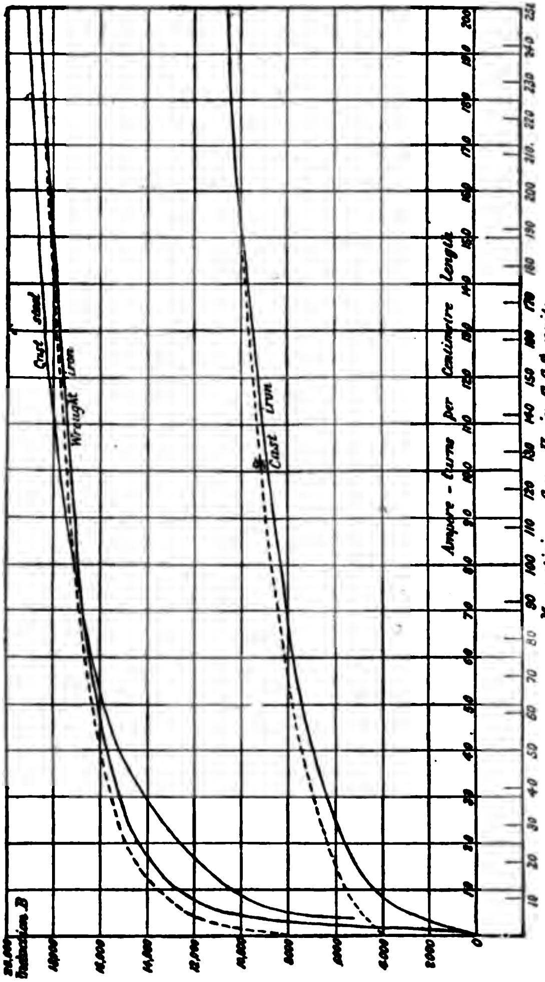

FIG. 111.—Curves of magnetisation connecting B and H.

tive scales to which B and H are plotted. Fig. 111 shows the curves of magnetisation of three different kinds of iron, the ascending curve being shown full and the descending curve dotted: in these the induction and magnetising force (the latter shown in the lower of the two scales along the horizontal axis) have been carried to much higher values than in fig. 110. The scale of H has been altered to obtain a convenient size of diagram, and it will be seen that now the bend of the curve for annealed wrought iron appears to be at B=14,000 instead of at B=11,000, as in fig. 110. The curves of the former figure serve to show the differences existing between the permeabilities of different materials used in the manufacture of dynamos. The most permeable material is charcoal iron of good quality, soft and well annealed, or very pure wrought iron; but mild cast steel containing a small percentage of carbon may almost equal or even surpass ordinary annealed wrought iron; as is shown by the curve in fig. 111 for a mild cast steel containing 0·8 per cent. of combined carbon.

The permeability of iron is largely affected by the presence of impurities, intermixed or in combination with it, such as phosphorus, sulphur, tungsten, or antimony, and also by the closely-connected physical quality of 'hardness.' Wrought iron when rolled into thin sheets or drawn into wires may usually be relied upon as being soft, owing to the processes of manufacture through which it has to be carried, and, further, in this form it admits of a very thorough annealing. When, however, such sheets are stamped or punched out into the shapes suitable for dynamo purposes, it is usual to subject the stampings to a further re-annealing, since by the process of stamping they become, to a certain extent, hardened. In cast iron the effects of impurities and of hardness are again very marked. The iron used for castings should be specially soft and pure, and all hardening or chilling of it after casting should be avoided by allowing it to cool as slowly as possible, or in other words by annealing it. Roughly, it may be said that, for ordinary values of H between 50 and 200, the permeability of cast iron is only half that of annealed wrought iron; but different samples of cast iron show much more divergence among themselves in permeability than would be found in as many samples of wrought iron. The amount of 'retentiveness' and of 'coercive force' likewise varies greatly in different qualities of iron, and is affected by their purity and hardness. It will be seen from fig. 111 that when a high magnetising force is gradually reduced to zero, the residual magnetic induction which persists in annealed wrought iron is about 7,000 or 8,000, while for cast iron it is about 4,000.

The retentiveness (O R) of soft annealed wrought iron is greater than that of any other material, since as much as 80 or 90 per cent. of the induction may be retained. Its coercive force, O C, is, however, very small, and the residual magnetism is quickly reduced by a feeble demagnetising force, or by mechanical vibration, jarring, or tapping, especially when the iron circuit is incomplete. Hard iron and steel, although retaining less magnetic induction than soft iron, keep it much more strongly, and therefore permanent magnets, which are required to maintain their magnetisation, in spite of mechanical shocks or demagnetising influences, are made of steel, the most suitable alloy being tungsten steel, of which the coercive force is as much as 50 C.G.S. units. Were it not for the fact that a high magnetic induction cannot be obtained with hard steel owing to its low permeability, the field-magnets of dynamos would be made of steel permanently magnetised, and requiring no exciting current; but in default of a material which is at once permeable and retentive, and possessing considerable coercive force, we are compelled to employ soft iron or mild steel: these are permeable, but require the exciting current to be continuously maintained round them in order that they may not lose their magnetism entirely by reason of the mechanical vibration to which the dynamo is subject when running.

Reverting to the downward curve of fig. 110, it is seen that the changes in the magnetic state of the iron are not coincident with the changes in the strength of the magnetising force, but lag behind them: this is most forcibly exemplified by the residual magnetic induction which persists when the positive magnetising force has been reduced to zero, and by the fact that the induction only becomes zero after the magnetising force has reached a definite negative value. The physical fact here described is known shortly as the 'magnetic hysteresis' of the iron. In considering this phenomenon it should be carefully noted that it is not a lagging behind in point of *time*, since the actual time taken for the changes in the value of the magnetising force (provided they be not extremely rapid) is immaterial. The magnetic hysteresis depends merely on the order of succession of the different current strengths, and even long intervals of waiting, during which the magnetising force is kept constant, do not obliterate the distinction between ascending and descending curves. What is really implied by the term 'magnetic hysteresis' is that if the magnetising force is reduced from a stronger to some weaker value, the rate at which the magnetic induction becomes reduced with reference to strength of magnetising force, or the slope of the downward curve, is less than the rate at which it increased when the magnetising force was raised from the weaker up to the stronger value; thus, if at point R the magnetising force be again reapplied in a positive direction, the rate at which the induction is recovered is again less than the rate at which it was lost for a change of magnetising force within the same limits.

Let us now continue the descending curve from the point C onwards by increasing the negative magnetising force, and so reversing the direction of the induction. After reaching some point, E′, let us gradually reduce the negative magnetising force, reverse it, and increase its strength in the former positive direction. A new ascending curve marked III is thus traced, which again shows hysteresis: it differs in shape from our previous ascending curve, which started with the iron in a neutral or 'virgin' state, but is analogous to the descending curve II. By carrying the new ascending curve up far enough, it will eventually cut curve II., say at E, and we thus arrive at the same point whence we started to trace the descending curve. A complete *loop* has therefore been described, and the two curves Nos. II and III enclose a certain area depending upon the extent to which they diverge from one another between the points E and E′. The magnetising force has been taken through a cycle of changes in direction and value, eventually returning to the same point as that from which the cycle began, and the iron has similarly been taken from a positive magnetisation to a negative and back again. It is not, however, necessary that the magnetising force should be actually reversed in order that the curve of induction may describe a complete loop; it is sufficient to partially withdraw the magnetising force and then reapply it. Thus, at any point on the ascending cu

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value of magnetising force as corresponds with E on both the two ascending curves. Thus, for simplicity's sake, only three curves are shown, and the cycle of induction of curves II. and III. once traced is immediately capable of indefinite repetition so long as the same cycle of magnetising force is repeated. But even if this be not so, when the magnetising force is continuously varied between two fixed values in either direction, the magnetic effect soon also becomes cyclic, and the induction likewise varies between two values in each cycle. If, therefore, the loops obtained when the iron is carried from a strong positive induction to an *equally* strong negative magnetisation, and back again to the original starting-point, are determined for several different values of the maximum induction, it will be found that the area of the loops, and therefore the energy dissipated in heat in each complete cycle, depends upon the nature of the material and also upon the maximum induction up to which the iron is carried. The amount of this

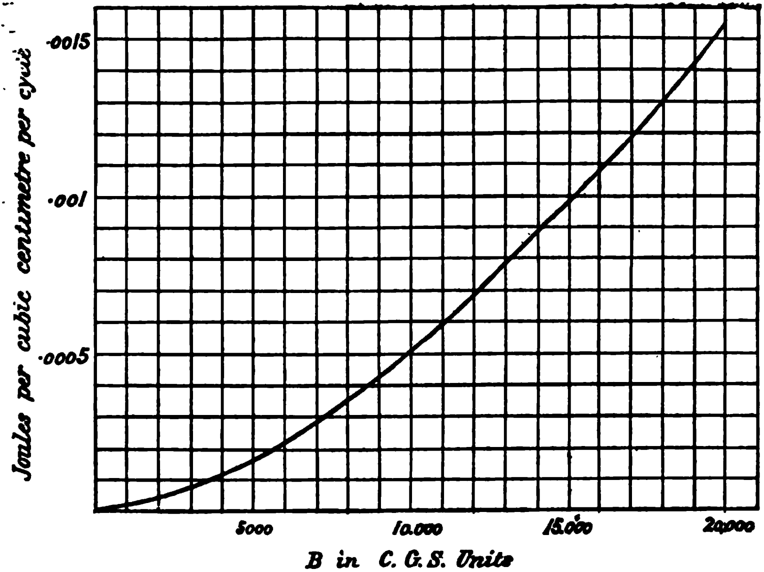

FIG. 112.—Dissipation of energy in iron through hysteresis.

slight increase of the displacement the whole group becomes broken up, and has to be partially or wholly rearranged and reconstructed after a new plan. From the nature of the group above described, it follows that when a group is subjected to a magnetising force which is gradually increased in strength from zero, the first effect is to produce a *stable* deflection of all members of a group, except those which lie exactly along or opposite to the direction of H, and the general lines of the group are still retained. This corresponds with the initial stage of magnetisation (*a*, fig. 110) when the induction increases at a slight rate almost proportional to the increase of the magnetising force. But now let the value of H be increased still further; the members of the group are still further deflected, until at last one or more members become *unstable*. The ties which bound them are then ruptured, and the intermolecular attractions and repulsions constrain it and others to entirely change their grouping, and take up some new configuration, the main lines of which agree more closely with the direction of H. In general, with a considerable number of different groups, this stage of instability will not be reached by them all simultaneously at one given value of H, but group after group will gradually become unstable and break up. This second stage, when the groups are one after another passing through an unstable condition, corresponds to the steep part of the ascending curve (*b*, fig. 110) when the induction increases at a rapid rate. If H be still further increased in strength, we have the third stage (*c*, fig. 110), in which the alignment of the molecules becomes gradually more and more perfect; as each molecule is pulled more and more into line with H, the iron becomes more and more 'saturated,' and its permeability decreases. The deflections of the molecules in their third stage are, however, again stable, as in the first stage.

But now, if after stage 2 the force H is gradually removed, the majority of the groups, having swung over to a new stable condition, retain their new configuration; hence, if the force be reduced to zero, there is still a considerable amount of residual magnetic induction, and it will require the magnetising force to be actually reversed to produce a condition of instability in the new groupings, and so cause them to be in their turn upset and replaced by fresh configurations. Thus the gradual removal of the magnetising force does not lead to the exact and complete repetition backwards of what happened when the magnetisation was being increased; in other words, the movements of the molecules are not reversible without qualification. Given, however, a sufficient number of groups composing the body, if the magnetising force be removed and reapplied, then, unless it be very weak, there are, in general, some groups which pass through a condition of instability. Especially will this be the case if the piece of iron be not perfectly homogeneous, and if, therefore, the lines of the different groups or chains are differently inclined at different places. This exactly corresponds with the observed facts, that hysteresis is always present in all cyclic changes unless the values of H are very small, but that its effect is much less marked for changes confined to the third stage of the curve.

The phenomenon of hysteresis is thus amply accounted for. It occurs whenever on the ascending curve a molecule is deflected from one stable position to another through a position of instability; since then, on the descending curve, the new position will persist until the change in the magnetising force becomes so marked as to cause it to again pass through a position of instability. Further, whenever a molecule passes through a position of instability, there is a dissipation of energy in the form of heat. To quote Professor Ewing's own words ('Magnetic Induction in Iron and other Metals,' § 180), 'when the molecule's equilibrium is upset, it tumbles over and acquires kinetic energy in falling towards a new position of equilibrium. It oscillates about that, and communicates oscillations to its neighbours, until the motion is damped out by setting up eddy currents in the surrounding conducting mass, and the energy of these currents is in turn converted into heat.'

That mechanical vibration should lessen the residual magnetic induction is easily explicable on the molecular hypothesis, since it will cause changes in the distances between the molecular centres. During the swinging thus set up as the magnets recede from each other, their stability is reduced so that they respond more easily to change of magnetising force, and hysteresis is lessened.

It only remains to consider the importance or otherwise of hysteresis in the design of dynamos. If we consider any dynamo with a disc armature, such as fig. 75, and suppose that the coils of the armature are wound on iron cores, it is evident that, as each core passes from one field to another, the magnetising force and induction in it are increased and reduced in the one direction, then reversed, and again increased and reduced in the other direction; hence it is taken through a complete cycle of induction, varying from a positive maximum to an equally great negative maximum, and back again. Further, midway between the two fields there is zero magnetising force, and the molecules of the iron are free to pass through a position of instability. The case is therefore exactly analogous to curves II. and III. of fig. 110. There will be hysteresis, and the determination of the amount of energy lost by hysteresis per cycle, as obtained from the curve of fig. 112 for the maximum induction in the armature, will be strictly applicable. Next, in the unipolar disc alternator of fig. 80 the magnetising force and induction in an iron core would be continually varied through a cycle of changes from a positive maximum to zero. Again, therefore, there would be hysteresis, although the amount of the loss by hysteresis would not be so great as in the former dynamo. Modern machines, however, with disc armatures seldom have iron cores, partly on account of the hysteresis and other losses that would be present in them, and consequently the above cases are not often met with. Now when we pass to ring, discoidal-ring, and drum armatures, whether ot alternators or continuous-current machines, it is true that, as the armature rotates, each molecule of iron must be twisted through an entire circle of 360° in each period; but this is effected by its gradual adjustment of itself to the changing direction of the lines, as it is carried round with the armature, and it is not clear how far the magnetising force is ever removed, so as to allow it to fall back through a position of instability. If the magnetising force always at each point of its path was of considerable strength, and was never removed, there might be no hysteresis. But this cannot be entirely the case, since when centrally situated under a pole, the inner portion of the core is permeated by few or no lines of induction.[1] Hence it is doubtful how far determinations of the hysteresis loss by the cyclic process of fig. 110 are strictly applicable to the case of ring, discoidal-ring, and drum armatures.

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[1] Since the above was written, Mr. Baily has found experimentally that with a rotating magnetic field and stationary armature (which is the exact equivalent of an armature revolving between stationary poles) the hysteresis diminishes when the induction is raised above a certain critical value: see *Electrician*, August 31, 1894.

There is, however, invariably some loss by hysteresis in all armatures containing iron, and, so far as its amount has been satisfactorily determined by direct experiment, it does not appear to differ much from the amount calculated by the use of such a curve as that of fig. 112. In all calculations, therefore, as to the heating of armatures, or as to the efficiency of dynamos, some allowance must be made for a loss of energy by hysteresis, and in default of more decisive data, it will suffice to determine its amount by reference to the curve of fig. 112. Since p in our formula of page 226 is equal to the periodicity of the dynamo, the rate in watts at which heat is generated in the core by hysteresis may be taken as kcp, where c is the total volume of iron in the armature in cubic centimetres. It will be found that even in the case of alternators with a periodicity of 100 this quantity is but small, and, even under conditions unfavourable to cooling, is incompetent to raise the temperature of the entire iron mass more than a few degrees Fahrenheit; it only becomes of importance as increasing the heat due to other and more serious losses.

CHAPTER XIII

ARMATURES

THE practical construction of armatures, and the methods by which the cores are built up and the inductors are wound on, must next claim our attention. The subject is too great to be fully treated within our limits, but in the present chapter certain general principles will be discussed by which in everyday practice the mechanical construction of armatures and commutators is governed.

Except in the case of certain disc machines, the armature winding is now almost invariably supported on metal cores, and these, in ring, drum, or discoidal armatures, are of iron. As mentioned in Chapter VII., the use of the metal supporting core either necessitates its rotation together with the rotating inductors, or, if the armature is stationary and the field magnet revolves, implies that its surface is swept over by the moving field of lines. From this fact there follows a most important consequence: whenever a metallic mass actively cuts lines of induction, it becomes the seat of induced E.M.F.'s, which will, unless prevented, set up electric currents; the direction of these in that part of their path where the E.M.F. is generated will be at right angles to the direction of the lines and to the direction of movement. If, therefore, the core whereon the inductors proper are wound be formed of a solid metal mass, the latter will itself act inductively, and although the E.M.F.'s induced in it may be small, yet, owing to the low resistance of the numerous metallic paths open to them, they will cause enormous eddy-currents to circulate round the core; the latter will then become heated and energy will be wasted equal in amount to the product of the E.M.F. and the strength of the eddy-current. In a solid ring or drum armature the eddy-currents will flow in two sheets along the length of the iron core, in opposite directions on the two sides. Their intensity will be greatest near the surface, where the rate of cutting lines and the induced E.M.F. is greatest, and throughout the entire length of the core, and especially at its ends, the currents will bend radially inwards to complete their circuit through the armature shaft, hub, or inner portion of the core. Analogously, in a solid discoidal-ring armature the eddy-currents will circulate round the core, running up one side of the flat ring, across and down the other side. In all cases their direction on the surface of the core will be similar to that of the useful currents in the inductors proper, since these latter are expressly arranged to obtain the best inductive effect. So long as the metal core rotates, or is swept by the lines, it remains impossible to prevent the induction of E.M.F.'s in its surface, but it is possible to interpose in the paths which the eddy-currents would follow such very high resistances as to prevent their attaining any appreciable strength. This is effected by dividing the core up into small portions, each of which is more or less completely separated from its neighbours by a thin layer or covering of some insulating material. The directions in which the subdivision is carried out must be so chosen that the insulation obstructs the chief paths of any induced currents. In the iron-cored armatures now under consideration this can only be effectually secured if the core be divided in planes parallel to the direction of the lines and to the direction of motion; the mass of metal must be broken up at the actual seat of the E.M.F. and transversely to its direction, and the subdivision should be especially perfect at the periphery, where the E.M.F. is greatest. In short, the core must be laminated, the direction of lamination being suited to the type of armature. In ring or drum armatures the planes of lamination should be at right angles to the axis of rotation, or to the shaft, as indicated in the sections of fig. 46, while in discoidal-ring armatures the divisions should be approximately concentric with the axis of rotation (figs. 50 and 74). In both cases the lamination is at right angles to the inductors themselves, and the *edges* of the divisions are presented to the lines of the fields. Small E.M.F.'s will still be induced transversely across the surface of each division, but they will be prevented from acting summationally or in series by the intervening insulation ; the currents that can be set up are thus reduced, and likewise the energy that is absorbed. The thinner the divisions, the less will be the amount of the eddies, and the limit to the advantage of further subdivision is only reached when questions of the cost of manufacture or of mechanical strength outweigh the advantage of a further slight gain in efficiency.

The requirements above laid down for ring or drum armatures are met by the use of thin discs or washers, either threaded on to the arms of a central hub or strung direct on the shaft. Each core-disc is well insulated from its neighbours on either side; hence they make little or no contact with each other except through their connection with the shaft, and the passage of eddy-currents along the surface or through the mass of the core from one disc to another is prevented. The same end is obtained, although somewhat less effectually, by using wire, wound to form a cylindrical core, the separate turns making but slight electrical contact with each other. The requirements of the discoidal-ring armature are fulfilled practically by a thin band or ribbon coiled on a supporting ring, adjacent layers being in either case lightly insulated from each other.

Of the three magnetic materials discussed in Chapter XII. it is at once evident that cast iron and cast steel are out of the question for armatures, since they do not permit of the core being sufficiently laminated. Thus, the only magnetic materials which are practicable for armature cores are soft annealed[1] wrought iron or very mild steel (which from its low percentage of carbon is practically pure iron) in the form of thin discs, thin ribbon, or wire. Charcoal-iron is often specified for the purpose, but the discs or ribbon most commonly in use cannot be regarded as charcoal-iron manufactured after the technical sense of the term.

The use of discs stamped out of thin sheet-iron is now almost universal for ring and drum armatures, and in the best machines their thickness does not exceed ·025″ or No. 24 (sheet-iron gauge). Each disc is either coated with insulating varnish or paint, or is effectively separated from its neighbours by an interposed thin sheet of paper cut out to the required shape; or the oxide, which forms on the discs after heating, and which is practically a non-conductor, is relied on to insulate the discs. In ring armatures the discs must be supported on a non-magnetic spider or hub, which is usually of gun-metal. Most commonly, key-ways are stamped on the inner periphery of the discs, to fit three or four arms, which project radially from the central sleeve of the hub: the latter is keyed to the shaft, and at one end abuts against a collar; the discs are then slid over the arms, compressed tightly together, and held in position by a gun-metal star, this latter being screwed up by a nut on the shaft (fig. 113). In some cases semicircular notches are stamped in the discs; through these, and also through the two end-wheels of gun-metal, run three or more long bolts, the whole being firmly tightened up by means of nuts at either end of the bolts.

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[1] See Chapter XII. p. 221.

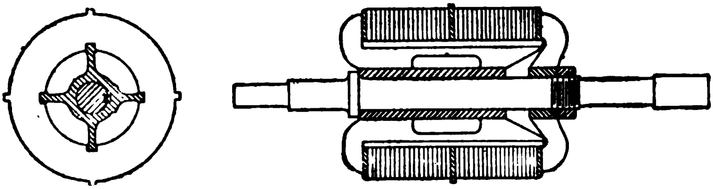

FIG. 113.—Ring armature of discs.

In drum armatures the discs are either supported on a hub, which, however, may be of cast iron, or are threaded directly on to the shaft; in the latter case they are usually held in position by a feather running along the length of the core (fig. 114), but not unfrequently they are simply compressed tightly between two end-flanges, one or both of which are screwed on the shaft (fig. 115).

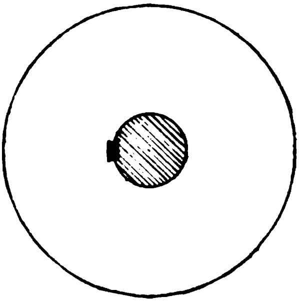

FIG. 114.—Keyed core-disc of drum armature.

In all cases it is advisable to lock the tightening-up nut or plate by a set screw, the point of which takes into the shaft, since otherwise there is a possibility of the nut and discs slacking back through mechanical vibration.

At either end of the ring or drum armature there are usually placed one or more stout iron core-discs (figs. 113 and 115), which serve to support the thinner discs where they are driven up against the end-flanges or end-arms of the hub. In order to avoid eddy-currents being set up in these end core-discs (by reason of lines curving round from the poles into their outer surfaces), the axial length of the core is frequently made slightly greater than the length of the pole-pieces parallel to the shaft.

Wire cores are now used only for small ring armatures : they are constructed by winding soft charcoal-iron wire either on a gun-metal flanged pulley, or between two flanges on the arms of an open spider, a cylindrical interior being in this case obtained by inserting, during the winding, wooden supports, which are subsequently withdrawn. The iron wire is either varnished or single-cotton-covered, or it is slightly oxidised in order to obtain moderate insulation between contiguous turns. Such armatures are, however, distinctly inferior to those composed of thin discs. The interstices which exist between adjacent turns of the round wire must be reckoned as lost space, and the actual volume of iron in a core of given cross-section is much less than could be obtained by the use of discs. Further, the lamination is carried out in two planes, whereas in the ordinary ring armature it is only required in one plane, and therefore more space is wasted by the insulation than is necessary. Lastly, the iron is magnetically

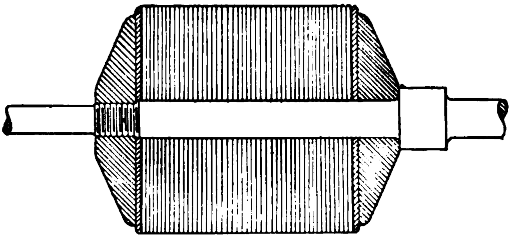

FIG. 115.—Drum armature of discs.

discontinuous in the direction which the lines take through the armature, and in consequence to reach the lower layers of iron the lines have to pass through several non-magnetic gaps. Hence the full benefit of the radial depth of iron is never obtained, the outer layers become saturated before the inner, and the total reluctance of the armature is considerably greater than even the actual sectional area of the iron would warrant.

To form discoidal-ring armatures, soft iron ribbon or tape about ·025″ thick is coiled on to a supporting wheel or foundation ring. The ribbon is at the outset riveted to the wheel, and is then wound on in a lathe, being drawn from between rollers so as to put considerable tension on it; a layer of thin paper of the same width as the iron is wound on simultaneously to insulate the successive layers. In order to give mechanical rigidity to the whole, long pins are in some cases passed radially through the centre of the iron ribbon and screwed into the rim of the wheel, or the latter is furnished with a central projecting flange, on either side of which the ribbon is wound. In small armatures it is preferable to use gun-metal arms or spokes for the foundation wheel, but in the case of multipolar armatures of large internal diameter the whole may be of cast iron, the spokes being so arranged that the points which they directly connect are of similar polarity. If the poles on either side of the flat ring are connected by a common pole-piece arching over the core, lines of induction enter the ring radially from the top as well as from the two sides, and it is therefore necessary to laminate the core in two planes —at least near the outer periphery. This is, to a certain extent, effected by adopting a narrow width of iron tape, and winding it on in three or more rows of turns side by side, while in other cases rectangular or square iron wire, lightly insulated, has been used.

All bolts, &c., used to hold the core in position must either be so placed as to be traversed by few or no lines of induction, or must be insulated from the core by fibre tubes and fibre or mica washers in order to obstruct the paths that eddy-currents generated in them would follow.

Lastly, care must be taken that the end-flanges, supporting spiders or hubs, are not the seat of eddy-currents: so far as possible they must be kept without the influence of the magnetic field. In drum armatures with discs of considerable radial depth very few lines pass across from inner side to inner side of the discs through the shaft; but in ring armatures worked at a high induction, not only may a small percentage of the lines of the field cross the internal opening of the discs, but a further number of lines crossing the ring internally will be induced by the current in the armature winding (see Chap. XVII.). Although the armature revolves, this internal field remains stationary; it is therefore cut by the arms and sleeve of a central hub, and the latter will become heated by eddy-currents. In order to minimise this waste it is important that the hub should be lightened so far as possible, while still maintaining sufficient mechanical strength (compare fig. 113).

Armature shafts are usually of mild steel, produced by the Bessemer or Siemens-Martin processes. The determination of the diameter required at different parts of their length is largely a matter of experience, but nevertheless in the main it requires to be based on the theoretical rules appropriate to shafts which are subjected to the combined stresses of torsion and bending. The calculation of the torsional stress due to the continuous and steady transmission of horse-power through a rotating shaft is an easy matter; if the shaft makes N revolutions per minute, and transmits HP horses' power, the twisting moment about the axis of the shaft is $T_m = 63{,}000 \frac{HP}{N}$ inch-pounds. For dynamo work this may be translated into a more convenient form. If the output of a dynamo be *kw* kilowatts, the actual power which passes into the shaft either through the pulley of a belt-driven machine or through the coupling connecting it directly to the crank-shaft of an engine, is $KW = \frac{kw}{\eta}$, where η is the coefficient of commercial economy, or the ratio between the useful power given out at the terminals, and the total power absorbed in obtaining it.[1] By substitution of the quantity KW, reduced to horse-power in the above equation, we obtain

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[1] In the case of a separately excited machine, the exciter of which is not driven from the main dynamo shaft, the power absorbed in the field should strictly be taken into account in calculating the commercial efficiency, although in our present connection such loss in the field stands outside the calculations of the strength of shaft.

$$T_m = 85{,}000 \frac{kw}{\eta N} \text{ inch-pounds} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The commercial efficiency of dynamos varies considerably with their type and size, but in the greater number of cases it ranges from 70 in small to 90 per cent. in large machines, which gives an average value for η of about ·8.

But torsion is by no means the only stress to which the armature shaft is subjected; there is also considerable bending stress, due to several causes, chief among which is the weight of the armature itself. In ring or drum armatures the weight is more or less uniformly distributed over the length of the shaft between the bearings; in discoidal and disc armatures the weight is more nearly concentrated at the centre, although even then the axial length of the supporting hub is usually a considerable number of inches. If W be the weight of the complete armature in pounds, and l be the span in inches between the inner ends of the bearings on either side of the armature, then if we assume the whole of the weight to be concentrated at the centre of the span, the greatest bending moment is $B_m = \frac{Wl}{4}$ inch-pounds. If, however, we assume the weight to be uniformly distributed throughout the entire length of the shaft, the greatest bending moment is reduced to $B_m = \frac{Wl}{8}$. Neither supposition is strictly true, and the best approximation is probably obtained if we adopt an intermediate value for $B_m = \frac{Wl}{6}$

Combining this by a well-known rule with the twisting moment, we obtain the equivalent twisting moment

$$T_e = B_m + \sqrt{(B_m^2 + T_m^2)}\,; \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and equating this to the moment of resistance of a circular section to torsion, or $T_e = 0{\cdot}196\, d^3 f$, we obtain $d = \sqrt[3]{\frac{T_e}{0{\cdot}196 \times f}}$, where d is the minimum diameter of a solid circular shaft for a given value of f, and f is the greatest safe working stress of the material of the shaft in pounds per square inch.

In engine-driven dynamos, where the heavy mass of the fly-wheel is supported between the inner bearing of the engine and outer or commutator bearing of the dynamo, it will be necessary to take this further weight into account as concentrated at a certain distance from the centre of the armature.[1]

In many classes of work the maximum safe working stress for steel may be put as high as 10,000 lbs. per square inch; but experience has shown that in dynamo shafts a large apparent factor of safety is necessary in order to allow for a variety of influences which in the above equations are entirely unrecognised. First amongst these may be mentioned the lateral pull caused from the pulley end by the tension of the driving belt or ropes. The pulley in belt-driven machines is usually placed outside of, but as close as possible to, one of the two bearings (fig. 2); and were the armature shaft rigidly fixed within the bearing, the bending moment due to the pull of the belt on the projecting shaft would be but small. The actual case is, however, somewhat different, and a certain amount of bending stress must be allowed for within the span of the shaft from the one bearing to the other, over and above that due to the weight of the armature. There may further be in single-magnet machines, such as those of figs. 2 and 123, a considerable downward or upward magnetic pull, the nature of which will be described in Chapter XIV.

Perhaps even more important than this is the necessity for the utmost stiffness in an armature shaft. In a rapidly rotating armature the slightest want of balance when running will tend to deflect the centre-line of the shaft away from the straight axis of rotation; unbalanced centrifugal force then comes into play, and vibration, which further increases the evil, is set up. The shaft must therefore possess ample stiffness to withstand this tendency, especially when the span between the bearings is long, and the armature is a large, heavy mass running at a high speed. Further straining actions are set up when the masses of the armature and fly-wheel are accelerated or retarded by reason of sudden changes in the armature load, such as occur, *e.g.*, in a generator supplying current to a tramcar motor. The influence of all these causes is to a large extent indeterminate, but it is owing to them that an armature shaft which is strong enough to transmit the requisite horse-power torsionally, and to resist the bending due to the armature weight, may still not be stiff enough to secure freedom from vibration and withstand the working shocks and strains. On this account a larger factor of safety is required than would be necessary were the conditions more exactly known; and hence in the above equations f will be found in practice to work out no higher than about 3,500–4,500 lbs. per square inch. Adopting some value between these limits, our equation may be used to determine the minimum diameter of the shaft at the centre of the armature, or at any part between the centre and the point of application of the driving-power. Further allowance must be made at the centre if a key-way is to be cut in the shaft for keying on the discs or their supporting hub; and in general to give stiffness at the centre, the shaft is usually there swelled out to a larger diameter than it has within the bearings. Since the useful horse-power is absorbed within the limits of the armature, the diameter of the shaft may be reduced within the further or commutator bearing; in many cases, however, for convenience of manufacture, the same diameter of journal is retained throughout, even though within the one bearing there is a surplus of strength. Any alteration in the size of shaft requires, of course, to be effected with a fairly large radius in order to avoid opening of the fibres of the steel at the edge, where the change of diameter is made.

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[1] See Unwin's *Elements of Machine Design*, vol. i. p. 212 (11th edition).

In order to ensure durability and coolness in working it is necessary in all high-speed machinery to make the journals of considerable length as compared with their diameter, and in belt-driven armatures, running at from 500 to 1,500 revolutions per minute, the ratio most frequently observed is $\frac{l}{d} = 4$, where d is the diameter of the journal. In engine-driven armatures, running at about 300 revolutions per minute or less, the proportionate length may be somewhat reduced, say, to three diameters.

In ring or drum armatures, end-play is usually limited by raised shoulders, which bear against the two inside ends of the brasses of the bearings : a slight amount of end-play is advantageous, since it secures a more uniform wear of the commutator surface, but it is seldom allowed to exceed $\frac{1}{16}$th of an inch. In discoidal and disc armatures, end-play must be definitely avoided by the use, at one or other end, of a thrust or collar bearing, since otherwise there is danger of the armature striking the poles at the sides.

Returning to the process of constructing an ordinary ring or drum armature, let us assume that the discs have now been built up into a cylindrical core. The next step consists in turning its surface true in a lathe. Unless carefully done with a sharp and fine-nosed tool, this operation is apt to burr over the edges of the discs : they are thus brought into contact on the surface, and the advantage of lamination is largely nullified, since eddy-currents can flow in sheets from end to end of the core. Hence some makers consider it advantageous to take the discs apart after turning, and grind the burr off on an emery-wheel, after which the armature is again built up.

When the armature core is completed, and all rough edges have been filed smooth, the surface is varnished with shellac and carefully insulated all over with Willesden or other stout paper and tape or calico to a thickness of ·040″–·070″, the whole being finally varnished and preferably dried in a stove in order to thoroughly rid it of all moisture. The ends and interior of ring armatures are similarly insulated, especial care being taken to insulate with fibre protecting-pieces the driving spokes or arms against which the internal wires are tightly pressed. In the case of alternators and high-potential machines in general, the core is often covered with strips of thin mica held down by varnished tape or with micanite sheeting.

Passing to the operation of winding, let us first consider the continuous-current ring armature. Each section of its winding is usually wound on separately, the ends being left temporarily open and subsequently connected up when the commutator is put on. The double-cotton-covered wire is cut off into lengths sufficient for an entire section of, say, two or three turns, and after being lightly varnished with shellac is allowed to dry. The winding of a section consists in passing one end of the wire through the interior of the ring, bending it round and over the core and drawing it tight from either end, so as to form a complete loop, the process being again repeated to form a second or third loop. In moderate-potential machines, giving 150 volts or less, there are seldom more than two, and usually there is but one, layer of wire on the outside; owing, however, to the smaller diameter of the central opening, and the loss of space by reason of the radial spokes, it is necessary to dispose the winding in at least two, and sometimes three or more, layers on the inside of the ring. The later-wound loops of a section are then made to ride over the first-wound, so that any section can be unwound and replaced without disturbing the rest of the winding. In order to secure an even and symmetrical winding, the ends of the core should at the outset be divided off by radial marks into the required number of compartments, so that no section may exceed its allotted space.

For large currents the required sectional area of wire is so great that it becomes difficult to bend it round the corners of the ring-core satisfactorily. Moreover, economy dictates the use of wire having a rectangular or trapezoidal section, so shaped that the inductors lying close by the side of each other form a nearly complete cylindrical envelope to the core; little or none of the valuable space round the periphery of the armature is then wasted by the interstices between the inductors. Hence in the bipolar ring armatures which were so commonly made in former days, if wound for large outputs at moderate pressures, the loops were not of round wire wound on the core, but of wire having a rectangular section: each **U**-shaped loop was formed separately on a mould, and was then sprung over the core. In some cases each loop was divided into an inductive and a connecting portion, and these two were of different section, the inductors on the surface being rectangular and the connectors trapezoidal. The wedge-shaped section of the latter enabled them to be conveniently packed into the interior of the ring in a single layer, but the multiplicity of riveted and soldered joints was a serious drawback to the method.

At the present time, however, in continuous-current bipolar machines, ring armatures are seldom employed for outputs above

15 kilowatts—unless their voltage exceeds, say, 200 volts, in which case the area of wire is small and many turns per section are required. In fact, the drum method of winding is now much more commonly used for all outputs over 10 kilowatts at moderate pressures. If the output be small the drum is wound with round wire in sections of, say, three or four complete loops.[1] Where the turns cross the end of the armature they require to be neatly and systematically disposed, so as to form a compact 'head' round the spindle, and in order to prevent any slipping they should be interlaced with tape.

While the term 'armature-winding' is very appropriate to the covering of the core with inductors in the case of small ring and drum armatures wound with round wire, and, indeed, took its origin from the fact that such were the earliest types made, it is somewhat of a misnomer as applied to large drum armatures. In these not only are the inductors large and heavy bars of copper of rectangular or trapezoidal section, but the end-connectors are usually entirely distinct, being thin but wide strips of copper, which are built up separately and apart from the rest of the winding.

The development of the armatures of figs. 100 and 101 will serve to illustrate the general principle of all such end-connections for drums. When the connectors for either end of the armature are arranged in place, the lugs at their ends stand out in two circular rows, the one near to the armature core and the other farther away, each connector being so twisted or so formed that within its length it passes across from the one row to the other. At each end of the armature the ends of the inductors are alternately long and short, and, by means of the spirally-bent connector, the long end of one inductor is connected to the short end of another, situated at the proper distance apart round the circumference of the core. Two advantages are thereby gained, as contrasted with wire-wound drum armatures. In these latter the loops, as they pass round the spindle at either end of the armature, must needs overlap each other, and the difference of potential between the overlapping loops, since it amounts to the full E.M.F. of the machine, is apt to break down the insulation, and cause short-circuits, while, further, the repair of any one loop usually involves the unwinding and rewinding of the whole armature. But now, when a separate system of connectors is arranged as described above, in the first place, the difference of potential between any two adjacent connectors is never more than the E.M.F. due to two inductors; thus in a 100-volt dynamo with 100 inductors it will not be more than about 4 volts (see fig. 100); hence there is little likelihood of the insulation between the connectors giving way. In the second place, if a bar be damaged, it can be unsoldered, taken off, and replaced without interfering with the rest of the winding, and therefore repairs can be much more easily effected.

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[1] See Chap. XI. pp. 204–206.

The various forms which have been devised for drum end-connectors are too numerous to permit of detailed consideration,[1] but certain of the best-known types will be described in Chap. XIX. In general, they are thin strips of copper about 1″ to 2″ wide, which are insulated with tape or calico; when built up apart from the armature or inductors, the connectors for each end form a compact and symmetrical mass, which is subsequently passed over the shaft into its place. When the connections for one complete loop have been marked so as to start the winding, an inductor is laid on the surface of the core, and its ends soldered to the lugs of the appropriate pair of connectors; bar after bar in continuous succession is thus connected up at both ends by soldered joints until the whole surface of the armature is covered, and the winding results in a closed helix of many loops.

Owing to the fact that any inductor must necessarily possess a certain width in the direction of rotation, its leading edge enters or leaves the field at the polar tips sooner than its rear edge, and in general, when moving through a field of varying density, one edge is cutting more lines of induction than the other. In consequence, when an inductor is either approaching or receding from the approximately uniform field under the centre of a pole-piece, a greater E.M.F. is set up along one edge than along the other, and if the inductor be solid, the unbalanced difference of these

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[1] For an exhaustive analysis of them the reader is referred to a series of articles on 'Drum Armatures,' by F. Weymouth, published in the *Electrician*, November and December 1890.

E.M.F.'s will cause a local eddy-current to pass up and down along the two edges and round at the ends, as shown in fig. 17. This production of eddy-currents in a solid inductor is naturally not very marked in the case of round wires, but if the inductors of any armature, whether ring or drum, continuous or alternating, are solid and of rectangular section, and have considerable width, the difference in the E.M.F.'s of the two edges of a bar will be appreciable and may lead to disastrous results. Thus in a bipolar machine on open external circuit, but with excited field, four times in each revolution, an eddy-current will circulate round each inductor-bar as it abruptly leaves or enters the field at the polar tips, and although the eddy-current E.M.F. may be comparatively small, yet since it is acting round a circuit of very low resistance a very large current may for a short time be generated: not only is mechanical energy thereby absorbed, but the temperature to which the bar is raised may become so high as to damage the soldered joints at its ends. If considerable current be taken out of the armature, no actual eddy-current will be produced, but the distribution of the current over the sectional area of the copper is no longer uniform; in consequence of the greater current density at the one edge the energy lost and the heat there generated become excessive, since the loss in watts is proportional to the square of the current. The value of the eddy-current E.M.F. for a given width of bar will vary directly with the peripheral speed and with the rate at which the density of field changes in the neighbourhood of the polar tips, while the strength of the eddy-current will itself vary directly as the E.M.F. With high speeds and strong fields abruptly entered, it will readily be understood that the eddy-currents generated in wide bars may seriously impair the efficiency of the machine, even if the whole armature-winding does not become overheated. Something may be done to neutralise this source of loss by so shaping the pole-pieces that the edge of the field is not parallel to the length of the inductor, but is inclined to it: the whole of the leading edge of the inductor does not then become immersed in the field before the rear edge. Or the entry into and departure from the field may be rendered more gradual if the bore of the pole-pieces be slightly elliptical, so that the length of air-gap is increased slightly at the pole-tips and the field is gradually shaded off. In spite of such devices with an average peripheral speed of no more than 1,500 feet per minute, and a fairly strong field, solid copper bars become inadmissible if their width much exceeds ·180″. A completely satisfactory solution of the problem is, however, obtained if each bar be divided into a number of thin laminations, lightly insulated from each other, and be then twisted at its centre through an angle of 180° (fig. 116); although the laminations are at either end joined together, their position relatively to the direction of rotation is thus transposed, the lamination which forms the leading edge along one half of the bar's length becoming the following edge along the other half of its length. The E.M.F. that is set up along the entire length of one lamination is then equal to that of any other, since if one half of it is moving through a denser field, the other half is moving through a weaker field.[1]

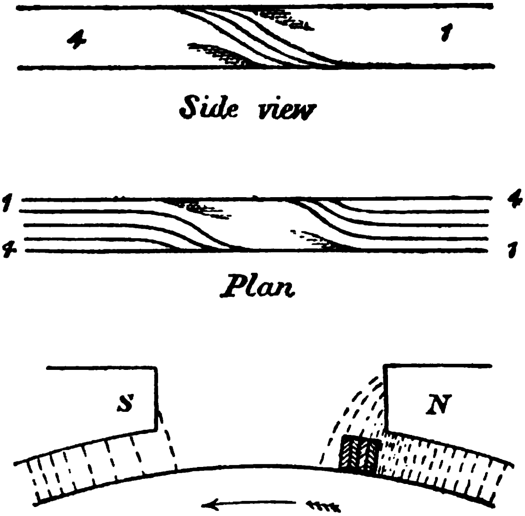

FIG. 116.—Laminated bar twisted at its centre to eliminate eddy-currents.

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[1] Several methods of carrying out this same principle are illustrated in the original patent of Crompton, No. 12,880. 1886.

In order to carry out this equalisation of the E.M.F. still more thoroughly, many makers use bars of stranded copper wire; the separate strands are varnished or oxidised, or otherwise lightly insulated from each other, so that little or no contact is made along their length, and the cable is then compressed into a bar of rectangular or trapezoidal section. By reason of the stranding, each separate wire continually passes from the one edge to the other of the composite bar, and the formation of eddy-currents is largely prevented.

When the winding of the armature sections is completed, the next step is to wind circumferentially round the armature several bands of binding-wire; by these the inductors are held securely in place, and the tendency for them to be thrown off the core by centrifugal force is counteracted. The materials used for binding-wire are German silver, phosphor bronze, hard brass, or in some cases non-magnetic steel, the requirements for it being great tensile strength with but little expansion under the heat of the soldering iron. The diameter of the wire varies from ·020″ (No. 25 S.W.G.) for small armatures, to ·040″(No. 19 S.W.G.) in large armatures. A band of tape or webbing covered with strips of thin mica is fastened round the armature, and on this the wire is wound under considerable tension to form a belt about ¾″ wide. Such bands are placed at intervals of about 2″ along the entire length of the armature (fig. 117), and finally each is soldered across in several places.

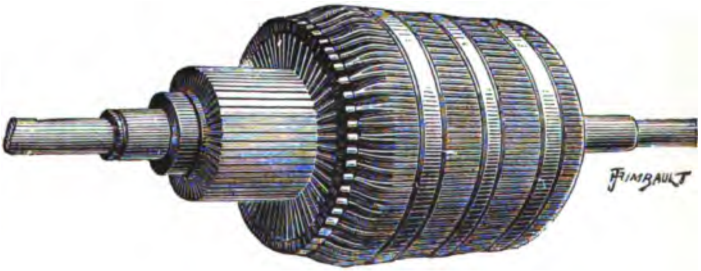

Fig. 117.—Completed ring armature.

Of the total power absorbed by a dynamo when in full work, fully nine-tenths are expended in overcoming the drag which the inductors experience as they move through the magnetic field. Since it is in the inductors that the actual conversion of the mechanical energy delivered to the shaft into electrical energy (whether useful or not) takes place, it is evidently of the utmost importance that the inductors should be so firmly supported on the armature that they are positively and effectively driven through the magnetic field, without any possibility of their slipping round the surface of the core. The magnetic pull to which each inductor of a dynamo is subjected can be calculated by our fundamental equation (8) of p. 66. In the bi- or multi-polar continuous-current dynamo, the pull, although intermittent, is always in one direction, relatively to the direction of rotation; and this is also the case with alternators, since when the direction of field is changed, the direction of the current in the inductor likewise changes, and therefore the direction of the pull remains the same. The pull on any inductor in a continuous-current dynamo may be deduced from the total energy absorbed per minute divided by the space moved through per minute and by the number of inductors under the pole-pieces (cp. p. 73); but in an alternator it increases and decreases roughly as the current rises and falls in value; and the maximum pull to which each inductor is subjected in each semi-period is proportional to the maximum value of the current. Hence the stresses to which the inductors of alternators are subjected are greater than in a continuous-current dynamo giving the same output with the same linear speed and same number of inductors.

The driving-power is in the first instance transmitted from the shaft to the hub, and discs or band-iron of which the armature is composed, and it is therefore evident that the armature-core must be satisfactorily driven from the shaft. This is secured when the hub and discs are keyed on to the shaft, and even when the discs are simply strung upon the spindle, they can be so tightly squeezed up between the end-plates (these being themselves keyed on) that there is no likelihood of any slip. The transmission of the power from the core to the inductors is, however, a more difficult matter. To a very large extent this is effected by simple friction; by means of the binding-wires the inductors are pressed down so tightly on to the surface of the armature that any slip of them relatively to the core would be opposed by a very large amount of friction. Again, in small armatures wound tightly with wire, the grip of the wires themselves materially assists in withstanding the pull on them, especially if they are ring-wound, since then the turns, threaded through the interior of the ring, are held in place by the arms of the spider.

But with larger armatures, especially if drum-wound, it becomes necessary to reinforce the influence of friction by some more positive means of driving, and various ways have been devised for effecting this. In many cases comparatively thick discs (say $\frac{1}{8}''$ thick) are inserted at intervals among the thinner discs of the core, and on the periphery of these are driving teeth which project up through the winding : several of these teeth are arranged in parallel line with insulation on either side of the row, or if they are few and dispersed over the surface of the core they are insulated separately with fibre ; in either case the inductors bed against them, and are thus positively driven through the field. (One such disc is shown at the centre of the ring armature of fig. 113.) In other cases small drivers of delta metal or other metal of high electrical resistance are let into the surface of the core ; or shallow grooves are milled longitudinally along the core, and into these are driven hard-wood strips which stand up level with the inductors and serve as drivers. In drum armatures the end-flanges sometimes have a row of grooves cast in them on their outer face ; into these, when insulated, the wire of the end-connections is wound, or every other bar is cranked to fit into them, so as in either case to secure positive driving.

A perfect system of driving is obtained by the use of toothed or grooved armatures, somewhat similar to the form originally used by Pacinotti. As first constructed, all the core-discs of such an armature had a number of projecting teeth, and hence when put together the surface of the armature was divided in a series of ridges and deep grooves running from one end to the other. The grooves are carefully lined with paper or calico and mica, and in them the inductor-wires are wound. Unless, however, the width of the grooves be narrow, the passage of the projecting teeth past the poles is liable to set up eddy-currents in the polar face.[1] A very large number of teeth are therefore required with correspondingly small slots between them, and it became usual to mill slots in the surface of the core after it had been built up (fig. 118). A further extension, which obviated the heating of the pole-pieces by eddy-currents, consisted in drilling a series of holes through a smooth-surface core close to the periphery, and threading the wires through the holes after they had been carefully lined with tubes of insulating material. In both cases, however, the cost of manufacture becomes considerable, and the advantage of the small air-space which is needed merely for mechanical clearance cannot be fully made use of (except by special design) owing to the sparking that results. One advantage, however, still remains, in that solid inductors can be used on a toothed armature without any liability to eddy-currents. The reason of this lies in the fact that the lines snap across the inductor from tooth to tooth with such rapidity that they cut the whole section of the inductor practically at the same instant,[2] and the time during which any differential E.M.F. acts is thus reduced to extremely small limits. In some drum armatures a few narrow grooves about $\frac{5}{16}''$ deep are milled from end to end of the core; into these, when insulated, are tightly driven special driving bars having a depth slightly greater than the rest of the inductors.

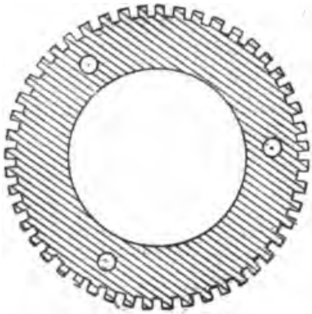

FIG. 118.—Toothed core-disc.

After the completion of the armature winding it remains to put the commutator in position, and connect up each of its segments to the winding, junction being made with the end of one section and the beginning of the next in succession. In the construction of the commutator various modifications are

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[1] Chap. XVIII. p. 414. See also *Electrician*, April 19, 1895.

[2] See a paper on 'Magnetism in its relation to Induced E.M.F. and Current,' by Elihu Thomson, reprinted in the *Electrical Engineer*, June 21, 1889.

possible, but, broadly speaking, two types may be distinguished. In the first (fig. 119) the wedge-shaped slips of copper and the intervening plates of mica are held in place by a sleeve or bush with a coned mushroom-shaped head and a coned collar (*c*), forced tightly up against the segments by a nut (*n*) screwing on to the bush. The latter may be of gun-metal or cast iron, the collar and nut being of wrought iron or gun-metal. Complete insulation between the segments and the supporting structure is obtained by the conical rings (*f f*), which are turned out of hard red vulcanised fibre, and are backed by thin slips of mica cut to the required curve and cemented together. The nut (*n*) is slotted, as shown at (*h*), so as to permit of its being tightened up by a horned spanner. During this process the segments are forced inwards under the sloping faces at either end, and thus bind on one another like the stones of an arch. It is still, however, possible, if their taper be but slight, that they may be driven inwards and put out of truth by an accidental blow. It is best, therefore, to support them on two rings of fibre (*r r*) threaded over the body of the sleeve, care being taken that they do not bed down on to the rings before they are thoroughly tightened up along their sides. In order to prevent the collar from twisting

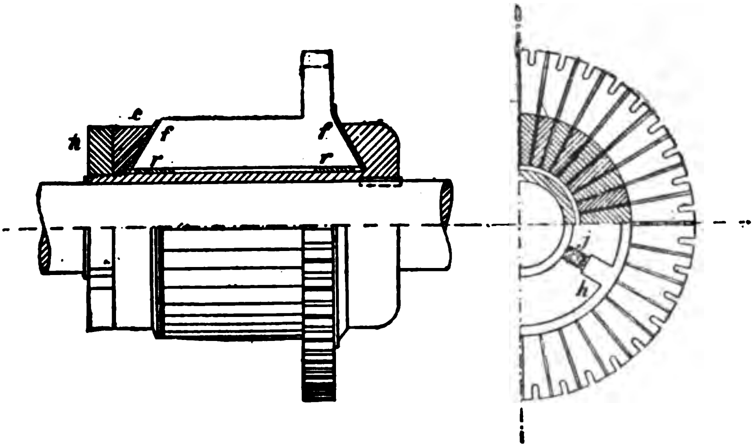

FIG. 119.—Commutator.

the segments round, a screw or pin in the collar is in some cases made to engage in a slot in the sleeve. Finally, to prevent the nut (*n*) from slacking back, two or three set screws (*j*) are put radially through the nut so that their points bite into the sleeve.

In the second type (fig. 120) the coned ends of both sleeve and collar are let into recesses in the segments. At both ends of the commutator a circular groove is turned, into which a turned ring of fibre or ebonite is fitted These rings are cut into three segments, so as to permit of slight contraction, and themselves have a coned surface. As the metal cones are driven home by screwing up the nut, they slide up the fibre and draw both fibre and segments inwards radially, until the segments jam against each other. Between the inner periphery of the copper and the sleeve is a clear air-space : should any segments receive a blow, they are prevented from being driven in radially by the support of the circular rings. In this type of commutator, sleeve, collar, and nut are usually of gun-metal.

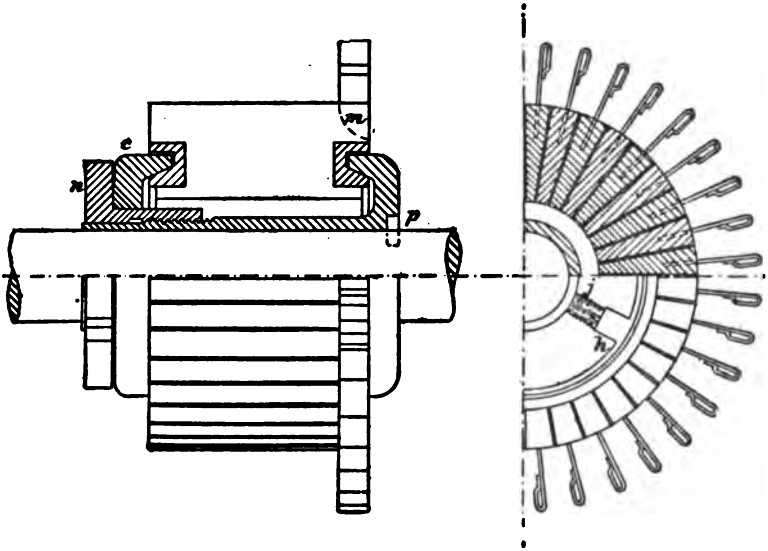

FIG. 120.—Commutator.

In either form of commutator, in order to prevent the sleeve twisting round the shaft it is secured either by a small sunk key under its head, as in fig. 119, or by a pin (*p*) driven into the shaft which engages in a slot in the sleeve, as in fig. 120.

It will be seen that for a given length of brush working surface commutators of the first type are slightly longer than those of the second type; on the other hand, the latter cannot be worn down below the level of the fibre rings, and therefore for a given diameter their radial depth of wear is less than in the first type.

The connections from the individual segments to the sections of the armature winding are most commonly made by thin strips of copper. A saw-cut is made in each segment, as shown at *m*, and into this one end of the strip is soldered; at the other, it is carried up to the level of the ends of the sections, and there embraces and is soldered to the armature wire, as in fig. 120. In some cases, cast segments of fine copper or gun-metal are used, and a projecting lug is directly cast on each segment, as in fig. 119; the wires are then soldered into a groove at the top of each lug. In such cases the mica is extended up to the full height of the lug; and an additional advantage claimed for the construction is that copper dust worn off the commutator by the brushes cannot be blown through the solid wall formed by the close-fitting lugs. On the other hand, a certain amount of ventilating effect is lost, which on the first method is secured by the moving blades of copper.

If equi-potential segments of the commutator are to be cross-connected, as explained in Chapter XI., the most convenient method is to string on the shaft, between the armature and the commutator, a number of brass or copper connecting washers, each provided with grooved lugs, into which the connecting wires which pass from the armature to the segments are soldered. The washers are all insulated from the shaft by a wooden sleeve, and each is also insulated from its neighbours on either side. Fig. 121 shows two such washers, each with two lugs for connecting together opposite segments in a 4-pole field.

For the insulation between the separate segments of the commutator mica is now almost universally used in plates of about $\frac{1}{25}$th of an inch thick. The difference of potential between neighbouring segments may be small, inasmuch as it is only the potential generated within the limits of one section of the winding. When, however, the segments pass under the brushes, sparking is liable to occur, and a small arc is formed which bridges the insulation. Under this action almost every insulating substance, except mica, is apt to char and become conductive, and even with mica insulation, if the thickness be less that $\frac{1}{32}$nd of an inch, small particles of copper may bridge over two adjoining segments. Air insulation is impracticable unless the insulating spaces be large, as in the open-coil machines of Chapter X., since otherwise they quickly become choked with copper dust. In closed-coil machines the segments should be as numerous as possible within reasonable limits, and, roughly speaking, the volts per commutator segment should not much exceed 15 to 20, if the machine is to be durable. The delicate nature of the commutator thus renders it practically impossible to build closed-coil machines to give more than 2,500, or at the outside 3,000 volts.

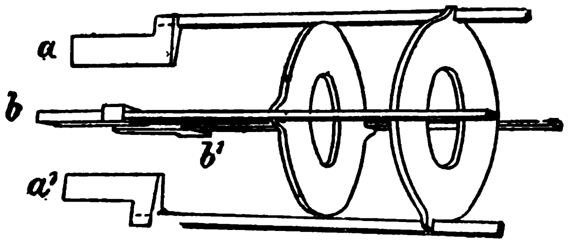

FIG. 121.—Cross-connection of commutator segments.

Owing to the absence of the commutator and the smaller number of coils, the construction of a ring or drum armature for an alternator is somewhat simpler than that of the analogous continuous-current machine. In winding the coils of a ring, since the turns are numerous, the length of wire in a coil is considerable, and hence it is usual to wind it on to two shuttles; the coil is then begun in the middle (p. 139), and one-half is wound from each shuttle, which is passed backwards and forwards through the interior of the ring. The coils of a drum armature are usually of rectangular wire wound on edge round a separate former; when completed, the central former is removed, and the coil is laid flatwise on the slightly curved surface of the armature and fastened securely in place.

Both in alternators and continuous-current machines precautions must be taken to prevent the oil from the bearings creeping along the shaft on to the commutator or collecting rings and destroying the insulation of the armature circuit. At either end of the shaft a ring is usually fixed, from the thin edge of which the oil is thrown off by centrifugal force.

The calculation of the electrical resistance of an armature from brush to brush is made as follows : From the dimensions of the core, the length of one inductor and of one connector by which it is joined to the next inductor in series can be estimated. Let l be this length in some unit, and let ω=the resistance of unit length of copper wire of the given sectional area ; then the entire length of wire with which the armature is wound is $\tau \times l$, where τ is the total number of inductors, and its resistance if extended out in series is $\tau \times l \times \omega$. If the armature be divided into two or more parallel circuits, let q be the number of parallels ; then $R_a = \frac{\tau \times l \times \omega}{q^2}$. Thus in the bipolar continuous-current machine the resistance of the armature is one-fourth of the resistance of the total length of wire if in series ; and, in general, in the multipolar continuous-current machine, if the armature be parallel wound, $q = P$, the number of poles, and $R_a = \frac{\tau \times l \times \omega}{P^2}$. While in a ring-wound armature l is the length of one complete loop, in drum machines it is the length of a half-loop. If the sectional area of inductor and connector be different, as is often the case in drums, it is simplest in calculating l to substitute for the actual length of the connector the equivalent length which would give its actual resistance supposing it to have the same area as the inductor itself : thus if the ratio $\frac{\text{area of connector}}{\text{area of inductor}} = 1{\cdot}25$, and l_r and l_c be the actual lengths of inductor and connector respectively, their combined resistance is $\left(l_r + \frac{l_c}{1{\cdot}25}\right) \times \omega$, where ω is the resistance per unit length of the inductor.

A high peripheral speed is in many ways desirable, as reducing the size and weight of a machine for a given output; but it must be limited by considerations of mechanical strength and durability, and therefore its permissible value depends largely on the method on which the armature is built up. But apart from the question of the mechanical strength of the rotating portion, its perfect balancing is of almost equal importance: with high speeds, even a comparatively light armature, unless accurately balanced, will set up such excessive vibration throughout the whole machine as will in the end considerably shorten its life. High-speed armatures are usually balanced on knife edges before leaving the workshop; but even this only secures a statical balance when at rest, and does not necessarily imply perfect balance under rapid rotation.[1]

With the ordinary ring or drum armature driven by belt, a peripheral speed of about 2,500 feet per minute is commonly found, and a well-constructed armature of this type should certainly allow of a speed of at least 3,000 feet. The large drum armatures of the Westinghouse alternator, two feet in diameter, are run at a normal speed of 1,030 revolutions per minute, or a peripheral speed of nearly 6,500 feet per minute. Discoidal rings of coiled band-iron may be constructed to permit of high peripheral speeds, and as much as 8,500 feet per minute is met with in the Kapp alternator of this type. In alternators of the disc type with separate bobbins supported radially on a ring, as in the Siemens and Ferranti machines, a peripheral speed of about 5,000 feet per minute is found.

The relative proportion of the length L to the diameter D of armature cores depends on the type of machine, and in each type considerable variations are permissible without transgressing the limits fixed by mechanical or economical considerations. In bipolar rings a general proportion is $L=D$, or $L=1{\cdot}25\ D$; but in the case of double-horseshoe fields, where a large diameter of ring is frequent, the length is often not more than half the diameter. The diameter of the internal hole in the core-discs is usually so chosen that after allowing for the internal insulation, and deducting the width of the supporting arms, the wires which form a single layer on the outside are conveniently disposed in two layers inside. This results in a radial depth of core, $a = \cdot 19$ D approximately, the proportion being somewhat larger for large ring armatures.

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[1] Hering's *Principles of Dynamo-electric Machines*, pp. 63, 64.

In bipolar drums the length is usually somewhat more than in a ring of the same diameter, and the ratio rises to $\frac{\text{L}}{\text{D}} = 1 \cdot 75$, or even higher. This, however, in armatures of large diameter leads to an inconveniently long span of shaft between the bearings, and therefore L is reduced to 1·25 or 1·5 D. The radial depth of core is limited only by the diameter of the shaft, unless internal spaces be desired for purposes of ventilation.

Discoidal rings are usually multipolar and of large diameter; in a 4-pole machine the core may have an almost square section, but with an increased number of poles, as in an alternator, the radial depth of core is usually considerably greater than its axial length.

The iron of which the core is composed being soft and highly permeable, a high induction is permissible in the armature without impairing the efficiency or economy of excitation. This is especially the case with ring armatures, since in them a high induction implies a small section of core for a given total number of lines, and a correspondingly short length of wire in each turn encircling it. In rings, therefore, the induction in the armature B_a is seldom less than 16,000, and may be taken as high as from 17,000 to 20,000 C.G.S. lines per sq. cm. of actual iron, allowance being made in estimating the net area of iron for the space occupied by the insulation between the laminations. If the induction be carried further, leakage occurs across the internal opening of the ring, and the loss by hysteresis becomes more important. In bipolar drums, owing to the greater radial depth of the core, the induction is seldom carried above $\text{B}_a = 17{,}000$, and if the discs be carried right down to the shaft, the induction may be as low as $\text{B}_a = 10{,}000$. In iron-cored alternators a low induction is usual, and $\text{B}_a = 9{,}000$ is seldom exceeded. Not only is the hysteresis loss greater, owing to the number of poles, but if the induction be high, the humming noise which is produced in the iron becomes disagreeably loud.

CHAPTER XIV

FIELD-MAGNETS

THAT the number of ampère-turns required per unit length of iron in order to produce a certain induction therein increases very rapidly as the induction is itself increased, is sufficiently shown by the magnetisation-curves of fig. 111. Thus in the case of wrought iron, while an induction B=16,000 may be obtained with 50 ampère-turns for each centimetre length of the circuit in the iron, it requires nearly treble that number of ampère-turns per centimetre length to obtain the increased induction of 18,000. The iron magnet of a dynamo necessarily has some length, and the number of ampère-turns required to propel the lines of induction through that length forms a considerable item in the total number required for the whole magnetic circuit. But the excitation of the field by magnetising coils implies, not only a certain first cost for the copper wire used therein, but also a continuous outlay during the working of the machine; for electrical energy is absorbed by the passage of the ampères through the turns, so long as the machine is at work. Evidently, therefore, there must be some approximate limits within which it is economical to keep the value of the induction in the iron. If the induction has less than the lower of the two limiting values, the iron magnet becomes too heavy and expensive; in a self-exciting machine there is the further disadvantage that the magnetism may be unstable (as will be more particularly described in Chapter XVI.), and the machine may become difficult to excite at all. On the other hand, if the induction in the magnet be pushed up to a very high figure, the ampère-turns required to produce it will bear an excessive proportion to the whole number, and will involve too large an expenditure either in the first cost of the copper or of watt-hours during the working of the machine; the result being that for the same total number of lines to be carried it would be more economical to increase the area of the iron so as to lessen the induction, and consequently the ampère-turns required to produce it. For wrought-iron magnets, the limits within which it is advisable to keep the induction may be set at $B_m=12,000$, and $B_m=16,500$, a good intermediate value being $B_m=15,000$; while for cast iron the limits are 5,500 and 9,000, an average value being $B_m=7,000$. Since, when designing a dynamo, the number of lines, Z_m, to be carried by the iron of the magnet is approximately known, we are enabled from the above considerations to determine approximately the area which the field-magnet must present for the flow of lines.

From the above values for the induction in cast iron and wrought iron respectively, it follows that it is not found advisable to magnetise cast iron more than about half as strongly as wrought iron; and consequently the relative weights of cast-iron and wrought-iron magnets to carry the same number of lines must be roughly as 2 to 1. Where considerations of weight and bulk are paramount, it is therefore essential to adopt wrought-iron forgings out of which to build up the magnet: these in their rough state as they come from the steam-hammer may be cheaper than castings of good soft iron, but when machined to the required dimensions their cost is so far increased that they become more expensive per hundredweight than cast iron; against this increased price, however, must be set the fact that a lesser total weight is required, and the balance, therefore, remains in favour of annealed wrought-iron magnets. On the other hand, castings of complex shape can be produced easily and cheaply, and of such accuracy of dimension that they require little or no further machining. Again, even if the form of magnet be simple, it is frequently desired to adapt the supporting framework or bedplate of the dynamo, so as to form a portion or the whole of the magnetic circuit, and such framework is most conveniently made in one single casting. Hence, for small belt-driven machines (up to an output of about 5 kilowatts) in which the cost of machining bears the largest ratio to the total cost of the machine, it is very usual to make the magnets of cast iron, and to cast them wholly or partially in one with the bedplate.

A cast-iron magnet, therefore, may be simpler and cheaper to manufacture, but must necessarily be heavier and bulkier than one of wrought iron ; furthermore, it is more expensive in copper, or exciting energy, or both. It will be seen from fig. 111 that inductions of 15,000 and 7,000 in wrought and cast iron respectively are produced by nearly the same number of ampère-turns per unit length ; hence, if the exciting coils on two equivalent round magnet-cores have the same length, and are composed of the same number of turns of the same wire, the length of each turn encircling the cast-iron magnet as compared with that encircling the wrought-iron magnet is as $\sqrt{2}$ to 1, that is, as 1·41 to 1 : the weight of copper used will therefore be 40 per cent. more on the cast-iron than on the wrought-iron magnet, and, further, the energy absorbed in the excitation will be correspondingly increased. If the latter is to be kept the same in the two cases, a further increase of the copper wire on the cast-iron magnet is necessary ; while if the weight of copper is to remain the same as on the wrought-iron magnet, the rate of loss of energy in the coils must be increased, and to meet this, as will be seen later, the length of the exciting coils, and consequently the length of the cast-iron magnetic circuit, must increased. Of recent years mild steel castings have come into greater use ; uniformity in their chemical composition and freedom from blow-holes can now be commercially assured, and when of good magnetic quality, they may even be superior in permeability to wrought iron, especially at high inductions. As they lend themselves to designs of complex shape with curved surfaces, they thus combine many of the advantages of both wrought and cast iron. They are commonly employed for large multipolar dynamos ; by casting the magnetic system in two or more large pieces, the necessity of joining together a number of wrought-iron forgings is avoided.

Lastly, it is very usual to find a composite magnet, built up partly of cast iron or cast steel and partly of wrought iron, the latter forming the 'cores' on which the magnetising coils are wound. The most common instance of this is to be found in the 2-pole dynamo of fig. 2, where a part of the bedplate forms the yoke, *y*, to which the magnet limbs of wrought iron are bolted. Again, the upper and lower pole-pieces of the 'Manchester' field of fig. 125 are usually castings, the magnet cores of wrought iron being let into them at either side. In both cases the portion of the magnetic circuits on which the exciting coils are placed con sists of wrought iron.

Since the permeability of any casting is much reduced if it be hard, it is important in the design of the shape of any casting to avoid introducing into the magnetic circuit any thin flange or narrow, outstanding edge : any such piece of small area is likely to chill when it is being cast, and so to become mechanically hard. Hence all corners and projections should be well rounded, or, if need be, cast massive, and subsequently machined to the required dimensions.

Thus, in conclusion, it may be said that wrought iron as forgings of best 'hammered scrap,' or mild steel castings, both being practically pure iron, are the most common materials in use for dynamo magnets.

The same economical considerations which determine approximately the sectional area of iron required to carry a given number of lines of induction, viz. the first cost of the copper coils and the allowable expenditure of energy in magnetising them, will also bear upon the geometrical shape or figure by which the required area is obtained—only, however, in that portion of the magnetic circuit whereon the magnetising coils are actually wound. Beyond the limits of the coils it is immaterial what shape the section of the iron takes. Since for a given area the circle has the least periphery, it follows that the theoretically best section which can be given to the magnet cores where the field-winding is placed is the circle, since then the length and resistance of the wire for a given number of turns embracing a given sectional area is a minimum.

For this reason, in the 'Manchester' dynamo (fig. 125), or in the multipolar alternator of fig. 74, the magnet cores, *m*, *m*, are usually of circular section. If, however, the magnet cores are of wrought iron and of circular section, it is often necessary to joint them into cast-iron pole-pieces or yokes, as in the above cases, or possibly also into the cast-iron bedplate or framework of the machine, which itself forms part of the magnetic circuit. Especially is it necessary to present a proper area of polar surface where the lines pass through the air-gap in order to minimise the reluctance of the interferric space between the iron of the pole and the iron of the armature core; and the cast-iron pole-pieces employed for this purpose, if not of large sectional area, may themselves perhaps increase the total number of ampère-turns required owing to their being magnetically inferior to wrought iron. It may therefore be more economical to give the magnet forgings a rectangular shape, which may be continued up to the armature, and so admit of being bored out to embrace the core, as in the case of the single horseshoe magnets of figs. 2 and 123. Or, again, a rectangular or trapezoidal section of magnet core may be arranged to give the required polar area at the air-gap, as in the alternators of figs. 72 and 75.

Next to the circle, the square must be ranked as the most economical form of section, its periphery being 1·13 times that of the circle containing the same area. Finally, the larger the ratio between the length and breadth of any rectangular figure, the more uneconomical becomes the shape as regards length of wire in each turn encircling it. If the two sides of the rectangle be as 2 to 1, the periphery becomes 1·2 times that of the equivalent circle, while for a ratio of 3 to 1 it increases to 1·3. If wound, therefore, with the same number of turns and the same size of wire, the weight of copper employed will be increased respectively 20 and 30 per cent. above that of the circular magnet, and the resistance being similarly increased, the rate of expenditure of electrical energy to produce the same exciting power will likewise be increased by 20 and 30 per cent. If, therefore, the same efficiency is to be retained in all cases, the area of the increased length of copper wire must also be increased, making in all, for a ratio of 3 to 1, an increase in the amount of copper of nearly 70 per cent. A larger ratio, therefore, is very uneconomical, and is seldom required in ordinary designs.

Having thus considered the sectional area and shape of the magnet cores, the question of the length of the magnet remains to be dealt with. Since the field-magnets of commercial dynamos are now invariably electro-magnets, this length must necessarily be such as to admit of the requisite magnetising coils. It has been said that the maintenance of the excited field during the working of the dynamo requires the continuous expenditure of energy in the magnetising coils at a certain rate in watts, such expenditure being simply that due to the passage of the magnetising current through the electrical resistance of the wire, and appearing as heat in the coils. This heat must be carried off by radiation, convection, and conduction in order to avoid such a rise of temperature in the coils as will endanger the insulation of the field-winding; and this is secured by so disposing the coils that they present a considerable cooling surface to the air. As soon as the dynamo is set to work, the temperature of the coils begins to rise above that of the surrounding air: this gradual rise continues until, finally, the rate at which the heat is generated in the coils is balanced by the rate at which it is dissipated under the combined action of radiation, convection, and conduction. After reaching this limit the temperature of the coils remains stationary so long as the conditions are unchanged. The effect of radiation &c. being dependent on the entire cooling surface of the coils, it is evident that the rise of the temperature of the coils above that of the surrounding air may be kept within any required limit by allowing the due proportion of cooling surface for each watt expended in the coils. The effective cooling surface of a given coil wound round an iron magnet will depend on a large number of conditions, difficult to calculate, since the mass of iron itself helps to dissipate the heat by conduction, and, again, the depth of a winding in many layers will largely affect the temperature of the turns forming the middle layer. Roughly speaking, however, the comparative cooling power of a coil may be taken as proportional to its external surface, reckoned as the product of its external periphery multiplied by its length, the cooling influence of the end-flanges or depth of winding in the coil being neglected. Thus, if the over-all dimensions of one such coil, as is shown in fig. 2, are a width of 14″, a depth of 8½″, and a vertical height or length of winding of 9½″, the external surface, $45'' \times 9\frac{1}{2}'' = 428$ square inches, will be a measure of its cooling power, and this may in our present connection be reckoned as its cooling surface. Having thus decided on the method upon which the cooling surface is to be estimated, experience will guide us to a certain ratio which the cooling surface must bear to the watts: the rise of temperature of the coil is not to exceed a certain assigned limit, and we can thence determine how far any coil complies with our requirements : of the values for this ratio more will be said in Chapter XVIII. If the rate in watts at which energy may be lost in the field-winding is approximately determined by the efficiency required in the machine, the length of the magnet coils must be such as to provide a suitable amount of cooling surface per watt dissipated in them. More generally, however, the settlement of the number of watts which may be regarded as an allowable rate of dissipation in the field-coils will be a matter of compromise between the efficiency of the machine and the first cost of the copper wire ; since the smaller the number of watts expended in the field-coils, the greater the weight of copper wire required. Further, the total number of ampère-turns required for the field excitation is not exactly known until the length of the magnetic circuit has been determined ; experience, therefore, alone can furnish guidance for a first approximation to the required length of coil, and a method of trial and error must be resorted to in order to exactly determine the ampère-turns on the field, the loss of energy in magnetising the field, the dimensions of the coils, and their weight of copper. When, however, the designer has assigned an adequate length to those portions of the magnetic circuit whereon the coils are to be placed, so as to provide sufficient cooling surface for the watts which it is approximately known will be expended in the field, he will complete the magnetic circuit in as direct a line as possible ; since any length beyond the minimum thus required unnecessarily adds to the reluctance, and is therefore uneconomical in both iron and copper.

The most compact form which the single horseshoe-magnet can have is that in which it is bent round into a more or less circular shape, as in fig. 39, the magnet-coil being wound uniformly all over its curved length. The length of the magnetic circuit in the iron is then a minimum, and the form is both economical in iron and light in weight. It is, however, expensive to wind, and it is therefore customary to concentrate the winding into one or more straight coils : these may be wound directly on to the iron magnet-cores, the wire being of course entirely insulated from the iron ; but more often, for convenience of manufacture, they are wound separately on sheet-iron or brass bobbins or 'formers,' which are subsequently slipped on over the magnet.

We thus arrive at the simple C-shaped magnet of fig. 122, formed of two curved castings screwed together at one faced joint under the single coil. The same form may also be used with a wrought-iron core let into two curved cast-iron pole-pieces; in such cases, owing to its greater permeability, the wrought iron may, as already pointed out, be of considerably less sectional area than the cast iron, and therefore the length of each turn of magnetising wire and the consequent weight of copper are largely reduced. A simple modification of this type leads us to the peculiar field-magnets of the unipolar alternator of figs. 78 and 80; there the two end-castings are branched out to give the several fields required, and the steel or wrought-iron magnet-core *m m* is screwed up tightly between the two end-castings by means of nuts on the spindle. In general, it may be said, whenever wrought iron forms a portion or the whole of the magnet, that, owing to the difficulty of forging and machining complicated shapes, the necessity of joints becomes more pressing, and the coils are usually more numerous. The form of single horseshoe-magnet shown in fig. 2 with wrought-iron magnet-limbs and cast-iron yoke is perhaps the most commonly adopted type of a composite magnet; the armature being above the yoke, it may be called the 'overtype' single horseshoe, in distinction to the 'undertype' magnets of figs. 123 and 124. The advantage in these latter of placing the armature under the yoke is that the axis of rotation is kept low, and the necessity of high pedestals or standards to carry the two bearings is obviated; hence this form lends itself admirably to direct coupling to the engine crank-shaft. Since the whole machine is usually supported on a cast-iron bedplate, it is necessary that the magnet-poles should be carried on a base or footstep of non-magnetic material, such as gun-metal or zinc (*n n*); since otherwise the magnetic circuit would be directly closed by the iron bedplate, and many of the lines of induction would be shunted

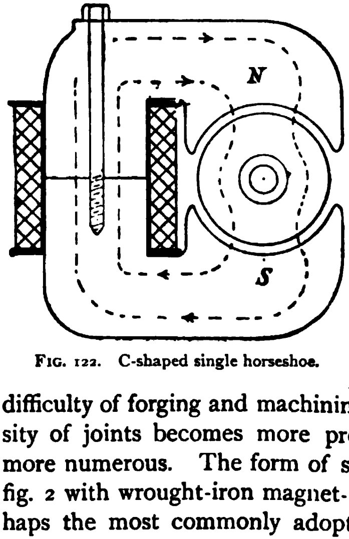

FIG. 122. C-shaped single horseshoe.

across from pole to pole through the bedplate, instead of passing through the armature as required. The vertical depth of the non-magnetic base must, in fact, be such as to avoid excessive leakage of the lines into the bedplate, and should therefore be roughly proportional to the length of the interferric air-gap. A common proportion is for the depth of the base to be about eight times the length of the single air-gap, which with an armature of 12″ diameter leads to a depth of from 4–5 inches.

In fig. 123 the whole of the magnet is formed of wrought-iron forgings of rectangular cross-section bolted together at the top; in fig. 124 the wrought-iron cores are let into a common cast-iron yoke, and also into the two cast-iron pole-pieces, and may therefore themselves have a circular section. The length of each of the two coils on the single horseshoe-magnets above described varies usually from 1 to 1·5 times the diameter of the armature.

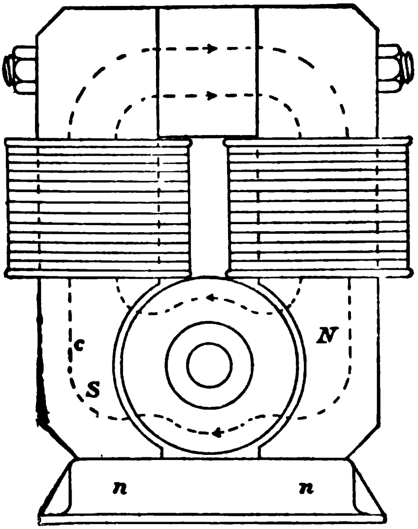

FIG. 123.—Undertype single horseshoe with rectangular magnet-cores.

A further modification consists in dividing the single magnet into two portions, so arranged that their like poles abut on each other, and have a common polar face (figs. 125 and 126). Each half of the double magnet carries half the total number of lines that pass through the armature, the right-hand magnet carrying the lines that pass through the right-hand half of the armature core, and the left-hand magnet those that pass through the left-hand half. Both halves of the field require, therefore, to be exactly similarly magnetised. The 'Manchester' field of fig. 125 is in reality a pair of C-shaped magnets (like that of fig. 122) placed together, each magnetic circuit requiring its own magnetising coil, which is placed on the yoke, and these coils being wound so as to produce 'consequent' poles at N and S. In the same way, the double magnet of fig. 126 consists of two horseshoe-magnets, each similar to that of fig. 123, with two magnetising coils on its limbs, but carrying half the total number of lines which pass through the armature.

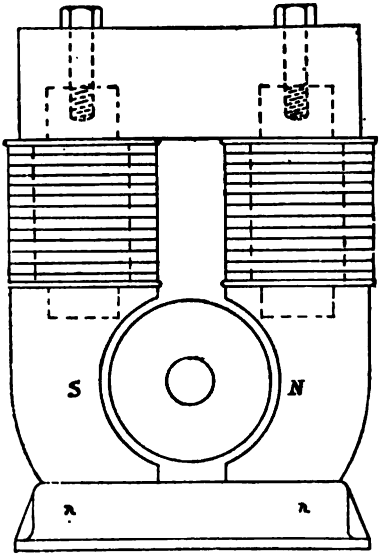

FIG. 124.—Undertype single horseshoe with cylindrical magnet-cores.

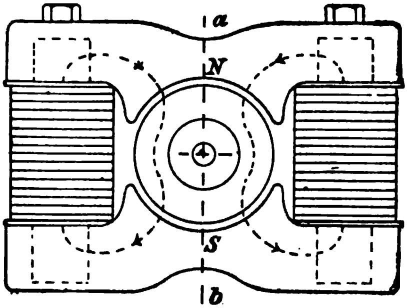

FIG. 125.—Double horseshoe 2-pole field, 'Manchester' type.

The advantages gained by such 2-pole double-magnets are complete symmetry of the field, and in certain cases avoidance of the necessity for massive and heavy pole-pieces; the leakage of lines by paths other than that through the armature is slightly greater in the double than in the single magnet, but approximately for the same size of armature and the same total flow of lines through it, the cross-sectional area of each double magnet limb need be but slightly more than half that of the single magnet. Two such equivalent fields may be compared by means of figs. 126 and 123, and on calculating the respective weights of iron in the two, it will be found that the single magnet is about 1·4 times heavier than the double magnet, chiefly owing to the thin section which is allowable in the poles of the latter on the line *a b*. Against this advantage, however, must be set the fact that the exciting power on *each* of the two circuits of the double magnet must be the same as that on the single magnet, and that therefore the double magnet is expensive in copper. If the magnet-cores of the two dynamos to be compared are both circular (as in figs. 125 and 124), and if (neglecting any effect due to increased leakage of lines) the area of each core in the double magnet is half that of the single magnet, the length of wire required by the double magnet will stand to the length of wire required by the single magnet in the ratio of $2 : \sqrt{2}$, or 1·41 to 1; an increase of 40 per cent. in the weight of copper. If the magnet limbs of both are formed of rectangular slabs (as in figs. 126 and 123), and the two dimensions of the cross-section are as 4 : 1 in the double magnet, and 4 : 2 in the single magnet, the respective lengths and weights of wire in the two machines are as 20 : 12, or as 1·66 to 1.

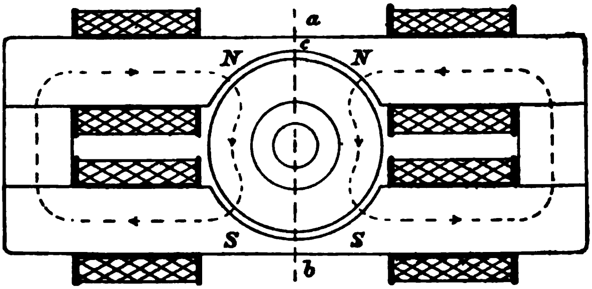

FIG. 126.—Double horseshoe 2-pole field.

The rate in watts at which energy is dissipated in the coils of the double magnet will also be correspondingly increased by 40 per cent. in the case of circular cores and 66 per cent. in the case of the above rectangular shapes, but it will still bear the same ratio to the square inches of cooling surface of the coils, and will not therefore be detrimental as regards their heating. If, however, the same efficiency is to be attained in the two machines, the weight of wire must further be increased: this may, however, again be followed by a proportionate shortening of the length of the coils, and a consequent shortening of the magnetic circuits, the net result being that the double magnet may be light in weight, but is expensive in copper, and must therefore rely for its advantage chiefly on its complete symmetry of field distribution, the importance of which will be again alluded to later.

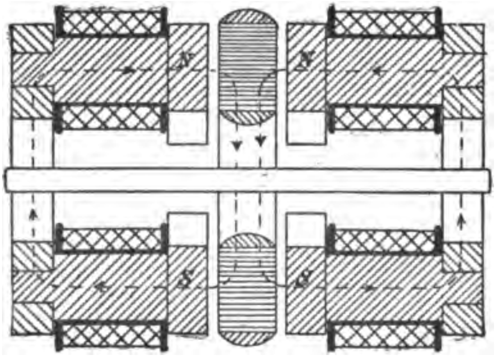

FIG. 127.—2-pole field with divided magnetic circuits.

Another form of divided field is shown in fig. 127, and is adapted to flat-ring machines; the like poles, which face each other on either side of the armature, may or may not be connected together by pole-pieces arching over the core, and in any case may be reckoned as one polar surface. The lines which emanate from one such joint polar surface and pass in to the armature core are divided into two groups, one on either side of the armature: the circuit of each group requires to be similarly magnetised by its own magnetising coils. Since the lines of each joint pole are bifurcate before they pass through the exciting coils, the machine is a double-magnet 2-pole dynamo, exactly similar to those of figs. 125 and 126.

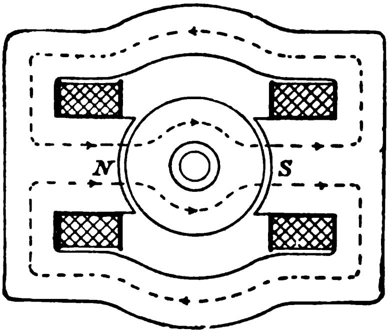

FIG. 128.—2-pole 'iron-clad' dynamo.

In fig. 128 we revert to a single-magnet dynamo. In this type the yoke is divided, and passes above and below the armature if the poles are horizontal, or on either side if the machine is turned so that the poles are vertically above and below the armature. It has been called an 'iron-clad' dynamo, since the field-magnet incloses the magnetising coils. From it is easily derived the 4-pole form shown in fig. 129: this type is frequently made of two castings bolted together, the lower magnets, lower half of the yoke ring, and bedplate being cast in one, while the upper magnets and upper half of the yoke-ring form the second casting. A wrought-

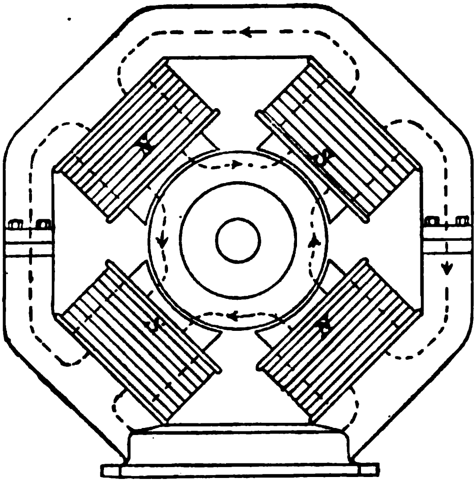

FIG. 129.—4-pole dynamo.

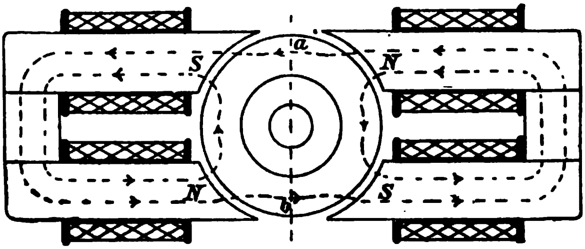

FIG. 130.—4-pole field with two single horseshoes.

iron magnet-core may also be used, fastened on to the yoke-ring by screws, and fitted with cast-iron pole-pieces; in this case the copper of the field-winding is reduced, and at the same time it is easier to secure the due area of polar surface, although the cost of machining is increased. Thence it is but a step to the multipolar forms of figs. 72 and 73, the latter having the yoke-ring internal and the magnet-cores projecting radially outwards.

An entirely different type of 4-pole field with only two horseshoes is shown in fig. 130; it is usually constructed of wrought-iron slabs, the magnets being laid horizontally on their sides, and supported by gun-metal brackets at a suitable height above a common bedplate. The paths of the lines are shown dotted. It will be seen that the group of lines issuing from one pole divides, part returning at once through the armature core to the neighbouring pole of the same magnet, and part passing through the armature on to a pole of the second magnet.

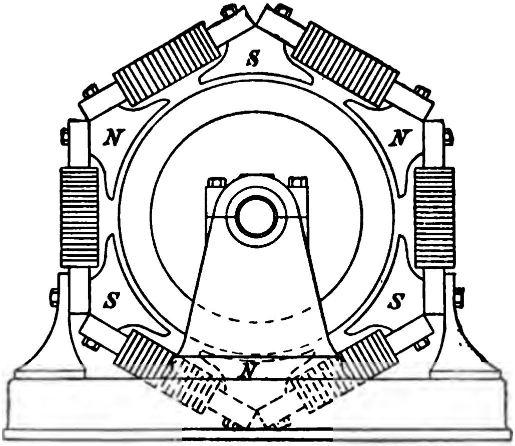

FIG. 131.—Thury 6-pole dynamo.

The principle of the divided magnetic circuit is often introduced in multipolar fields; thus fig. 131 shows a 6-pole dynamo which may be evolved out of the 2-pole dynamo of fig. 125, while from fig. 127 may be evolved figs. 74 and 132. The magnetic system of fig. 131, which is identified with the name of Thury, is composed of wrought-iron slabs bolted to wrought-iron pole-pieces, and is therefore light in weight but expensive to manufacture. In figs. 74 and 132, as in fig. 127, the cast-iron yoke-rings form at the same time the supporting framework of the machine. In all such cases of divided magnetic circuits, where the lines of induction of each pole are divided before passing through the magnetising coils, the weight of copper on the field must necessarily be more than in machines where the coils are wound on the poles themselves.

That the field-magnet and armature core of any dynamo should form approximately a closed magnetic circuit, only broken by the two interferric air-gaps, has already been stated at the end of Chap. III., and only needs to be again enforced in our present context. The aim of the dynamo-designer being to produce a good magnetic circuit, economical in its manufacture and in its working, the requirements which he has to consider may be summed up under the heads of compactness and mechanical strength, suitable proportion of the areas of cross-section at different parts of the circuit, small proportion of the leakage relatively to the useful lines, and lastly symmetrical distribution of the field; while, in certain special cases, lightness of weight may become of first importance.

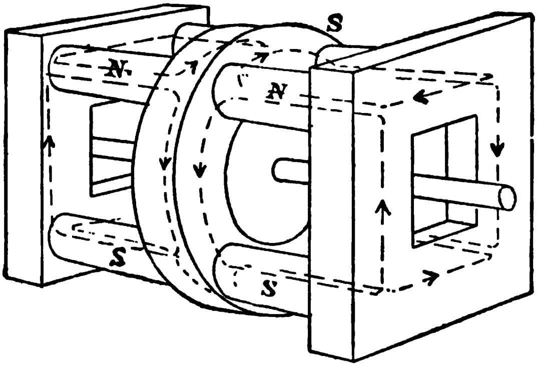

Fig. 132.—4-pole dynamo with divided magnetic circuit.

How far the requirement of compactness is met in the usual designs of field-magnets will be evident from the preceding figures; in older designs, originated when the principles of the magnetic circuit were not so fully understood as now, long columnar magnets were frequently adopted, and great use was made of divided magnetic circuits with consequent poles. Experience, however, has shown that the simple U-shaped magnet with comparatively short limbs of massive size stands pre-eminent in point of simplicity, economy, and compactness, while the double-magnet fields of figs. 125–127 occupy more room, and require more winding and more copper. For very large outputs or for alternators it becomes necessary to adopt a multipolar field; but again the simple forms of figs. 129, 72, and 73, which can be evolved out of the single horseshoe, must be given the preference on the score of economy.

As regards mechanical strength, it must be remembered that between the armature core and the polar surfaces of the magnet a very considerable attractive force acts; the magnet requires, therefore, to be rigidly and firmly supported, so as to preserve the requisite clearance and avoid any liability of the poles to collapse upon the armature. The attractive force acting between each square centimetre of a pair of divided surfaces is proportional to the square of the number of lines that pass through from one surface to the other, i.e. to the square of the induction in the gap B_g; and for a field of lines uniformly distributed over an area of A square centimetres the total pull[1] in pounds between the two surfaces is $P = \frac{B_g^2 A}{8\pi \times 981 \times 453{\cdot}6} = \frac{B_g^2 A}{11{,}183{,}000}$ acting along the direction of the lines, and tending to shorten their length. A pull of this nature exists, for example, between each half of the armature and the opposite pole-face in the dynamo of fig. 2, and has to be withstood by the mechanical rigidity of the magnet as a whole. Again, in the discoidal machine of fig. 74 each of the side magnet frames with its numerous poles experiences a direct pull, tending to draw it up against the rotating armature; the two yoke-rings with their poles must therefore be held down at the required distance apart, the armature being situated exactly midway between them. So long as the armature is central, and the flow of lines into the one side of the core is similar to the flow into the opposite side, the pull on the one side of it is precisely balanced by the pull on the other. Hence any difference between the strength of the fields on either side of the armature must be avoided, since it would produce an unbalanced end-thrust on the armature. Provided the two sets of fields are alike, there is no tendency for the magnet frames to move as a whole; but should they shift, owing to mechanical vibration while the machine is running, the air-gaps will be increased on the one side and reduced on the other side; the fields will be correspondingly reduced and increased, and the result is that for a very small displacement an unbalanced pull of considerable magnitude is set up, tending to still further displace the magnet frames. Again, in the multipolar disc machine of fig. 75 the two sets of magnets are attracted inwards towards each other, and must therefore be kept mechanically apart in order to retain the due clearance on either side of the armature. Apart, however, from any magnetic reasons, since the field-magnets form part of the supporting framework of most machines, the whole structure must be sufficiently rigid to withstand any mechanical stresses or vibration due to the working of the machine, and all jointed surfaces must therefore be securely bolted or screwed together. Absence or fewness of joints in the magnetic circuit has also been frequently regarded as desirable from magnetic reasons, and it is therefore necessary to consider shortly what is the exact effect of a joint on the magnetic reluctance of an entire circuit.

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[1] This equation is strictly applicable only when the opposing surfaces are large as compared with the gap between them, so that the induction in the gap is appreciably uniform; it is, however, sufficiently accurate for the ordinary purposes of dynamo calculations.

If a bar of iron be divided transversely in two, and the two pieces be then placed in contact and magnetised, it is found that the total reluctance of the iron is slightly increased owing to the influence of the joint, and this increase in the reluctance may be expressed as that of an air-gap equal in area to the cross-section of the iron, and having a certain width depending on the exact conditions of the experiment. If the two surfaces of contact are such as are produced by ordinary planing or other machine-tools, and the joint may be regarded therefore as, comparatively speaking, rough, the width of the equivalent air-gap is equal to about 0·005 cm., which is only reduced to 0·004 cm. when the two pieces are squeezed together under a very considerable pressure. If, however, the two surfaces are carefully scraped up, so that they are true planes, the equivalent width may be taken as 0·0035 cm. when the joint is not compressed, although under the influence of considerable compression the effect of the joint may be made almost entirely to disappear. It would seem that the increased reluctance due to a joint is in either case due partly to the interposition of a thin film of air between the surfaces, and partly to an alteration in the molecular structure of the iron at the joint, which decreases its permeability.[1] It will be seen, however, that the effect even of a comparatively rough joint, although appreciable, is not of great magnitude, and is especially unimportant in the case of the magnetic circuits of dynamos, which necessarily have air-gaps beside which the equivalent air-gaps of joints may be regarded as negligible. The conclusion to be drawn is that it is not worth while to use forgings of complicated shape, and therefore expensive to manufacture, in order to avoid joints in wrought-iron magnets, nor is it worth while to spend time and labour in carefully facing or scraping up any joints which it is decided to introduce into the design of a field-magnet. Absence or fewness of joints in a magnetic circuit must therefore be regarded as a minor consideration from an electrical point of view, and is only of importance as bearing on the mechanical strength of the design, and as lessening the first cost of the machine to the manufacturer.

The question of joints where different portions of a magnet, possibly one of wrought iron and the other of cast iron, are united, leads us naturally to the third question—of the suitable proportioning of the sectional areas at different parts of the magnetic circuit. It is highly important that the lines of induction should never be 'throttled,' as it is termed, by having to traverse in the course of their path a portion which is of insufficient area; any such 'throttling' implies a high induction, and therefore a large number of ampère-turns required for each inch in length of the contracted portion. If the disproportion between the areas at different parts of the magnetic path be carried to excess, the ampère-turns required to induce the lines through the area of the narrowest part may form so large a proportion of the whole number as almost to nullify the advantage of the larger section at other parts. In magnets composed throughout of iron of the same quality the area of section should be approximately identical at all parts of the circuit which carry nearly the same total number of lines; and, generally, in any portion of a circuit of the same magnetic quality, the minimum number of ampère-turns is required if it be worked at a uniform induction. Hence in designing, the distribution of the lines must be carefully studied. Thus, for example, in the wrought-iron magnet of fig. 123, a reduction of section is only allowable at the pole-pieces after a certain proportion of the lines have already passed into the armature; even then the sectional area of the pole-piece at *c* must be quite ample enough to carry half the total number of lines, since otherwise their path to the lower half of the armature would be throttled. When, however, we compare the single magnet with the double magnets of figs. 125 and 126, it will be seen that the area of the pole-pieces at *c* may be very much thinned down; in fact, if consistent with mechanical strength, the two horseshoes may even be separated along the line *a b*, and made to merely abut on each other over an area of very small depth; no lines need pass through the joint, since each magnet will separately feed its own half of the armature with lines.

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[1] Vide *Magnetic Induction in Iron and other Metals* (Ewing), pp. 273–280.

Next, when the lines pass from one material into another, as from wrought iron into cast iron, or *vice versâ*, as in figs. 2, 125, 74, the areas of the two parts should be so proportioned that both are worked at a suitable induction; hence it follows from p. 260 that the area of the cast-iron should be almost double that of the wrought-iron portion. Further, the nature of the joint should be such as to give ample area of contact, and afford an easy passage from the one metal into the other: this is secured in the dynamo of fig. 125 by sinking the magnet-cores into accurately turned holes in the cast-iron yokes, and drawing the two firmly together by screws. So also, in the dynamo of fig. 74, the wrought-iron cores are drawn tightly up against the cast-iron yoke by means of screws, a shoulder on each wrought-iron core bearing against the yoke-ring.

In applying the above rules it must be borne in mind that 'throttling' is a relative term, and under certain circumstances a high induction and somewhat contracted area of certain parts may be legitimate ; especially is this the case with those portions of the magnetic circuit which are actually encircled by the magnetising coils. It has been stated that the induction in an armature of the ring type may be given a high value, since by so doing the length of each turn of the copper wire, and consequently its resistance and weight, are lessened, and precisely the same consideration applies to the wrought-iron magnet-cores on which the field ampère-turns are wound. On the other hand, portions of the magnetic circuit which are not overwound with copper, especially if of cast iron, which is comparatively inexpensive, may often be advantageously given somewhat lavish proportions and worked with a low induction. Thus, in the dynamos of figs. 74, 75, and 127, it is economical to keep the induction in the magnet-cores at a fairly high figure ; the cross-area of the cast-iron yoke-ring

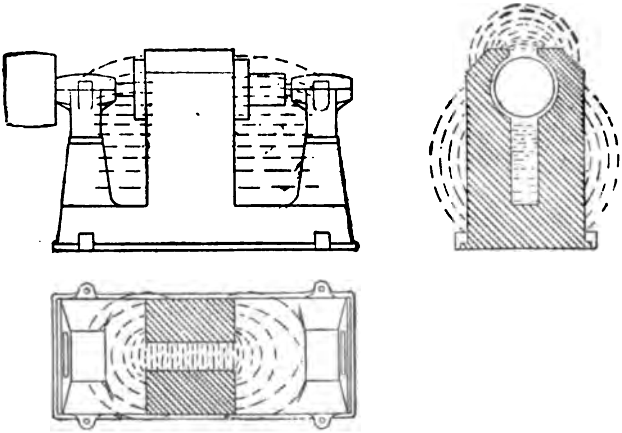
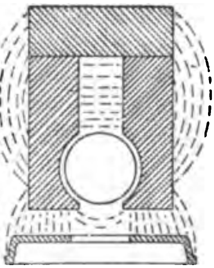


FIG. 133.—Magnetic leakage in overtype bipolar dynamo.

should then be at least equal to that of the magnet; the induction is thus halved, since the lines divide after passing through each core, half only passing in either direction round to the next pole. As, however, the material of the yoke is cast iron, and in this part gives strength and solidity to the whole structure, it may be economical to even enlarge its area to more than equality with the area of the wrought iron, so as to further lessen the induction and with it the ampère-turns on the field. In many other cases the yokes and pole-pieces, not being surrounded by wire, may advantageously be of comparatively large area.

In all calculations for the purpose of determining the proper area for any portion of the magnet, the question of leakage lines must not be left out of sight when estimating the induction. As was pointed out in Chap. III., lines of induction will flow between any two points or surfaces between which there exists a difference of magnetic potential, and consequently, in all dynamos, there is a considerable number of lines that leak across from one pole to another without passing through the armature core, where alone their presence would be of use. Thus, in the 2-pole dynamos of figs. 2 and 125, lines will leak across from one pole-piece to the other, from one magnet limb to the other, and even from one part to another part of the same limb: a few of these paths are traced out in figs. 133 and 134. The presence of these leakage lines in the air is shown in a disagreeable manner by the magnetic effects which they produce in watches, electromagnetic measuring instruments, or any piece of iron held in the vicinity of an excited dynamo.



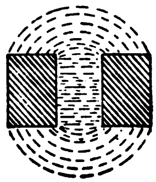


Fig. 134.—Magnetic leakage in undertype bipolar dynamo.

In calculating, therefore, the area of iron required at any part of the field-magnet, the induction will be reckoned too low if only those lines which pass through the armature, and are therefore useful, are taken into account. If the magnet is to be worked with uniform induction throughout its length, its area of cross-section should, strictly speaking, be continually varied as lines leak into or out of it. Since, however, the chief fall of magnetic potential occurs over the two air-gaps and the armature between them, it will be approximately sufficient, as was pointed out in Chap. III. pp. 35, 36, to regard all the leakage lines, ζ, as flowing between the one pole-piece and the other, and then to add them to the lines through the armature in order to estimate the total number of lines which pass through the magnet. The induction, therefore, in the magnet will be reckoned as equal to the sum of ζ and z_a divided by the area of the magnet; and, apart from any inferiority in the permeability of the iron of the magnet to that of the armature core, the area of the former should be larger than that of the armature.

Since it is the aim of the designer to reduce ζ to a small proportion of z_m, the total number of lines passing through the magnet, he will avoid bringing any large mass of iron into close proximity to the pole-pieces. In especial the magnetic leakage will be largely affected by the nearness or otherwise of the bedplate, bearings, or the flywheel of the machine, since these by their comparatively high permeability lessen the total reluctance presented by any leakage path. The effect of the bedplate has been already pointed out, in the case of the undertype single horseshoe, as rendering its leakage greater than in the corresponding overtype machine. In the Manchester field it is very usual to support the bearings of the armature shaft on pedestals formed by extensions from the lower cast-iron pole-piece, and due to this there is considerable leakage from the bearings into the upper pole-piece. Even in the overtype single horseshoe, where the bearings are supported from the yoke of the magnet, lines leak from the sides of the pole-pieces into the pedestals and bearings on either side of the armature. For the same reason, in the single magnet of fig. 128 the over-arching halves of the yoke must be kept at some distance away from the armature and poles in order to avoid considerable leakage. When this type is adopted for multipolar machines, as in fig. 129, the leakage from the yoke becomes less important, but the close neighbourhood of the poles leads to considerable leakage across the air from one to the other; and this is almost inevitable in any case where there are numerous poles of alternate sign presented to the armature. The great advantage in this respect of unipolar machines, such as those of figs. 78, 79, and 80, over multipolar machines, such as those of figs. 74 and 75, has already been mentioned on pp. 154, 164, and in any unipolar machine, since all the poles on one side of the armature are of the same sign, the leakage may be regarded as practically negligible.

Lastly we come to the symmetrical distribution of the field: by this requirement we are to understand that if a line be drawn through the centres of a pair of fields existing between the armature and two separate polar surfaces, and each of the fields be thus divided into two halves, the distribution of the lines of induction in the two halves on the one side of the dividing line should be as nearly as possible similar to their distribution in the two halves on the opposite side. Thus in fig. 44 it is requisite that the two halves, *a* and *c*, of the fields above the line N S should be symmetrical with the two halves, *b*, *d*, below that line as regards their distribution of lines. It is not implied that the two halves, *a*, *b*, of one and the same field must be identical; and, in fact, in the working of the dynamo (as will be explained in Chap. XVII.) this cannot be secured, for at the corner of *b* the lines will be more densely crowded together, and the field will be stronger than at *a*. But granting this to be so, what is implied is that the field at the corner *c* should be stronger than that at the corner *d*, to the same extent and in the same way as *b* is stronger than *a*, so that the entire distribution above and below the dividing line is symmetrical. The importance of this is due to two reasons, one mechanical and the other electrical. Taking first the case of the overtype single-magnet dynamo (fig. 2), it is evident that there is a tendency for the lines of induction to crowd into the lower half of the armature, since the path which they then follow is shorter and of less reluctance than that of the lines which pass on into the upper half of the armature. Especially will this be the case under two conditions: first, if the sectional area of the pole-piece across the neck at S or N be too small, and the passage of the lines to and from the upper half of the armature be thereby throttled; or, secondly, if the armature be of large diameter, and the length of iron traversed by the lines flowing to and from the upper half of the armature be therefore considerably greater than that traversed by the lines of the lower half. The general direction of the lines through the air-gap being more or less radial (*vide* fig. 44), the attractive force of each half-field upon a quarter of the armature may be regarded as concentrated along a line inclined at an angle of about 30° to the line N S; it has therefore a vertical component in an upward direction in the case of the two upper quarters of the armature, and downwards in the case of the two lower quarters. Now, owing to the density of the field at *b* being greater than at *a*, as described above, the downward component of the pull, due to the field at *b*, may be greater than the upward component of the pull due to *a*; yet if the distribution of the field be symmetrical on either side of the line N S, according to our previous definition, this difference will be balanced by the greater upward pull of the field at *c* as compared with the downward pull of *d*. The result is that the armature as a whole experiences no unbalanced pull, either upwards or downwards. But if the field be unsymmetrical, and the quarters *b*, *d* are more crowded with lines than *a* and *c*, the downward pull of *b*, *d* will more than counterbalance the upward pull of *a* and *c*, and there will thus be an unbalanced pull downwards. This will cause the armature of fig. 2 to exert a steady pressure on its bearings, greater than that due to its mere weight; and if the want of symmetry of the upper and lower halves of the fields be great, the downward pressure may reach such dimensions as to cause undue friction, and perhaps serious heating of the bearings. Finally, as the brasses of the bearings wear down, the armature sinks and becomes decentralised in the bore of the pole-pieces: this shortens the length of the air-gaps at the lower quarters and increases those of the upper quarters, thus still further weakening the upper halves of the fields and strengthening the lower halves.

Mutatis mutandis, the whole of the above applies equally well to the undertype single horseshoe; but in this case the unbalanced pull will be upwards, and tend to lift the armature, so taking its weight off the bearings; in fact, in some cases of large drum armatures placed underneath a single 2-pole magnet the distribution of the fields has been so arranged that the upward pull almost balances the weight of the armature. We may, indeed, conceive of an undertype armature revolving in mid-air without any bearings, its only support being the unbalanced pull exerted by the upper polar surfaces. Generally speaking, however, it is desirable that the fields in which both ring- and drum-wound armatures are to be placed should be symmetrically distributed, and the extreme importance of this in the case of ring-wound armatures has already been pointed out in Chap. XI. (p. 207). Now the chief merit of the double-magnet type of 2-pole dynamo is that complete symmetry of field-distribution is much more easily attained; from an inspection of figs. 125 and 126, it is evident that, provided the armature be placed concentrically within the bore of the pole-pieces, there will be complete similarity between the two halves of the fields on the right of the dividing line, *a b*, and the two halves on the left of that line; the double-magnet field is therefore specially suited to ring armatures of large diameter. Symmetry of field may also be accredited to the single magnet form of fig. 128.

In the case, however, of the simple horseshoe magnet especial care must be taken in the design that the area of the pole-piece be not so much reduced at its middle or thinnest part as to throttle the lines and cause the majority of them to take the shortest path through the armature by passing through that half which is nearest to the yoke. The area of the pole-pieces must be well maintained up to their furthest extremities, so that, as the lines gradually filter out into the armature, the reduced density of those that remain lessens the reluctance and compensates for the greater length of the path which they follow. In fact, by careful shaping of the pole-pieces, much may be done to secure a uniform distribution of lines in each air-gap between the polar face and the armature core. Special devices have also been adopted in order to equalise the reluctances of the two paths—the upper and the lower. The simplest method is to place the armature eccentrically in the bore, its centre in the overtype dynamo being farther away from the yoke than the centre from which the pole-pieces are bored out. Another method is to make the width of the gap between the pole-pieces farther from the yoke less than that of the gap nearer the yoke; or, again, the rectangular magnet slabs may have polar extensions forged on them at their further ends (c_1 c_2, fig. 135), the place of these extensions at the ends of the polar surface nearer the yoke being taken by cast-iron 'horns,' c_3 c_4, screwed on to the wrought iron; the inferior permeability of the cast iron then makes the path through the half of the armature nearer the yoke present equal reluctance to that of the path farther from the yoke.[1]

The same principles may easily be extended to multipolar fields. If a dividing line be taken through the centres of *any* pair of fields, the distribution of the fields on the one side of the line should be symmetrical with those on the opposite side. The distribution of the fields in the magnetic systems of figs. 129 and 131, being multipolar offshoots from those of figs. 128 and 126, will be perfectly symmetrical provided that the armature be exactly central; but as the bearings of the armature-shaft wear and allow the core to sink slightly, the air-gaps at the lower poles will be shortened, and those at the upper poles lengthened, thus producing, on the whole, stronger fields in the lower half than in the upper half. The mechanical result will be an increasing pressure on the bearings, but there is a further electrical result which is even more important. In the case of all parallel-wound armatures, whether ring or drum, the inductors which at any time form the lower parallels of the winding will continually produce a slightly higher E.M.F. than those which form the upper parallels; the result is that an unbalanced E.M.F. acts through the resistance either of the entire winding in series, or of certain of the parallels as joined together by the wires connecting together sets of brushes; on open circuit this produces large local currents, and consequent waste of energy, while on closed circuit the currents are unequally distributed among the different parallels, which likewise reduces the efficiency of the machine. This detrimental action in large multipolar fields having ring or drum armatures with radial poles either inside or outside the armature only applies to the drum winding when it forms more than two parallels, and is entirely removed if the drum armature be series-wound to form only two parallels, since then the zigzag winding passes continuously round in series from the weaker poles to the stronger poles (p. 210).

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[1] See Esson, *Journal Inst. Elec. Eng.* vol. xix. No. 85, p. 127.

When, however, we pass to the multipolar form of fig. 132, with flat-ring armature, no such objection can be raised, even though the armature is ring-wound and in several parallels; the armature may sink relatively to the fields as wear takes place in the bearings, but the effect of such decentralisation on the voltage of the parallels will be practically negligible.

CHAPTER XV

THE AMPÈRE-TURNS OF THE FIELD

IN our previous equations for the internal E.M.F. induced in an armature, one of the three determining factors has been z_a, or the number of lines of induction which enter into the armature-core from one pole or leave the core to pass into another pole. We have now to determine the ampère-turns of exciting power which must be be placed on the magnet in order to produce z_a lines in that part of the magnetic circuit where they are required, viz. in the armature, so that they may be cut by the inductors. The actual process has been already illustrated in Chap. III., and consists in the subdivision of the entire magnetic circuit into such lengths as may be considered to be of the same sectional area and permeability, and to carry the same number of lines, the calculation of the magnetic differences of potential required to drive the lines through each of these lengths, and their subsequent summation as one magneto-motive force. In nearly all cases, any magnetic circuit in a dynamo may be divided into the three principal parts : (1) the air-gaps, (2) the iron of the armature, (3) the iron of the field-magnet ; the latter may further require to be subdivided into two parts, viz. (a) the magnet-cores or limbs, and (b) the yoke, and in some cases, where the pole-pieces differ much in area and quality from the rest of the magnet, a third subdivision becomes necessary, viz. (c) the pole-pieces. The subscript letters, g, a, m, y, and p, will in future be used to denote these several parts. Owing to leakage the total flux of lines will vary in different parts of the circuit, but, as explained in Chapter III., on the supposition of the leakage paths being all in parallel with the armature and air-gaps, it will suffice to consider z_a lines as flowing through the armature, and a larger number, Z_m, as flowing through the magnet.

From the ring experiments described in Chaps. III. and XII. it was shown that to produce a given induction, B, in any material in a definite physical state, a certain magnetising force, H, was required. The 'magnetising force,' H, is, in fact, the 'difference of magnetic potential' which must exist between opposite faces of a centimetre cube of the substance in question, in order that B lines may flow through it.

If Z lines flow through a portion of a magnetic circuit, having a total area of cross-section of a square centimetres normal to their path, the value which H must have is entirely independent of the total flow of lines, and is solely determined by their density per square centimetre, or the induction $B = \frac{Z}{a}$; that is to say, whatever be the total area normal to the flow, if a difference of magnetic potential, H, be maintained between two surfaces one centimetre apart, B lines will flow through each square centimetre of the cross-section. Each centimetre length of the substance, therefore, requires a specific difference of magnetic potential, H (dependent on B), to be maintained between its ends, and if the same induction, B, be continued over a portion whose length is l centimetres, the total fall of magnetic potential between opposite faces of the portion will be $H \times l$, and that difference of potential maintained between its ends will cause a total flow of $Z = Ba$ lines through it. Reckoning, therefore, the lengths of the three portions of the magnetic circuit in centimetres, and l_g being the length of a single air-gap, we may express the fundamental equation for the ordinary bipolar dynamo with its two air-gaps, one on either side of the iron armature, in the following form :—

$$\text{Total M.M.F.} = H_a l_a + H_g 2 l_g + H_m l_m,$$

where H_a, H_g, and H_m are the magnetising forces required to produce B_a, B_g, B_m in the armature, air-gap, and field-magnet respectively. Now H has been shown to be simply a function of the induction, so that the equation may equally well be written—

$$\text{Total M.M.F.} = f(B_a) \,.\, l_a + f(B_g) \,.\, 2 l_g + f(B_m) \,.\, l_m.$$

And finally, since $B = \frac{Z}{a}$,

$$\text{M.M.F.} = f\left(\frac{Z_a}{a_a}\right) . l_a + f\left(\frac{Z_a}{a_g}\right) . 2l_g + f\left(\frac{Z_m}{a_m}\right) . l_m.$$

Now curves of magnetisation, such as fig. 111, are curves which connect together corresponding values of B and H for various substances. Having calculated, therefore, the different values of B_a, B_g, B_m for a given value of Z_a, all that is necessary is to look out in the magnetisation-curve of the substance in question the particular value which B now has in it, and then to read off on the scale of abscissæ the corresponding particular value which H must have for that induction: this value can then be substituted for the specific H in the above equation.

So far we have dealt with C.G.S. units of magneto-motive force, their number being equal to 1·256 times the ampère-turns; but for our immediate purpose we require an equation expressed directly in ampère-turns of exciting power (X), or

$$X = X_a + X_g + X_m,$$

where, as before, the three right-hand terms are the exciting powers required respectively by the armature, air-gaps, and field-magnet. The magneto-motive-force equation will require to be divided throughout by 1·256 in order to express it in ampère-turns; but since the values of H in the curves of magnetisation were originally derived from a measured number of ampère-turns per centimetre length, it is still simpler to show on the horizontal axis of abscissæ a scale of ampère-turns per centimetre length (as well as a scale of H in C.G.S. units), and thence read off the exciting power required per centimetre length to produce a flow of B lines per square centimetre. This has been done in the upper of the two horizontal scales of fig. 111, whence, therefore, we can read off the new value which the function of the induction takes when expressed in ampère-turns, or

$$f'(B) = \frac{f(B)}{1{\cdot}256} = 0{\cdot}8 f(B).$$

We thus obtain

$$\text{Total ampère-turns} = X = f'(B_a) . l_a + f'(B_g) . 2l_g + f'(B_m) . l_m.$$

It may here be objected that no curve of magnetisation appropriate to the air-gaps of the dynamo has been given, and that therefore $f'(\mathrm{B}_g)$ cannot be at present determined. When, however, it is remembered that the permeability of air is strictly constant, or, in other words, that the exciting power required per cm. length of path in the air is directly proportional to the induction, it is evident that the curve of magnetisation of air would be simply a straight line inclined at a certain angle to the axes, and that it is therefore unnecessary to plot it. As stated in Chap. III., the permeability of air on the C.G.S. system$=1$, so that to produce an induction of one line per square cm. in air requires $\frac{1}{1{\cdot}256}={\cdot}8$ of an ampère-turn, and to produce any other induction B_g requires ${\cdot}8\times\mathrm{B}_g$ ampère-turns per cm. length. The air-gap of a dynamo is made up partly of air and partly of cotton-covered copper wire, but since the permeability of the latter two materials is sensibly the same as that of air, the whole interferric space between the iron of pole-piece and iron of armature core may be treated as a gap of air alone. Hence for $f'(\mathrm{B}_g)$ in the above equation we may at once substitute the value ${\cdot}8\mathrm{B}_g$, and

$$\mathrm{X}_g={\cdot}8\mathrm{B}_g.2l_g, \text{ or } {\cdot}8\frac{\mathrm{Z}_a}{a_g}.2l_g.$$

As hinted on p. 37, in calculating a_g a certain additional area must be reckoned over and above the actual area of the polar face in order to allow for the spreading out of the lines in a fringe, which increases the effective area of the air-gap. The quantitative value for this increase will be discussed later.

The final expression now obtained for the total ampère-turns of excitation, viz.

$$\mathrm{X}=f'\left(\frac{\mathrm{Z}_a}{a_a}\right).\,l_a+{\cdot}8\frac{\mathrm{Z}_a}{a_g}.\,2l_g+f'\left(\frac{\mathrm{Z}_m}{a_m}\right).\,l_m, \quad . \quad . \quad (16)$$

may also be reached by another more circuitous route, which will, however, serve to emphasise the principles underlying it. Since the product of the reluctance of any portion of a magnetic circuit, and the total number of lines flowing through it, gives the difference of magnetic potential which must exist between its ends, the fundamental equation for the magnetic circuit of a dynamo may be expressed as

$$\text{Total M.M.F.}=0{\cdot}4\pi\ \mathrm{AT}=\mathrm{Z}_a\mathrm{R}_a+\mathrm{Z}_a\mathrm{R}_g+\mathrm{Z}_m\mathrm{R}_m,$$

where R_a, R_g, and R_m are the reluctances of the armature, air-gaps, and field-magnet respectively. These several reluctances may also be expressed as the quotients of their length in centimetres, divided by the product of their permeability and area in square centimetres, or

$$0 \cdot 4\pi \text{ AT} = Z_a \frac{l_a}{a_a \mu_a} + Z_a \frac{2 l_g}{a_g \mu_g} + Z_m \frac{l_m}{a_m \mu_m}.$$

But the number of lines divided by the area through which they flow, or $\frac{Z}{a}$, is equal to B, or the induction at that part ; therefore

$$0 \cdot 4\pi \text{ AT} = B_a \frac{l_a}{\mu_a} + B_g \frac{2 l_g}{\mu_g} + B_m \frac{l_m}{\mu_m}.$$

Further, $\frac{B}{\mu} = H$, and therefore

$$\cdot 4\pi \text{ AT} = H_a\, l_a + H_g 2 l_g + H_m l_m.$$

Thence, by the same steps as before, since H is a function of the induction, and in the case of air, of which the permeability μ_g is 1, is equal to the induction, we return to our previous equation—

$$X = \text{AT} = \cdot 8 f(B_a) \,.\, l_a + \cdot 8 B_g 2 l_g + \cdot 8 f(B_m) \,.\, l_m$$
$$= f' \left(\frac{Z_a}{a_a}\right) . l_a + \cdot 8 \frac{Z_a}{a_g} 2 l_g + f' \left(\frac{Z_m}{a_m}\right) . l_m . \quad . \quad . \quad . \quad . \quad (16)$$

Both methods of arriving at this equation are equally correct, and their apparent difference is due to the fact that the former deals more directly with magnetising forces, while the latter starts with reluctances. Hence the former is at once simpler and more appropriate, since it is seldom requisite to calculate the actual reluctance of any portion of the magnetic circuit of a dynamo. The actual process in designing will be to determine the inductions, B_a and B_m, and then to read off from a magnetisation-curve the corresponding number of ampère-turns required per cm. length ; while even in the case of the air-gap it is useful to calculate B_g as affording something of a clue to the probable amount of losses by eddy currents which may be expected in the armature winding and surface of the core.

The magnetisation-curves from which the values of the specific exciting powers are read must be suited to the exact nature of the materials of which the dynamo's magnetic circuit is composed. Thus, for the ampère-turns required over the length of path in an iron armature-core, the curve for annealed wrought iron will be used, since the armature core is invariably made of wrought-iron discs or ribbon or wire, while for the field-magnet, either the wrought-iron or the cast iron or steel curve will be used, according to its material. On referring to fig. 111, it will be seen that if the magnet be worked with a low induction, say, *e.g.* 10,000 in wrought iron or 6,000 in cast iron, the value of H for a given induction is indeterminate, and may have any value lying between the ascending and descending curves; in the cases supposed, the limits will be 1·5 or 4 ampère-turns per cm. length for wrought iron, and 16 or 25 for cast iron. *Vice versâ*, with a given magnetising force, the induction may vary between certain limits according as it has been reached from a higher or lower value. Hence a certain number of ampère-turns on the field may induce a slightly larger number of lines at one time than at another, if the excitation be raised to a higher value and then decreased; and the student may therefore feel a doubt as to what value of exciting power he is to select. In practice, however, any such effect of hysteresis resulting in a variation of the voltage given by a dynamo when running at a certain speed is hardly perceptible; not only is a dynamo seldom worked with a very low induction in the iron of the field-magnets (and with higher values for the induction the vertical difference between the ascending and descending curves becomes negligible), but also the exciting power expended over the reluctance of the field-magnet or X_m forms only a certain proportion, usually less than half, of the total number of ampère-turns, $X = X_a + X_g + X_m$, and therefore an increased number of ampère-turns is required for the armature and air-gaps, if an increased number of lines pass through the armature. Further, in actual working the excitation is never varied through a very wide range; in fig. 111, the magnetising force per cm. length of iron has been carried up to a high figure, and then gradually reduced, so producing the greatest possible difference between the ascending and descending curves. Such a condition would seldom occur, except in the conduct of workshop or laboratory experiments on dynamos; in the ordinary course, the field-magnets become gradually excited as the machine is set to work, and then the excitation is maintained fairly constant until on the stoppage of the machine the magnetism dies away. Hence in the design of dynamos it is sufficient to invariably use the ascending curve of magnetisation; any difference, due to the exciting power having been differently reached, may be neglected, since its only effect will be to slightly reduce the speed required to generate a given voltage.

There still remains one point on which our previous equation requires to be supplemented. When a considerable current is passing through the armature of any continuous-current bi- or multi-polar dynamo, it is necessary to move the brushes forward through a small angle in the direction of rotation in order to prevent sparking at the commutator; the reason for this shifting of the brushes will be more fully explained in Chap. XVII. Suffice it here to say that as the current flows through the armature it magnetises the iron core, and that when the brushes are given the requisite forward 'lead' in order to obviate sparking, the magnetic effect of the current flowing in the belt of armature inductors inclosed within twice the angle of lead (fig. 153) is directly opposed to the magnetisation of the core by the ampère-turns of the field. In fig. 153 those wires in which the current, whether of the field or of the armature, is directed towards the observer are marked with a dot, and those in which the current is away from the observer are shown crossed; and from this figure it will be apparent by application of the rule of the hand that the current-turns between *k l*, *m n*, are opposed to the current-turns of the field, and tend to magnetise the circuit in exactly the reverse direction: they are, in fact, 'back ampère-turns,' and the result is that a certain number of extra ampère-turns have to be added to the field ampère-turns in order to neutralise the demagnetising effect of the armature current. Let this additional number $=x_b$; then the complete equation for the ampère-turns required from the field coils must contain this additional term, and thus becomes

$$x = x_a + x_g + x_b + x_m. \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

When the armature current is very small, x_b may be regarded as negligible, but for any but small currents the appropriate number of compensating ampère-turns must be added, their number increasing as the armature current and its demagnetising effect are increased.

In the case of alternators we have shown that a principal effect of self-induction is to lower the resultant voltage of the armature below the impressed; now this statement is but another method of taking into account the demagnetising effect of the armature current, and if, therefore, the effects of self-induction are separately considered, the term X_b must be omitted from the above equation when applied to the magnetic circuits of alternators.[1]

The reason for the insertion of X_b in the third place needs further explanation. It has been said that the flow of leakage lines takes place in the main under the magnetic difference of potential existing between the poles of the dynamo, and even if this be not true of all the leakage, allowance can be made for the inaccuracy involved in the assumption. Now before Z_m or B_m can be determined, the amount of leakage ζ requires to be estimated. In order to do this, the joint permeance of the leakage paths is calculated; then it is only necessary to multiply the magnetic difference of potential at the poles by this joint permeance to determine ζ. The magnetic difference of potential between the poles is equal to 1·256 times the ampère-turns required to produce the flow of lines between the poles; let these be expressed as X_p; then $\zeta = 1·256\ X_p \times P$, where P is the joint permeance of the leakage paths. Thence $Z_m = Z_a + \zeta$, from which the magnet induction B_m and the ampère-turns required to produce it, X_m, can be determined.[2]

Now X_b, or the ampère-turns required to counterbalance the back ampère-turns of the armature current, take effect between the poles, and consequently must be reckoned in when estimating the difference of magnetic potential between the poles : thus

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[1] See Swinburne, *Journal Inst. Elec. Eng.* vol. xx. part 92. Of the two methods of viewing the armature reaction in alternators, that which considers the term X_b is the more strictly correct in relation to the design of the magnetic circuit, since if the number of lines required are deduced from the impressed, and not from the resultant E.M.F., the iron of the magnet is given larger dimensions than necessary; the difficulty, however, of its application renders it unsuitable for a first treatment of alternating currents.

[2] Cp. Chap. III. p. 36. The principle of the above method of treatment was first definitely applied to the leakage of dynamos in 1887 by Mr. Kapp in his paper on 'The Predetermination of the Characteristics of Dynamos,' *Jour. Soc. Tel. Eng.* vol. xv. part 64. The whole paper and the discussion which followed will be found worthy of careful study.

$$\mathrm{X}_p = \mathrm{X}_a + \mathrm{X}_g + \mathrm{X}_b$$

and

$$\zeta = 1{\cdot}256\,(\mathrm{X}_a + \mathrm{X}_g + \mathrm{X}_b) \times \mathrm{P} \quad . \quad . \quad . \quad (18)$$

Our final and complete equation for the ampère-turns of the field will thus take the form

$$\mathrm{X} = f'\left(\frac{Z_a}{a_a}\right) . \, l_a + {\cdot}8\mathrm{B}_g \;\; 2l_g + \mathrm{X}_b + f'\left(\frac{Z_m}{a_m}\right) . \, l_m \quad . \quad . \quad (19)$$

Simple though this equation is in its form, its application presents certain difficulties which render it well-nigh impossible to predict the ampère-turns required with absolute and complete accuracy. Apart from the somewhat indefinite nature of the leakage paths, the lengths of the paths in the iron portions of the magnetic circuit can only be calculated approximately, a mean having to be struck between the longest and the shortest; or, again, the area of cross-section normal to the lines of induction may be continuously varying (as, for example, in the pole-piece), and this in practical work necessitates a mean being taken which it requires considerable judgment to estimate accurately. A second source of error is that our calculations are based on certain magnetisation-curves, yet the particular class of iron used may not be exactly similar to that for which a curve has been obtained; cast iron in particular varies considerably in its magnetic properties. Still, when tested experimentally, a dynamo should certainly not require to be run at a speed differing by more than 5 per cent. from the designed speed in order to give the designed voltage.

In order to render the above methods of calculation clearer by an actual example, they will now be applied to the design of an 8-kilowatt shunt-wound machine, giving an output of 62 ampères at a pressure of 130 volts: the chief dimensions of the iron carcase which forms the magnetic circuit are given in fig. 135, from which it will be seen that it is a bipolar overtype horseshoe with wrought-iron limbs bolted up to a cast-iron yoke in the centre of the bedplate. The armature core is 9″ in diameter × 12″ long, and a preliminary estimate will lead us to allow a loss of about 6 volts over the resistance of the armature, and to determine upon 234 inductors. The speed is not to exceed 800 revolutions per minute, and therefore, by the equation (11) of p. 212,

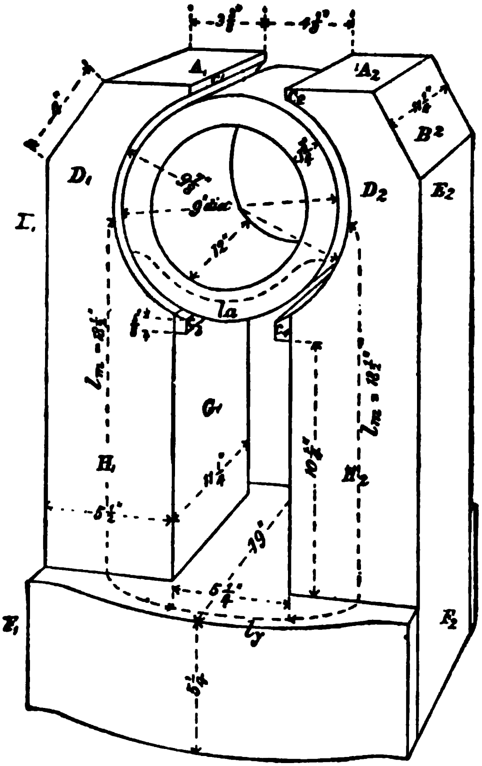

FIG. 135.—Typical 2-pole field.

$$E_a = 136 \text{ volts} = Z_a \times 234 \times \frac{800}{60} \times 10^{-8},$$

whence $Z_a = 4,359,000$, or, allowing a slight margin in speed, say $Z_a = 4,400,000$, at 790 revolutions per minute.

Taking the separate elements of the magnetic circuit in the order of our equation (19), we have

(1) The armature, requiring X_a ampère-turns.

The area of the iron in the armature core through which all the lines z_a pass is twice the area of the iron at any one part, since the two halves of the ring are in parallel, and half the lines pass along each of the two paths. Hence, if $a =$ the radial depth of the discs of which the armature is built up, and $b =$ the length of the core parallel to the shaft, the cross-sectional area of the ring is $2ab = 2\,(1 \cdot 75 \times 12)$ sq. inches. The whole of this is, however, not filled with iron, since allowance must be made for the insulating paper or varnish between the separate thin discs of iron. About 13 per cent. of the space is thus occupied by the insulation, and therefore the net sectional area of iron in the ring is $2ab \times \cdot 87 = 36 \cdot 5$ sq. inches, or $36 \cdot 5 \times 6 \cdot 45 = 235$ sq. cm. The maximum induction in the armature core is thus

$$B_a = \frac{4,400,000}{235} = 18,700.$$

The mean length of path in the armature, as shown by the dotted line, l_a, in fig. 135, is about 8½″, or 22 cm.

By reference to the wrought-iron curve of fig. 111, it will be seen that to produce an induction of 18,700 lines per sq. cm. requires an exciting power of over 200 ampère-turns per cm. length of path. It is, however, only over about 3½″, or 9 cm., between the polar tips, that the maximum density obtains: along the horizontal diameter, B_a is practically 0, and midway between these extremes, where $B_a =$ about 10,000, the necessary exciting power is only about 4 ampère-turns per cm. length of path. We may therefore with fair accuracy regard the whole of the exciting power required by the armature as expended in producing an induction of about $B_a = 18,000$ over a path of 12·5 cm., or a length somewhat more than the distance between the polar tips $c_1\ c_2$, or $c_3\ c_4$. For such an induction $f'(B_a)$ is 155, and therefore

$$X_a = f'(B_a) \,.\, l_a = 155 \times 12 \cdot 5 = 1,940 \text{ ampère-turns.}$$

Possibly this may prove a slight over-estimate, since the permeability of well-annealed charcoal-iron discs may be superior to that of the wrought iron in the curve of fig. 111; but an error here will produce but little error in the total result.

(2) The air-gap, requiring X_g ampère-turns.

Owing to the spreading of the lines of induction as they issue out of the polar face and pass into the armature core, the area of the air-gap is greater than the area of the bored face of the pole-piece. Not only do lines pass in a sloping direction into the core from the vertical edges of the 'horns,' or extended tips of the poles (fig. 116), but some also curve round into the armature from the outer faces of the pole-pieces at the edge of the bore, and this is especially the case when, as here, the length of the armature core is slightly greater than the width of the poles parallel to it. A certain 'fringe' must therefore be added to the area of the polar face, all round its perimeter. From the experiments of Dr. Hopkinson,[1] the effective width of this fringe may in our present case be taken as equal to four-fifths of the distance from iron of pole-piece to iron of armature. If, therefore, D be the diameter of armature core, and l_g the length of one air-gap, the length of arc subtending the polar angle at the mean radius of the air-gap is

$$\lambda = \pi \left(D + l_g\right) . \frac{\text{polar angle}}{360^\circ} ;$$

and the effective area of the air-gap is

$$(\lambda + \cdot 8 \times 2l_g)\,(w + \cdot 8 \times 2l_g), \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

where w is the width of the pole-piece parallel to the armature shaft. The actual area traversed by the lines entering the armature is even slightly larger, but the above is the effective area (a_g), assuming that the density of the lines over it is reduced to a uniform value of $B_g = \frac{Z_a}{a_g}$, and that all the lines pass radially into the core, traversing a minimum distance of air.

In the present case the bore of the pole-pieces is $9\frac{7}{8}''$, whence, the diameter of the armature being 9″, $l_g = \frac{7}{16}$″. The angle subtended by the polar face is 130° : hence

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[1] Vide *Journal Soc. Tel. Eng.* vol. xv. part 64, pp. 561, 565, 566, and Hopkinson's reprinted *Original Papers on Dynamo Machinery*, pp. 95, 137, 142.

$$\lambda = \pi\,(9\tfrac{7}{16}'')\,\frac{130^\circ}{360^\circ} = 10{\cdot}7''.$$

The effective area of the air-gap is therefore

$$(10{\cdot}7 + {\cdot}7)\,(11{\cdot}25 + {\cdot}7) \times 6{\cdot}45 = 880 \text{ sq. cm.,}$$

and the induction when averaged over the whole of it is

$$B_g = \frac{4{,}400{,}000}{880} = 5{,}000,$$

and $$X_g = {\cdot}8 B_g \,.\, 2l_g = {\cdot}8 \times 5{,}000 \times 2{\cdot}22 = 8{,}880.$$

(3) The back ampère-turns, X_b.

The method by which these are calculated will be reserved for Chap. XVII., but for the machine now under consideration they may be reckoned at full load as $=580$.

The result so far is

$$\begin{aligned} X_a &= 1{,}940 \\ X_g &= 8{,}880 \\ X_b &= \underline{\ \ \ 580} \\ \text{Sum} &= 11{,}400 = X_p, \end{aligned}$$

or the exciting power acting at the poles of machine.

The next step necessitates a knowledge of P, the leakage permeance : for the present this may be taken as $= 63{\cdot}8$, the method by which this numerical value is obtained being explained on pp. 302–5. Thence the leakage lines

$$\zeta = 1{\cdot}256 \times 11{,}400 \times 63{\cdot}8 = 913{,}000$$

and $$Z_m = Z_a + \zeta = 5{,}313{,}0000.$$

(4) The magnet limbs, requiring X_m ampère-turns.

The area of the wrought-iron limb is

$$5\tfrac{1}{2} \times 11\tfrac{1}{4} \times 6{\cdot}45 = 400 \text{ sq. cm.,}$$

and the average induction in them is

$$B_m = \frac{5{,}313{,}000}{400} = 13{,}300.$$

By reference to the wrought-iron curve of fig. 111, we see that when $B_m = 13{,}300$, the ampère-turns required are 13 per cm. length, or slightly less on the descending curve. The length of path in each magnet-core under the exciting coils is $10\frac{1}{4}''$, and at the yoke, where the lines bend round into the bedplate, their curved path to the edge of the wrought iron may be taken as 4·3″ long. Within the pole-piece the induction decreases slightly, for the passage of the lines outwards is hardly counterbalanced by any corresponding decrease in the cross-sectional area of the iron. Hence the equivalent length of the mean path in the pole-piece may be taken as 4″. The average length of path within each limb may therefore be reckoned as

$$10{\cdot}25+4{\cdot}3+4=18{\cdot}5'',$$

and the total length within the two as

$$18{\cdot}5\times 2\times 2{\cdot}54=94 \text{ cm.}$$

Hence

$$X_m=f'(B_m)\,.\,l_m=13\times 94=1{,}220 \text{ ampère-turns.}$$

(5) The yoke, requiring X_y ampère-turns.

The area of the cast iron $=19''\times 5\frac{1}{4}''\times 6{\cdot}45=643$ sq. cm.;

and

$$B_y=\frac{5{,}313{,}000}{643}=8{,}270,$$

whence from the cast-iron curve of fig. 111, $f'(B_y)=84$. The lines spread somewhat in the yoke, and their mean length of path is about 16 cm., whence

$$X_y=84\times 16=1{,}350.$$

The total excitation required is therefore

$$X=11{,}400+1{,}220+1{,}350=13{,}970, \text{ say } 14{,}000 \text{ ampère-turns}$$

We have now to return to the question of magnetic leakage, ζ, and in the first place to the calculation of the leakage permeance, P, of any dynamo. In the following pages the method of Prof. Forbes[1] is described with slight modification. Since we have to do with a large number of paths, which are assumed to be all in parallel with one another, it is simplest to deal with their permeances, for these can be immediately added together to discover the joint permeance of the leakage paths. The lines are assumed to follow certain directions in the air according to the situations and distances of the two surfaces between which they flow, and to meet these different cases three general propositions are, as a rule, sufficient.

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[1] *Journal Soc. Tel. Eng.* vol. xv. part 64, p. 555.

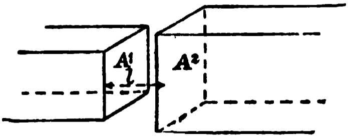

FIG. 136.

(I) In the case of two parallel surfaces facing each other, the areas of which are approximately equal, the lines of induction may all be assumed to pass straight across from the one surface to the other, and the permeance of the air-gap between the two surfaces is then equal to the mean of their areas divided by their perpendicular distance apart (fig. 136); or

$$p = \frac{\frac{1}{2}(A_1 + A_2)}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

(II) In the case of two equal rectangular surfaces situated near each other in the same plane, with their neighbouring edges parallel, the lines of induction may be assumed to be semicircles described about a central line drawn between the two surfaces; the permeance of the air-path from one to the other is then

$$p = \frac{a}{\pi} \log_e \frac{r_2}{r_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where r_1 and r_2 are respectively the distances from the central line to the nearest and farthest edges of either rectangle, and a is the depth of each rectangle at right angles to r, i.e. along the parallel edge (fig. 137).

(III) In the case of two equal rectangular surfaces situated similarly in one plane, but at some distance apart, the lines of induction may be assumed to be quadrants connected by straight lines, the quadrants being described from the neighbouring edges of the rectangles as lines of centres: the permeance of the air between the two surfaces is then

$$p = \frac{a}{\pi} \log_e \frac{\pi w + b}{b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where a is again the depth of each rectangle, w its width, and b is their distance apart (fig. 138).

To these may be added other extensions of the same principles, but in general the three main propositions above considered will meet the most common cases, or in default of any other guidance it must suffice to map out a probable course for the leakage on the above lines, and thence by scaling the mean areas of the two surfaces and the mean length of path between them to deduce the permeance $= \frac{\text{area}}{\text{length}}$.

It must be remembered that in all the foregoing equations the dimensions are in centimetres and the logarithms are to the Napierian base ϵ: hence, for the benefit of English dynamo-designers, it may be useful to give the equivalent equations when the dimensions are in inches, and the areas in square inches : they are

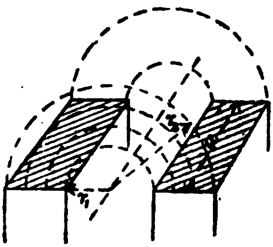

FIG. 137.

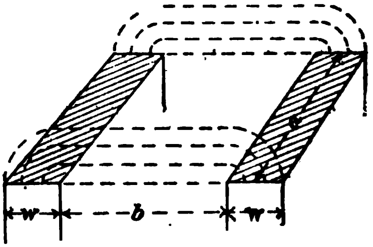

FIG. 138.

$$\text{(I)}\ p = 2{\cdot}54 \cdot \frac{\frac{1}{2}(A_1'' + A_2'')}{l''} = \frac{1{\cdot}27\,(A_1'' + A_2'')}{l''} \quad . \quad . \quad . \quad . \quad (21)$$

$$\text{(II)}\ p = \frac{2{\cdot}54\,a''}{\pi} \log_{10}\frac{r_2''}{r_1''} \times 2{\cdot}3 = 1{\cdot}86 \times a'' \times \log_{10}\frac{r_2''}{r_1''}. \quad . \quad (22)$$

$$\text{(III)}\ p = 1{\cdot}86 \times a'' \times \log_{10}\frac{\pi w'' + b}{b''}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

We are now in a position to calculate the leakage permeance of our dynamo (fig. 135), taking the permeance of the several paths in succession, and assuming them to be approximately as shown in fig. 133.

(1) Between the flat tops of the pole-pieces, A_1 and A_2, taking

the paths by proposition III. as partly quadrants and partly straight lines,

$$p_1 = 1{\cdot}86 \times 11{\cdot}25 \times \log_{10} \frac{\pi \times 3{\cdot}375 + 4{\cdot}125}{4{\cdot}125},$$
$$= 11{\cdot}6.$$

(2) Between the sloping surfaces, B₁ B₂, at the top of the pole-pieces, by proposition III., the mean direct distance between the two being 14″, but the lines curving round in a zone outside the lines of region (1).

$$p_2 = 1{\cdot}86 \times 11{\cdot}25 \times \log \frac{\pi \times 4 + 14}{14}$$
$$= 5{\cdot}85.$$

This may be roughly checked by directly scaling the mean length of the supposed path and dividing it into the area of either surface in centimetre measure, thus :

$$p_2 = \frac{10 \times 28{\cdot}5}{50} = 5{\cdot}7.$$

(3) Between the neighbouring edges of the opposing polar tips, c_1 and c_2, and the corresponding surfaces, c_3 and c_4, below the armature. By rule I. the permeance between c_1 and c_3 will be

$$2{\cdot}54 \times \frac{{\cdot}625 \times 11{\cdot}25}{4{\cdot}125} = 4{\cdot}35,$$

and the joint permeance of the two paths is therefore $p_3 = 8{\cdot}7$.

Many of the lines which issue from the extreme edge of the polar projections converge upon the armature core, and are therefore not to be reckoned as lost by leakage ; but when the leakage across between the lower surfaces of the horns c_3 and c_4 is taken into account, the permeance, as calculated above, may be regarded as sufficiently accurate.

(4) Between the sides of the pole-pieces, D₁ and D₂, and the corresponding surfaces at the further end of the magnet. Each surface may be treated approximately as a rectangle 3⅜″ wide × 10″ deep, the two being separated by a mean direct distance of 9″. Therefore, by proposition III., the permeance between the one pair D₁ and D₂ is

$$1{\cdot}86 \times 10 \times \log \frac{\pi \times 3{\cdot}375 + 9}{9} = 6{\cdot}3,$$

and therefore the joint permeance on both sides of the magnet is $p_4 = 12{\cdot}6$.

(5) From the back of the pole-pieces into the yoke, i.e. from E_1 to F_1, and E_2 to F_2. The surfaces E_1 and F_1 may be regarded as approximately of equal area, and the paths followed by the lines may be taken as quadrants joined by straight lines; hence, by proposition III., the permeance between E_1 and F_1 or

$$p_5 = 1{\cdot}86 \times 11{\cdot}25 \times \log \frac{\pi \times 7 + 10{\cdot}25}{10{\cdot}25} = 10{\cdot}4.$$

Now the magnetic difference of potential between the two surfaces, E_1 and F_1, is approximately half that existing between the pole-pieces, for the yoke may be regarded as at zero potential, and the limbs as gradually increasing in potential to a positive and negative maximum respectively, as we proceed from the yoke to the poles. The lines, therefore, which flow through each limb, due to the leakage now under consideration, are

$$\zeta_5 = 1{\cdot}256 \frac{X_P}{2} \times p_5.$$

If, therefore, p_5 be calculated as above, and we wish to express the leakage on the assumption that it is all due to the difference of magnetic potential at the poles, the value p_5 must be halved before it can be regarded as in parallel with the preceding permeances; when so halved it may be directly added to them, and, therefore, for our purpose it will be necessary to take the value $\frac{p_5}{2} = 5{\cdot}2$.

(6) From limb to limb. This leakage is divisible into two portions: (i) straight across from the inside of the one limb to the inside of the other, i.e. from G_1 to G_2; and (ii), in curves from the neighbouring sides of the magnets, i.e. from H_1 to H_2, and between the corresponding surfaces at the further end of the magnet.

The permeance of path (i) is

$$2{\cdot}54 \times \frac{10{\cdot}25 \times 11{\cdot}25}{5{\cdot}25} = 56,$$

and the joint permeances of path (ii) is

$$1{\cdot}86 \times 10{\cdot}25 \times \log \frac{\pi \times 5{\cdot}5 + 5{\cdot}25}{5{\cdot}25} \times 2 = 24,$$

whence we obtain $p_6 = 80$.[1]

Now the leakage along these paths from limb to limb takes place under varying differences of potential, as we pass from the yoke to the pole-pieces; the magneto-motive forces of the ampère-turns are uniformly distributed along the two limbs, but the difference of magnetic potential between the two limbs being the gradually increasing sum of these forces, less the fall of potential over the iron, rapidly rises in value as we proceed from the zero potential of the yoke towards either pole-piece. Approximately, it may be assumed that the average difference of potential acting between the two limbs is about half the maximum difference of potential, i.e. $= 1{\cdot}256 \frac{X_P}{2}$. Let $\zeta_6 =$ the actual number of lines that leak across under this average difference of magnetic potential; then

$$\zeta_6 = 1{\cdot}256 \frac{X_P}{2} . p_6.$$

Now these lines are distributed over the space between the limbs, and do not all pass through the entire length of limbs and yoke; the total number of lines carried by the limbs is, in fact, continually varying along their length, as lines leak out of or into them. But our object is to determine a mean value for the number of lines carried by the iron; thence we aim at obtaining a corresponding mean value of the induction B_m, so as to deduce the number of ampère-turns required for the field-magnet. If, therefore, ζ_6 lines be regarded as flowing through the entire length of the limbs, the induction will be over-estimated for part of their length. We may, however, assume without much error that within the limits $z_m = z_a + \zeta_1 + \ldots + \zeta_5 + \zeta_6$ and $z_m = z_a + \zeta_1 + \ldots + \zeta_5$ the number of ampère-turns required by the magnet will closely correspond to a mean number of lines, $z_a + \zeta_1 + \ldots + \frac{\zeta_6}{2}$. The question, therefore, is, what permeance will give $\frac{\zeta_6}{2}$ lines when a magnetic difference of potential of 1·256 X_P acts over it, and this is evidently $\frac{p_6}{4} = 20$.

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[1] In the case of round magnet-cores, or limbs, placed side by side, as in fig. 124, the permeance between them may be calculated by means of a method due to Professor S. P. Thompson, and set forth in his Cantor Lectures on the Electro-magnet before the Society of Arts, Lecture II., reprinted in the *Electrical Engineer*, November 21, 1890.

All the permeances have now been reduced to such values as may be regarded as in parallel with the armature and air-gaps, and may therefore be immediately added together; and our final result is that the joint permeance

$$P = p_1 + p_2 + p_3 + p_4 + \frac{p_5}{2} + \frac{p_6}{4} \quad . \quad . \quad . \quad . \quad (24)$$

$$= 11·6 + 5·7 + 8·7 + 12·6 + 5·2 + 20 = 63·8,$$

and the average number of lines passing through the entire length of the magnet is $z_a + \zeta$, where, as on p. 298,

$$\zeta = 1·256 \; X_P \times P$$

In addition to the several paths already considered, there are others which would have to be taken into account if greater exactitude were required, such, for instance, as from the pole-pieces to the bearings and their supporting pedestals on either side of the magnet. Again, the surfaces already taken into account may be paired differently; for example, we might calculate the permeance between A_1 and B_2, or between B_2 and F_2; but if this be done, it would be incorrect to add the several permeances together, since they are not all in parallel. In their actual distribution the lines of induction never cross one another or intersect; further, if two sets of lines are regarded as traversing the same air-space for a part of their course, *e.g.* leakage from A_1 to A_2, and leakage from A_1 to B_2, the fall of magnetic potential over that part of their course which is common to both must correspond to their joint density, and not to that of either set taken separately. Hence the addition of fresh paths would involve a distribution of the lines different from that assumed in our first approximation and a recalculation of their amount, and such rearrangement might result in a slightly higher value for the leakage permeance. Still, if the leakage field be regarded as distributed mainly after the fashion indicated above, a close approximation may be made to the ampère-turns actually found to be required by the field-magnet when the machine is tested. A certain amount of judgment is required in the first instance in selecting the direction of the several paths, none of which intersect; but to guide us in this actual experiment may be called to our aid, if a machine with the required type of field-magnet is at hand.[1]

A partial compensation for the difficulty of accurately determining the somewhat vague and indefinite factor, P, lies in the fact that a considerable percentage error in its determination produces, under ordinary conditions, but a small percentage of error in the total number of ampère-turns required to produce the useful field through the armature; for this total number of ampère-turns is the sum of those required for the armature and air-gaps and those required for the magnet, and an error in our estimate of P only affects the latter. Evidently, the smaller the proportion which the ampère-turns required for the magnet form of the total number, the less will an error in their determination affect the whole. Hence the percentage of error thus introduced will depend on the relative amount of X_m as compared with the total X, and will be greater in the case of a magnet which has a greater length of circuit or is worked with a higher induction. But, further, it will in especial depend upon the degree of saturation of the field-magnet, since the higher the induction the greater is the difference between the ampère-turns required to drive the supposed and the actual number of lines Z_m through the magnet.[2]

If the value of P for a particular machine of a given type has once been determined by calculation, and checked by experiment, certain further conclusions may be drawn applicable to other machines of the same type, but differing in their dimensions. If the linear dimensions of one machine were simply magnified n times in the design of another, the new value for P would be n times its former value; the lengths of the leakage paths would be increased n times, but their cross-sections would be increased n^2 times. More often, however, the ratio of the two chief dimensions of the dynamo, viz. the diameter and length of the armature core, is altered, and this involves a more detailed consideration of the way in which the leakage permeances are affected by any change in these dimensions. The most important principle must be the fact that an increased or reduced length of armature core involves no further alteration than an equal increase or reduction in the width of the magnet parallel to the armature, while, on the other hand, a change in the diameter of the armature usually involves a more or less proportionate change in the depth and length of the magnet limbs, for the alteration in the area of iron in the armature necessitates an alteration of the area of iron in the magnet, and also of the length of the exciting coils. Now, considering the various leakage paths of the dynamo above discussed, it will be found that the permeances between A_1 and A_2, B_1 and B_2, E_1 and F_1, G_1 and G_2, are almost solely dependent on the length of the armature, and vary directly with it; for if its diameter alone be altered, the length between them is increased, but so also to the same extent is one of the two dimensions which go to make up their area. On the other hand, the permeances between D_1 and D_2, or H_1 and H_2, are almost solely dependent on the diameter, being unaffected by any change in the length of the armature; and, further, they vary directly as the diameter of the armature, for although an increase in the diameter lengthens their paths, it increases both dimensions of their cross-section. Lastly, the permeance between the edges c_1 and c_2 of the pole-pieces varies directly as the length of armature and inversely as the diameter, for, with an increased diameter, the width of the gap between the pole-pieces would be increased, but the vertical depth of the edges would be scarcely altered. Hence an approximate formula for the total leakage permeance would take some such form as $P = k_1 L + k_2 D + k_3 \frac{L}{D}$, where k_1, k_2, k_3 are suitable constants for the particular type of machine. By this differentiation of the leakage permeances it is possible to judge of the relative effect of any alteration in the diameter or length of armature, and by the substitution of the new values of L and D to approximate to the resulting permeance of the new dynamo. In the case of the ring overtype dynamo, proportioned somewhat after the fashion of fig. 135, our previous calculations will lead us to the approximate formula

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[1] For such actual experiments, vide *Journal Inst. Elect. Eng.* vol. xix. part 86, pp. 244–246 (Trotter).

[2] *Vide* Esson, 'Some Points in Dynamo and Motor Design,' *Journ. Inst. Elect. Eng.* vol. xix. part 85, p. 123.

$$P=3\,L''+2\,D''+7\frac{L''}{D''}, \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

where L'' and D'' are in inches; and from this it will be seen that an increase in the length of the armature produces a considerably greater increase in the leakage permeance than the same increase in the diameter. If the ratio $\frac{L}{D}=1\cdot33$, as in our case, over one-half of the leakage takes place from surfaces the permeances of which are solely affected by the length of the armature, over one-quarter will be affected by a change in the diameter, and the remainder is affected by a change in either dimension.

In the case of a similar overtype dynamo with drum-armature, for a given diameter of core, the depth of iron will be greater than in the ring; hence the depth of the magnet will be greater, the second term which has to do with leakage between the outside faces, D_1 and D_2, H_1 and H_2, will acquire greater prominence, and k_2 must be increased. The equation will now take some such form as

$$P=3\cdot25\,L''+2\cdot5\,D''+7\frac{L''}{D''}, \quad . \quad . \quad . \quad . \qquad (26)$$

and in general the relative importance of the two dimensions depends on the thickness and length of magnet which suits a given design of armature.

The leakage of undertype dynamos is so largely dependent on the exact method by which they are supported on the bedplate, and the shortest distance between the iron of the bedplate and the iron of the magnet, that it is more difficult to give even approximate values for the constants k_1 k_2 and k_3. Roughly, it may be said that the leakage permeance varies between $P=4L''+2D''+8\frac{L''}{D''}$ and $P=5L''+3D''+8\frac{L''}{D''}$; in both cases the normal distance between the bedplate and magnet has been taken at about 8–9 times the air-gap on one side between armature and pole-piece, but in the former case no sheet of metal runs under the pole-pieces; in fact, a continuous stretch of iron lying under the pole-pieces should, as far as possible, be avoided, or broken up by a large central air-gap, so as to force the lines to pass round through a considerable length of cast iron.

The above equations will, however, serve to show the varying importance which must be attached to the two dimensions L and D in different types, and from the form in which they are cast they are better adapted to prevent us from being misled when any radical alteration of dimensions is made than other and simpler equations which have been from time to time proposed.

Since the leakage permeance of any given dynamo is almost entirely due to the air-spaces between certain surfaces, its value is practically a constant quantity for that dynamo, and independent of the actual amount of leakage. Now, since the difference of magnetic potential between the poles of the dynamo increases as the number of useful lines through its armature is increased, it is evident that the leakage ζ will also continuously increase as z_a is increased; and, further, will increase faster than z_a; for as the armature core becomes more and more highly saturated, an increase in the ampère-turns expended over the armature will produce less and less increase in the lines through it, while the leakage still continues to increase directly as the difference of magnetic potential between the poles. Hence the proportion which the leakage lines bear either to the useful or to the total number of lines through the magnet will continuously increase, and for each degree of magnetisation of the dynamo $\frac{z_a + \zeta}{z_a}$ or $\frac{z_m}{z_a}$ will have a different value. In the case of the ring overtype dynamo above considered, $z_m = 1{\cdot}21\, z_a$ when it is magnetised as it would be under ordinary working conditions, so as to obtain an induction of about 18,000 in the armature core; in other words, the leakage is about 21 per cent. of the useful lines, or 17 per cent. of the total flux. Strictly speaking, the factor by which the useful lines through the armature must be multiplied in order to obtain the greater number of lines flowing through any other part of the circuit will vary all along the length of the magnet, as lines leak into or out of it. With sufficient accuracy, however, we may assume the flux of lines to be constant over each of our main subdivisions of the magnetic circuit, so that $z_m = v_m z_a$, $z_y = v_y z_a$, $z_p = v_p z_a$; the different parts of the magnet are denoted by the subscript letters as before, and each part has its appropriate factor, v, greater than unity. Our fundamental equation for the ampère-turns of a simple magnetic circuit may now be expressed as

$$\mathrm{x} = f\left(\frac{z_a}{a_a}\right) \cdot l_a + \cdot 8\,\frac{v_g z_a}{a_g} \cdot 2l_g + \mathrm{x}_b + f\left(\frac{v_p z_a}{a_p}\right) \cdot l_p$$
$$+ f'\left(\frac{v_y z_a}{a_y}\right) \cdot l_y + f'\left(\frac{v_m z_a}{a_m}\right) \cdot l_m,$$

and such was, in principle, the original form of the equation as first published by Drs. J. and E. Hopkinson in their classical paper on *Dynamo-electric Machinery*.[1] Having established the theoretical equation, they further measured experimentally the number of stray lines of induction that leak through different portions of the air-space about the field-magnets of two dynamos of different types, each of which was excited with its normal magnetising current. Thence they were enabled to deduce the value which the factors v_g, v_p, &c. have for the given dynamos when magnetised to their working degree of saturation.

The first factor v_g requires further explanation. It has been said that, owing to the difference of magnetic potential which exists between opposite sides of an armature under the two pole-pieces, a certain number of lines pass diametrically across the interior of the armature through the shaft without passing round through the core between the two pole-pieces; hence, if z_a be reckoned as the number of lines actually flowing round through the two halves of the core, and passing through sections of the core taken midway between the pole-pieces, the number of lines flowing through the air-gaps must be greater than z_a; for the additional lines passing diametrically across the shaft must have passed through the air-gaps, and must, therefore, be taken into account. This is effected when z_a is multiplied by the factor v_g. In the case of drum armatures it is unnecessary to introduce this factor, owing to the lower induction which is usual in them and the absence of internal wires; but in rings strongly magnetised, the lines across their interior may become of sufficient importance to warrant their being taken into account as they are cut by the internal wires. Thus, in the ring armature tested by the Drs. Hopkinson, which was magnetised to a very high induction at the part of maximum density, the internal leakage was then found to amount to as much as 5 per cent. of the lines through the armature, or $v_g = 1{\cdot}05$. The dynamo, upon the ring armature of which the above experiment was carried out, was of the 'Manchester' double-magnet type (fig. 125), and it was found that the number of lines flowing through a section of the upper pole-piece near the magnet core, when expressed in terms of z_a was $= 1{\cdot}13 \frac{z_a}{2}$, and through the lower pole-piece $1{\cdot}24 \frac{z_a}{2}$, the difference between the two being due to leakage from the magnet-cores into the bearings, which were supported on extensions from the lower cast-iron pole-piece. The greatest number of lines flow through a section at the centre of each magnet-core, the value gradually decreasing from this point up to their ends; the *mean* number of lines flowing through either magnet was $1{\cdot}49 \frac{z_a}{2}$, or $v_m = 1{\cdot}49$. In designing the field-magnet, it is this latter factor which is in reality the most useful to us, since we wish to obtain some mean value of z_m whence we may calculate the ampère-turns required to overcome the reluctance of the magnet, assuming that that number pass through the entire length of the magnet core. Thus, in the above case of the 'Manchester' double-magnet field, if we require z_a lines through the armature, 49 per cent. more lines must be allowed for in calculating the area and ampère-turns required for the magnet-cores, since out of the total number of lines through them only about two-thirds pass through the armature core as well. Again, in the case of the undertype Edison-Hopkinson machine, with drum armature, which was also examined by Drs. J. and E. Hopkinson, it was found that out of 100 lines passing through the cross-section of each magnet limb at the middle of its length, only about 75 reach the armature core, or $v_m = 1{\cdot}32$. Out of the observed loss of 25 per cent., nearly half, or 11 per cent., was due to leakage through the zinc base into the cast-iron bedplate which stretched directly across below the pole-pieces; the leakage between the limbs and between the upper edges of the pole-pieces was 8·3 per cent., the remainder being accounted for by leakage between the lower edges of the pole-pieces, from their sides, and from their backs into the yoke.

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[1] *Phil. Trans.* 1886, part i. p. 331, reprinted in *Original Papers on Dynamo Machinery*, pp. 79–133. This important paper may be said to have first laid down a sure groundwork of theory for the scientific design of dynamos, and raised it from the level of empirical 'rules-of-thumb.' A study of it is essential to every student.

In most cases we may without much error assume the different factors for the different parts of the magnet to be identical, and reckon the number of lines through any part of it to be $z_m = vz_a$, where v may be called the 'leakage coefficient' of the dynamo. The values of v and the percentage distribution of the leakage for several types of dynamos have been experimentally determined and recorded by different observers;[1] but, unfortunately, little or no information has at the same time been given of the degree of saturation of the armature and magnet, beyond the fact that they were magnetised to their normal working extent.

Such recorded values of v may be used for the purpose of approximately calculating the area and ampère-turns required by the field-magnet. Thus the usually adopted value of v for the 2-pole overtype dynamo of fig. 2, with cast-iron pedestals on either side of the magnet, is about 1·25, which agrees closely with our previous calculation as to the amount of leakage when normally excited with 14,000 ampère-turns; and for the similar undertype of fig. 123, $v =$ from 1·3 to 1·35. For the Manchester field of fig. 125, with the armature supported on cast-iron extensions from the lower pole-piece, v was, as stated above, $= 1{\cdot}49$. In almost all multipolar dynamos, v is comparatively high, and for figs. 129 and 74 may be taken at from 1·4 to 1·5; and in the case of an alternator of the type shown in fig. 75, without iron in its armature, v has been found to be as high as 2; in other words, only half the total flux passed through the armature, the remaining half leaking directly across from one pole to the neighbouring pole on either side without being cut by the rotating inductors. Implicit reliance cannot, however, be placed on any such values of v. As already mentioned, it varies decisively if the degree of saturation be considerably varied, and since this is so, it varies with different values of the 'back ampère-turns,' i.e. with different values of the armature current, even though the total number of ampère-turns on the field remains unchanged; for the armature current directly affects the degree of saturation of the core. On this account the first method of obtaining the ampère-turns, which involves a term $\zeta = \frac{X_{(a+g+b)}}{\cdot 8} \times P$, is decidedly preferred by the authors to the second form, which contains the factor v. In default of such experimental determination of the values which it takes (as was, in fact, made by the Drs. Hopkinson before they put into use their original equation), we are more liable to be misled by false values of v than by errors in determining P. The somewhat laborious task of determining P, even if it be, at best, only an approximation, is thus very strongly to be recommended, especially in the case of the adoption of a new type of field-magnet.

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[1] *Vide* Esson, 'Some Points in Dynamo and Motor Design,' *Journal Inst. Elec. Eng.* vol. xix. part 85; 'Magnetic Leakage in Dynamos and Motors' (Ives), reprinted in the *Electrical Review*, January 22 and 29, 1892; and 'Magnetic Data of Sprague Motor' (Parshall), *Elec. Eng.* June 13, 1890; also (Puffer) *Electrical Review*, April 15, 1892.

Finally, it may be remarked that in all bi- or multi-polar machines, and to a much less degree in all unipolar machines, a certain amount of leakage is an inevitable necessity, and, so long as it be kept within due limits, is but a small evil. If the energy required to magnetise a dynamo is from 3 to 5 per cent. of its output, as it is in most modern dynamos, magnetic leakage cannot very greatly affect the efficiency and cost of working, for a supposed complete absence of leakage would but slightly decrease this percentage; and the practical endeavour to reduce it appreciably may reduce the weight of iron and wire, but can only lead to a considerably increased first cost of the machine, owing to a too lavish use of non-magnetic materials, such as gun-metal.

In dealing with the divided magnetic circuits of double horseshoe fields (figs. 125 and 126), it is simplest to calculate the ampère-turns that must be placed on one of the two horseshoes, since the ampère-turns on the other must be precisely alike. z_a being, as before, the number of lines that leave or enter one polar surface, the magnetising coils on one horseshoe must furnish sufficient exciting power to cause half that number to pass into and out of one side of the armature through two air-gaps, each of which has an area half that of the entire air-gap. We arrive, therefore, at the same inductions, B_a, B_g, whether we consider the entire number of lines, z_a, passing through the areas of the air-gaps and armature as a whole, or half that number passing through half those areas. Let ζ=the number of leakage lines for each magnet-core, and a_m= its area; then

$$X=f'\left(\frac{z_a}{a_a}\right).l_a+\cdot 8\frac{z_a}{a_g}.2l_g+X_b+f'\left(\frac{\frac{z_a}{2}+\zeta}{a_m}\right).l_m; \quad . \quad (27)$$

and this number of ampère-turns must be placed on each horseshoe.

The case of fig. 127 is exactly similar; two opposite poles are reckoned as one polar surface, out of which, or into which, flow z_a lines as required by equation (11) of p. 212, and these z_a lines within the armature core follow two paths in parallel, since they pass round the ring in opposite directions. In this case, however, not only is the group of z_a lines split up into two before they pass through the magnetising coils, but, further, each sub-group of $\frac{z_a}{2}$ lines again bifurcates when passing through the two separate paths of the yoke frame; the induction B_y in the yoke is therefore equal to $\frac{\frac{z_a}{2}+\zeta}{2a_y}$, where a_y is the area of either half of the yoke. Hence the number of ampère-turns that must be placed on each pair of coils on the one side of the ring is

$$AT=f'\left(\frac{z_a}{a_a}\right).l_a+\cdot 8\frac{z_a}{a_g}.2l_g+X_b+f'\left(\frac{\frac{z_a}{2}+\zeta}{a_m}\right).l_m+f'\left(\frac{\frac{z_a}{2}+\zeta}{2a_y}\right).l_y, \quad (28)$$

and the same must be placed on the opposite pair of magnet-cores.

If such a dynamo be multipolar, as fig. 132, and, in accordance with the symbols of p. 145, p = the number of pairs of poles on one side of the ring $=\frac{P}{2}$, the ampère-turns on the poles of one side of the armature$=p\,AT$, and the total number for the whole machine $=P\,.\,AT$.

In the case of the single magnet with divided yoke (fig. 128), the area of each half being a_y, it is evident that the number of ampère-turns on the pair of coils is

$$\text{AT}=f'\left(\frac{z_a}{a_a}\right).\,l_a+\cdot 8\,\frac{z_a}{a_g}.\,2l_g+\text{X}_b+f'\left(\frac{z_m}{a_m}\right).\,l_m+f'\left(\frac{z_m}{2a_y}\right).\,l_y. \quad (29)$$

and when the same magnetic system is multipolar, as in fig. 129, the total number of ampère-turns $= p\text{AT}$.

In the 4-pole field of fig. 130, each of the two horseshoes must be wound with sufficient ampère-turns to propel z_a lines through two air-gaps, half z_a through the armature core from S to N $=l_a$, and the remaining half from S to a, and from N to b, i.e. through l_a also. The original equation, therefore, for a single circuit applies exactly to each horseshoe.

In the case of the disc multipolar dynamo (fig. 75), which usually has no iron in its armature, reckoning l_g as the length of each air-gap between opposite poles, each pair of coils must be wound with

$$\text{AT}=\cdot 8\,\frac{z_g}{a_g}.\,l_g+\text{X}_b+f'\left(\frac{z_m}{a_m}\right).\,l_m+f'\left(\frac{z_m}{2a_y}\right).\,l_y. \quad (30)$$

It has been shown on p. 299 that the number of ampère-turns required to give 4,400,000 lines through the armature of a particular dynamo is 14,000 when the armature current has its full value (i.e. the sum of 62 ampères in the external circuit and about 4 ampères in the shunt circuit of its field-magnet, or 66 ampères in all). If a number of different values be assigned to z_a, and the ampère-turns required in each case be determined (always on the assumption that the armature current has its full value), a curve may be plotted, connecting together the corresponding values of z_a and X, and this curve may be called the 'curve of magnetisation' of the particular dynamo for its full current, or more strictly for the full value of its armature ampère-turns. Curve 3 in fig. 139 shows such a full-current curve of magnetisation for our 9″ × 12″ dynamo, C_a being maintained throughout at its full value of 66 ampères. The corresponding values of z_a and X may either be worked out by the same methods of calculation as have been previously applied, or, if the machine be already built, they may be determined experimentally, as will be shown later: in either case the brushes are throughout assumed to be adjusted to the point of minimum sparking, and the abrupt termination of the curve marks the point

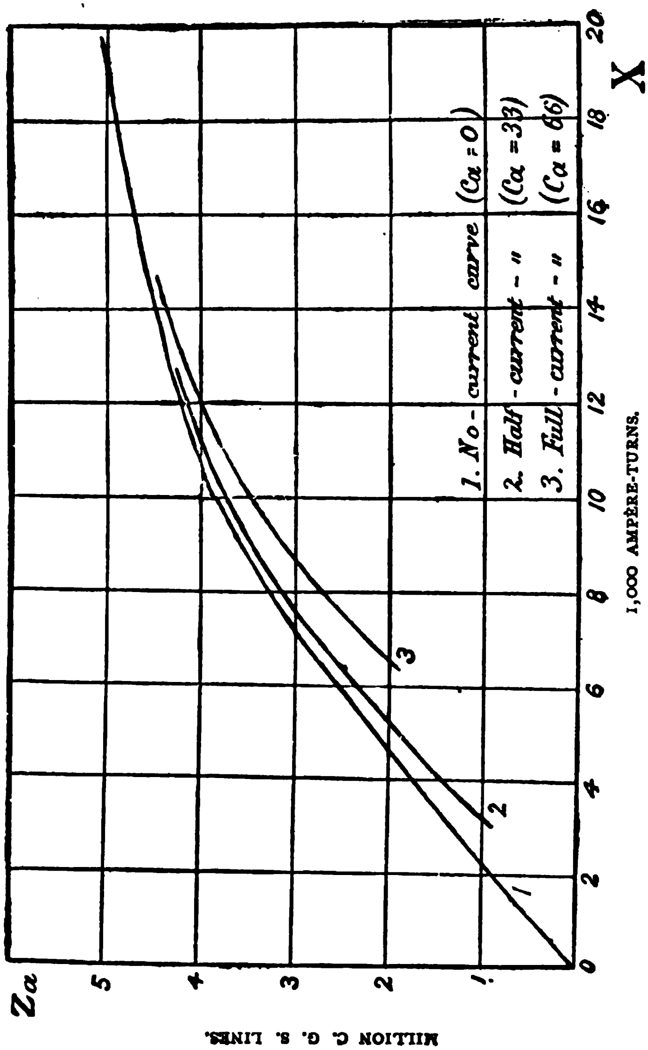

FIG. 139.—Curves of magnetisation of 9" × 12" dynamo.

below which it is impossible to reduce the magnetisation without causing excessive sparking at the brushes. If we further carry out the same process for a different and smaller value of c_a, say half its maximum value, or 33 ampères, a second curve of magnetisation will be obtained for half-current: this will fall higher than curve 3, inasmuch as the back ampère-turns are less, and therefore fewer ampère-turns are required on the field to produce a given number of lines through the armature. Finally, if c_a be taken as $=0$, there is no reaction of the armature-current on the field, the back ampère-turns, x_b, are zero, and the highest or 'no-current' curve of magnetisation (1, in fig. 139) is obtained. It should be observed that the horizontal distance between the curves for no-current and full-current at, say, 4,400,000 lines is more than the direct value of the back ampère-turns, viz. 580, inasmuch as these latter increase the leakage, and therefore the ampère-turns required over the iron of the magnet; further, the horizontal distance between curves 1 and 3 increases as the total number of lines diminishes, owing to the fact that with the weaker field the brushes require a greater forward lead, and the back ampère-turns are increased even for the same value of c_a. The use and importance of these curves will be more apparent in the next chapter.

The method by which the field-magnets of dynamos are excited admits of several variations, according to the sources whence the magnetising current is derived; and these will be explained in the following chapter. Apart, however, from such differences of source, it remains to determine the necessary gauge and weight of copper wire required for the winding of magnetising coils which are to give a certain number of ampère-turns, x: these may form either the whole or a part of the total number of ampère-turns required by the machine, the following being a general solution of the problem applicable to all cases. The data which we have at our disposal may vary in different cases, but in general it will be found that our starting-points will be a knowledge, direct or indirect, of (1) the voltage that will be applied to the ends of the wire, and (2) the mean length of one turn of the coils. Knowing the thickness and width of a rectangular magnet limb, or the diameter of a round magnet-core, it is easy to estimate fairly closely the mean length of one turn of a coil encircling either the one or the other : a certain allowance must be made for the depth of the winding, by reason of which the mean length of a turn in the central of several layers of winding will be greater than the actual perimeter of the magnet itself ; and the correctness of this allowance must be subsequently checked when the winding has been determined on. Experience, however, forms an easy guide on this point, and we may therefore assume that the mean length of one exciting turn is known when the dimensions of the iron encircled by the coils are known. Let this mean length$=l_x$ and let E_x=the voltage which will be applied to the ends of the exciting coils. Let ω=the resistance in ohms of unit length of the required wire at a certain standard temperature, say 60° F. ; and let A and T=the two factors, the magnetising current in ampères, and the number of turns, by which the excitation X is obtained. Then the resistance of the magnetising turns at the standard temperature is$=T \times l_x \times \omega$; but, as explained in Chap. XIV., when a current is passed through the coils, their temperature will rise until the rate of generation of heat in them is equal to the rate at which it is dissipated, and in consequence the resistance of the magnetising coils when they have attained their normal working temperature after a run of some hours will be higher than their resistance at starting. In order, therefore, to find the resistance of the coils when they are at the maximum temperature which they will normally attain, it will be necessary to multiply their resistance at 60° F. by some coefficient, k, dependent on the rise of the temperature of the coils in working, and on the temperature of the surrounding atmosphere from which that rise is reckoned ; hence $R_x=T \times l_x \times \omega \times k$. The maximum rise of the temperature of the outside of the coils above that of the surrounding air will depend on the ratio which the cooling surface of the coils bears to the rate of generation of heat in them ; hence from a knowledge of this ratio, and also of the temperature of the surrounding air, we can determine the maximum temperature attained by the outside of the coils. The value of k will, however, vary with the average temperature of the whole mass of the coils, and will therefore depend, not only on the maximum temperature of their external surface, but also upon the depth of the winding. To give an idea, therefore, of the practical value of k it may be mentioned that in an ordinary 2-pole dynamo of about 20 kilowatts output with field-coils $1\frac{1}{2}$ inches deep, k will be about 1·15 if the maximum temperature attained by the outside of the coils be about 50° F. above that of the surrounding air, and the latter be assumed to be 60° F.

$$\text{Now } \mathrm{A}=\frac{\mathrm{E}_x}{\mathrm{R}_x}\text{; therefore } \mathrm{X}=\mathrm{AT}=\frac{\mathrm{E}_x}{\mathrm{R}_x}\times\mathrm{T}$$

$$=\frac{\mathrm{E}_x\times\mathrm{T}}{\mathrm{T}\times l_x\times\omega\times k}$$

$$=\frac{\mathrm{E}_x}{l_x\times\omega\times k}$$

whence

$$\omega=\frac{\mathrm{E}_x}{\mathrm{X}\times l_x\times k};$$

or if l_x be reckoned in feet and ω' be the resistance of 1,000 ft. of the required wire

$$\omega'=\frac{\mathrm{E}_x\times 1000}{\mathrm{X}\times l_x\times k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The area and diameter of the required wire having a resistance of ω' ohms per 1,000 feet is easily obtained by reference to any table of the resistance of copper wires,[1] or by direct calculation. Copper wire, as now supplied to dynamo manufacturers, has a conductivity hardly, if at all, inferior to that of the pure copper of Matthiessen's standard; and the resistance at 60° F. of 1,000 feet of a No. 1 (Birmingham or standard wire gauge) wire of 100 per cent. conductivity is ·115 ohm, its diameter being ·3 inch, and its area ·0707 square inch. Hence the area of the required wire $= \cdot 0707 \times \frac{\cdot 115}{\omega'} = \frac{\cdot 00813}{\omega'}$ square inch; or combining this with the above equation,

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[1] Such as may be found in Munro and Jamieson's *Pocket-book of Electrical Rules and Tables*, p. 227, 6th ed. If such a table be not at hand, a very convenient *memoria technica* is afforded by the fact that at 60° F. the resistance of 1,000 feet of wire, of 100 per cent. conductivity, and having an area of ·09 sq. inch, is ·09 ohm: or the resistance of a wire of area ·05 square inch is very nearly ·05 ohm per 100 yards.

$$\text{area of wire} = \frac{\cdot 00813 \times \mathrm{X} \times l_s \times k}{\mathrm{E}_s \times 1000} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

If the wire is to be rectangular, the two dimensions which go to make up this area may be chosen to suit our own convenience in winding; but if it be round, the necessary diameter is

$$\sqrt{\frac{4 \times \text{area}}{\pi}} = \sqrt{\frac{4 \times \left(\frac{\cdot 00813}{\omega'}\right)}{\pi}};$$

whence

$$d = \frac{\cdot 1015}{\sqrt{\omega'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

From the above formulæ it is evident that if E_s and l_s be fixed, there is but one area or diameter of wire which will satisfy the equation and give the required number of ampère-turns, and, further, that this area or diameter is entirely independent of the actual number of turns in the coils. This result may at first sight seem surprising, but is easily followed when it is remembered that if the number of turns be, for instance, doubled, the resistance of the coils is also doubled, which, with a given E_s, halves the current through them, and therefore leaves the total number of ampère-turns unaltered. If the number of turns be doubled by winding twice as many layers on the same length of bobbin, it is true that l_s is increased, since the depth of the winding is doubled; but the effect of this upon the necessary diameter of wire is more or less counterbalanced by the reduction which must be made in the value of k. When the number of turns is doubled, the current and the rate at which it generates heat are halved; more than this, the cooling surface is itself increased, owing to the perimeter of the coil being greater; and therefore, for both reasons, the temperature attained by the coils will be less than before. The lesser value of k thus almost compensates for the increased value of l_s.

In order, therefore, to determine the actual number of turns which must be used, or the weight of wire, the necessary area or diameter of which has been determined, it is essential that some other factor of the problem be known to us. This may be either the number of watts, W, to be lost in the field-winding under consideration, or the current A; the latter case is, in reality, identical with the former, since $A = \frac{W}{E_x}$, and in many cases W and A form two of the data given at the outset, whence $E_x = \frac{W}{A}$ is at once derived. From our knowledge of W or of A, the total number of turns is at once fixed as

$$T = \frac{X}{A} = \frac{X \times E_x}{W},$$

and the total length of wire required is $T \times l_x$ feet. The weight of one foot of a No. 1 B. or S.W.G. wire is ·273 lb., therefore the weight of wire is

$$= T \times l_x \times \cdot 273 \times \frac{\cdot 115}{\omega};$$

whence by simple substitution

$$\begin{aligned} \text{weight in lbs.} &= T \times l_x \times \cdot 273 \times \frac{\cdot 115 \times X \times l_x \times k}{E_x \times 1000} \\ &= \frac{X}{A} \times l_x \times \cdot 0314 \times \frac{X \times l_x \times k}{E_x \times 1000} \\ &= \frac{X^2 \times l^2_x \times k \times 3 \cdot 14}{W \times 100{,}000} \quad \ldots\ldots \quad (34) \end{aligned}$$

It should be observed that if our object is simply to form an estimate of the weight of wire required from the above data, it is unnecessary to first determine ω, or the actual number of turns and layers of the wire. The final settlement of the winding will, however, require the latter to be determined, and after determination of the number of layers, it will be well to check the correctness of the assumed depth of winding underlying our first estimate of l_x, or the mean length of a turn. It may be worth while to mention that if a continuous length of round wire be wound on to a bobbin in several layers, the turns of the second or any subsequent layer do not bed into the hollows between the turns of the layer underneath, since the wire has in each turn of the upper layer to cross over the turn of the lower layer; hence if $d + \delta$ be the diameter of the round wire with its insulating covering, and n be the number of layers, the depth of winding is $n\ (d+\delta)$. Finally, it may be stated that the round wire in ordinary use for the field-winding of dynamos is double-cotton-covered, the thickness of the covering being 7½ mils a side, or 15 mils in all (1 mil=one-thousandth of an inch); while for rectangular wires it is usually somewhat thicker, making 20 mils in all. The bobbins or magnet-cores on which the wire is wound are insulated with several thicknesses of shellacked paper or calico, and their end-flanges are usually further provided with insulating pieces of wood or vulcanised fibre.

CHAPTER XVI

SERIES, SHUNT, AND COMPOUND WINDING

By the principles stated in the previous chapter the designer is enabled to calculate the number of ampère-turns required to excite the magnetic system of a dynamo. There are, however, several different methods of supplying the exciting current, and since these have different effects on the working of the machine, the four principal modes require to be considered in detail. The excitation of the electro-magnet of a dynamo is, in general, effected either (1) by coils forming a shunt to the external circuit, the dynamo being then known as a 'shunt' machine; or (2) by coils in series with the external circuit, when the dynamo is called a 'series' machine; or (3) by both shunt and series coils in combination, a method known as 'compound winding'; or lastly (4) by coils connected to a separate source of current, the machine being then said to be 'separately excited.'

The excitation of a machine from a separate and entirely external source of electrical energy is the most obvious method, and was the one first adopted in practice, a small dynamo with permanent magnets of steel being used to furnish current for exciting the field-magnet of a larger dynamo. Fig. 140 shows a separately-excited machine, the electric circuit of the magnetising coils being entirely distinct from the circuit of the main dynamo: this latter may be either an alternator or a continuous-current dynamo, as in the diagram, where the main external circuit R_e is indicated by incandescent lamps strung in parallel across from the positive to the negative lead. The source of the magnetising current is represented by a battery of cells, but may of course equally well be a separate continuous-current dynamo.

The second step in the order of development was made when it was suggested that a part of the electrical energy generated in the main dynamo might be used to maintain or increase the magnetism of its own field-magnet. This was, however, only possible if the machine were furnished with a commutator by which the current was commuted into a steady flow in one direction, and was thus rendered suitable for magnetising purposes. Two distinct methods were then invented, by which this suggestion was realised.

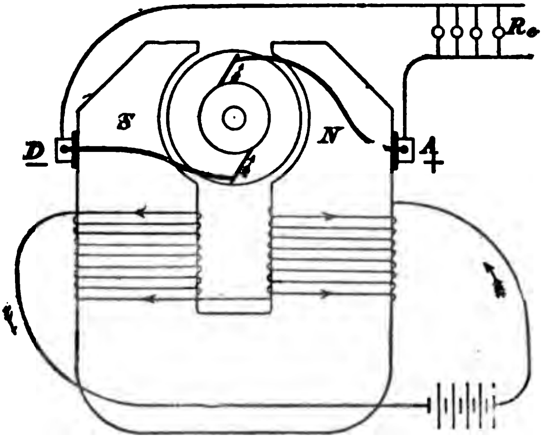

FIG. 140.—Separately-excited dynamo.

By the first, magnetising coils were arranged as a shunt to the external circuit proper; thus, in fig. 141, from the brushes of the dynamo two paths, R_e and R_s, are presented, and the armature current divides into two portions, the relative magnitudes of which will vary inversely as the resistances of R_e and R_s; while the one portion of the total armature current flows through the external circuit, R_e, wherein the useful electrical energy is manifested, the other is shunted through the magnetising coils, and both reunite to flow through the armature. The voltage on the shunt is, of course, the same as that on the external circuit, since the same terminals, A and D, serve for both. If the resistance of the shunt, R_s, be relatively high as compared with R_e, only a small proportion of the total energy developed will be absorbed in exciting the field-magnet ; hence the shunt coils are usually composed of a large number of turns of small wire, and are represented by fine lines in contrast to the thicker lines, which indicate the lower resistance of the external circuit.

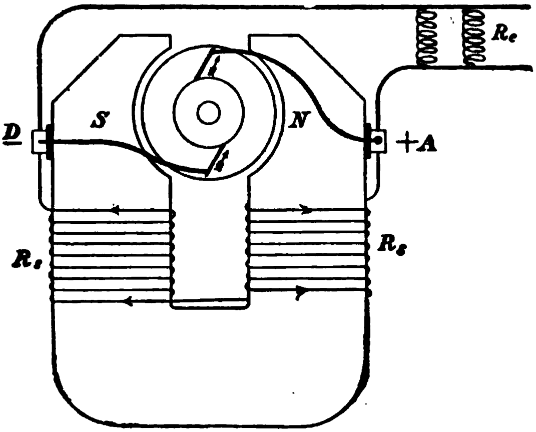

FIG. 141.—Shunt-wound dynamo.

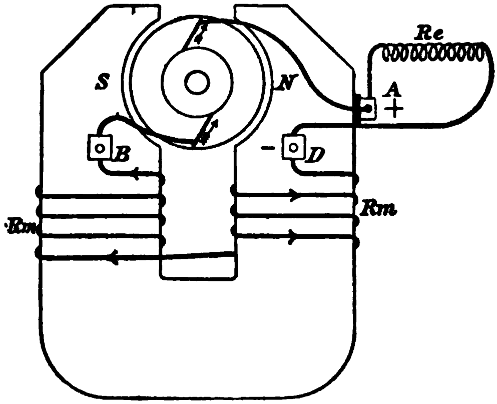

FIG. 142.—Series-wound dynamo.

By the second distinct method, magnetising coils are arranged in series with the external circuit, and the whole of the armature current passes alike through the turns of the field-magnet coils,

R_m, and the external circuit, R_e (fig. 142). A portion of the total pressure developed at the brushes is expended in the magnetising coils, and the remainder is available for useful work at the terminals, A, D, to which the external circuit is applied. If the resistance of the series coils, R_m, be low, as compared with the resistance of the external circuit, the percentage of energy absorbed in the field will be small as compared with the useful output, and hence, in this case, the series coils are represented in our diagram by a few turns of thick wire. The number of ampère-turns on the magnet of the dynamo may be the same whether it be shunt- or series-wound, since in the one case a small current flows through a large number of turns, and in the other case a large current flows through a few turns; and in both cases the amount of energy absorbed in securing any given number of ampère-turns is simply a question of the amount of copper in the field-coils.

A further and most important step was now found to be practicable, viz. the self-excitation of the machine, whether shunt- or series-wound. The field-magnet of a dynamo is always slightly magnetised, even when the machine is not running: it may then appear to be perfectly demagnetised by its failing to attract, say, a bunch of keys held near the poles, but a more delicate test will show that both wrought and cast iron retain a certain amount of magnetism (p. 222), and if the field-magnet be approached with a small compass needle, one pole of the machine will attract and the other will repel the N. pole of the delicately poised needle. Now the presence of this feeble residual magnetism is sufficient to start the process of self-excitation, since it implies that a few lines of induction are passing round the magnetic circuit of the dynamo through the armature, even when at rest. Hence, when the armature of either fig. 141 or fig. 142 is rotated, the inductors cut the lines due to the residual magnetism, and a small E.M.F. is thereby set up within the winding of the armature. If the brushes are 'down' on the commutator, and a closed circuit is thereby made, this small E.M.F. sends a feeble current through the magnetising coils; the ampère-turns of the latter, although as yet they may be small, still serve to increase the number of lines passing through the armature, and these, as they are cut by the inductors, produce an increased E.M.F.; the exciting current is thereby in turn increased, and its increase is again followed by a further increase in the magnetisation and E.M.F. Thus the magnetism gradually grows, the increments becoming less and less as the iron of the field-magnet becomes more and more saturated. The time taken by the 'building-up' process will vary from a few seconds in a small machine to a minute or more in a large magnet, but after a short time a small increase in the exciting current produces very little increase in the induction; finally, for a given speed of rotation, the voltage and exciting current reach a settled state, and the machine will run for any length of time, maintaining its own constant magnetisation. Thus the presence of residual magnetism renders it unnecessary to impart to the field-magnet any initial excitation, and the continuous-current machine becomes self-exciting by the mere rotation of the armature. One difference, however, between the shunt- and series-machine will be apparent from their respective diagrams. In the latter case, the external circuit must be closed before the process of excitation will begin, since the circuit of the field-coils is only completed through the external circuit. In the former case, if an external circuit of very low resistance be closed on the brushes, so small a portion of the feeble initial current will be shunted through the magnet-coils that the machine may fail to excite. Hence, in the shunt machine, the excitation is most quickly and surely obtained if the external circuit be left open until the magnet is thoroughly excited.

It only remains to remark that even when a machine has just come from the workshops, and is run for the first time, there is usually sufficient residual magnetism in its iron to enable it to excite itself. The first excitation may, however, require a higher speed than will be subsequently necessary, and in cases of large shunt machines, running at slow speeds, separate excitation may have to be resorted to in the first instance.

When once the machine is normally excited, and is running under settled conditions, the energy spent in the magnetising coils is entirely converted into heat; but during the process of creating the magnetic field, a certain amount of energy is absorbed which is given back as the field demagnetises. If the circuit of a series-wound dynamo be opened while it is running, the stored-up energy reappears in the form of a spark at the switch; the rapid collapse of the lines of induction which circle through the turns of the field-coils generates a high E.M.F., which tends to keep up the strength of the current, and if the field be powerful and the turns encircling it be numerous, the intensity of the spark becomes very considerable. In the case of a shunt-wound dynamo the opening of the external circuit does not interrupt the flow of the shunt current round the coils; in fact, these latter themselves afford a bye-pass for the discharge of electrical energy. Owing, however, to the large number of turns in the shunt, the effect of suddenly breaking the magnetising circuit is even more marked than in a series machine. Thus, if a person accidentally lifts the brushes of a large shunt machine running on open circuit, the self-induced E.M.F. may rise to thousands of volts, may damage the insulation of the machine, and, further, may give perhaps a fatal shock. If the machine be running on a closed circuit, say, of incandescent lamps, the person would be less liable to receive such a severe shock, since the so-called 'extra-current' is discharged through the external circuit, and causes the lamps to momentarily flash up.

The application of the formulæ of Chapter XV. to the design of the field-winding of the shunt machine is easy. Let E_a be the terminal or external voltage which the machine is required to give when supplying its full external current; then the difference of potential on the ends of the shunt is likewise E_a, and it is only necessary to substitute its value for E_s in the equations 31–34 in order to determine the necessary area or diameter of wire. In order to further determine the number of turns and weight of wire, it is necessary to know, at least approximately, the rate in watts at which energy is to be expended in the field-coils.

While, however, the necessary size of wire is rigidly determined, there is, in fact, no hard-and-fast rule for deciding the weight that is to be used, and this must be left to the designer's judgment. If a large quantity be used, the weight and cost of the machine are increased (double-cotton-covered round wire costs approximately £4 per cwt.), while if a smaller quantity be used, the heating of the field-coils is greater, and the efficiency of the machine is decreased. A further disadvantage which arises when the weight of copper is reduced is, that there is a greater difference between the resistances of the shunt when cold and when hot, owing to its rise of temperature being greater: this difference causes a difference in the shunt current, and therefore in the exciting power and total induction produced, so that when the dynamo runs at a constant speed, the E.M.F. induced is greater at starting than it is after it has run for several hours continuously, and has attained its final temperature. If the dynamo be supplying incandescent lamps direct, and this difference be great, it will necessitate an alteration in the speed of rotation in order to maintain the correct voltage on the lamps. The settlement of the rate in watts at which energy may be expended in the shunt coils depends, therefore, upon their allowable heating and upon the efficiency which the dynamo is to have: from both considerations combined, experience enables us to fix upon a preliminary estimate from which the completed design need differ but little. In practice, the loss of energy in the field of a shunt machine varies from about 1·5 per cent. of the output in a 100-kilowatt machine to 7 or 8 per cent. in a 4-kilowatt machine. The heating question will be more fully discussed in Chapter XVIII. If it be settled that w watts may be lost in the field-winding, then $w = E_s \times C_s$ watts, and $C_s = \frac{E_s}{R_s}$, whence the resistance of the shunt and its composition can be at once determined. The preliminary result thus arrived at should only require such slight revision as will lead to the winding forming a complete number of layers.

Thus in the case of the dynamo of p. 295, $E_s = 130$; assuming the depth of winding to be a little under $1\frac{1}{2}''$, we can, from the given dimensions of the magnet-core (fig. 135), determine l_s to be $39'' = 3{\cdot}25$ ft., while if the temperature attained by the surface of the coils be about 45° F. above the normal temperature of the air, k will be about 1·13 (cp. p. 396). Thence by equation (31)

$$\omega' = \frac{130 \times 1{,}000}{14{,}000 \times 3{\cdot}25 \times 1{\cdot}13} = 2{\cdot}52,$$

and the necessary diameter of the shunt wire is ·064″.

In the present case, the over-all length of each magnet bobbin will be 10″, and the cooling surface of the two together (as explained in Chap. XIV.) may be reckoned as $44 \times 9\frac{1}{2} \times 2 = 836$ square inches, and (as will be explained in Chap. XVIII.) an allowance of about 460 watts will be well within the mark as regards the heating of the coils. After a preliminary trial, we arrive at a winding which forms a complete number of layers as follows: The diameter of the double-cotton-covered wire will be $\cdot064 + \cdot015 = \cdot079''$, and after allowing for the insulating flanges at either end of the 'former,' there will be 118 turns of wire per layer. With seventeen complete layers the total number of turns on the two will be $T_s = 4{,}012$; 13,040 feet of wire will be required, and the resistance of the shunt when cold will be $13\cdot04 \times 2\cdot52 = 32\cdot86$ ohms; or, when hot, $32\cdot86 \times 1\cdot13 = 37\cdot1$ ohms. $C_s = \frac{130}{37\cdot1} = 3\cdot5$ ampères, and $X_s = C_s \times T_s = 14{,}000$, as required. The watts expended in the field-coils will be 455, or nearly 5¾ per cent. of the output.

If a dynamo be run at a constant speed, and the resistance of its external circuit be varied so as to alter the value of the external current, the curve connecting simultaneous values of the terminal E.M.F., E_e, and the external current, C_e, for a given speed of rotation is known as the 'external characteristic' of the machine for that speed; since from it the behaviour of the machine under varying conditions of load can be graphically studied. The volts and ampères of such a curve are usually plotted to the same scale, horizontal distances representing ampères, and vertical ordinates the corresponding volts. These two may either be obtained by direct measurement, voltmeter and ammeter being read off simultaneously, or the curve may be derived from the magnetisation-curves of the machine, and the advantage of so deriving it is that the speed during the tests to obtain the magnetisation-curves is varied, and does not require to be kept steadily at a constant value. In the case of a shunt-wound machine the curve of external E.M.F. and current will be found to take the characteristic form shown in fig. 143, O L K D. The manner in which it may be derived from the magnetisation-curves has now to be explained, and first it will be necessary to consider the process by which the three curves of fig. 139 may be determined by direct experiment on a shunt-wound dynamo.

Considering, for simplicity, a bipolar machine, the total number of lines passing through the armature is $z_a = \frac{E_a \times 10^8 \times 60}{N \times \tau}$ (eq. 11), and the ampère-turns on the field-magnet are $X_s = C_s \times T_s$. $C_s = \frac{E_b}{R_s}$ may be obtained by measuring the E.M.F. at the brushes and dividing it by the resistance of the shunt; but for greater accuracy it is preferable to measure it directly, since the resistance of the shunt is continually altering if the machine is in process of warming up during the tests. In a shunt machine

$$C_a = C_s + C_e : E_a = E_b + C_a R_a ;$$

whence
$$z_a = \frac{(E_b + C_a R_a) \times 10^8 \times 60}{N \times \tau} \quad . \quad . \quad . \quad . \quad . \quad (35)$$

When the external circuit is open, the only current flowing is that through the shunt. As a rule, this is so small that $C_s R_a$ may be neglected, and E_b and E_a are practically identical. If, therefore, the armature be run at different speeds, and a series of simultaneous readings of the speed, the E.M.F. at the brushes, and the shunt current be taken, a curve can be plotted from the readings which will connect z_a and X_s; and since the armature ampère-turns due to the shunt current alone are very small, their influence on the field may practically be neglected, and the curve thus obtained is the highest or 'no-current' curve of magnetisation of fig. 139. To obtain the lower curves of magnetisation for either 'half-current' or 'full-current,' the same readings must be taken; but between the readings at each different speed the resistance of the external circuit must be altered, so that when the measurements are made for either curve, the current through the armature ($C_a = C_s + C_e$) has the constant value required.

Since $E_a = \frac{z_a \times N \times \tau}{60 \times 10^8}$, it follows that $E_a \propto z_a$, if the speed be kept constant; further, $X_s = \frac{E_s}{R_s} \times T_s$, whence E_s, or the potential difference applied to the terminals of the exciting coils, is $\propto X$ when R_s is constant, as it is when a machine has been running for some time, and has attained a steady temperature. If, therefore, we possess the magnetisation-curves of fig. 139, these can be converted into curves connecting armature E.M.F. and exciting voltage when some fixed speed is assumed.[1] The full-line curves

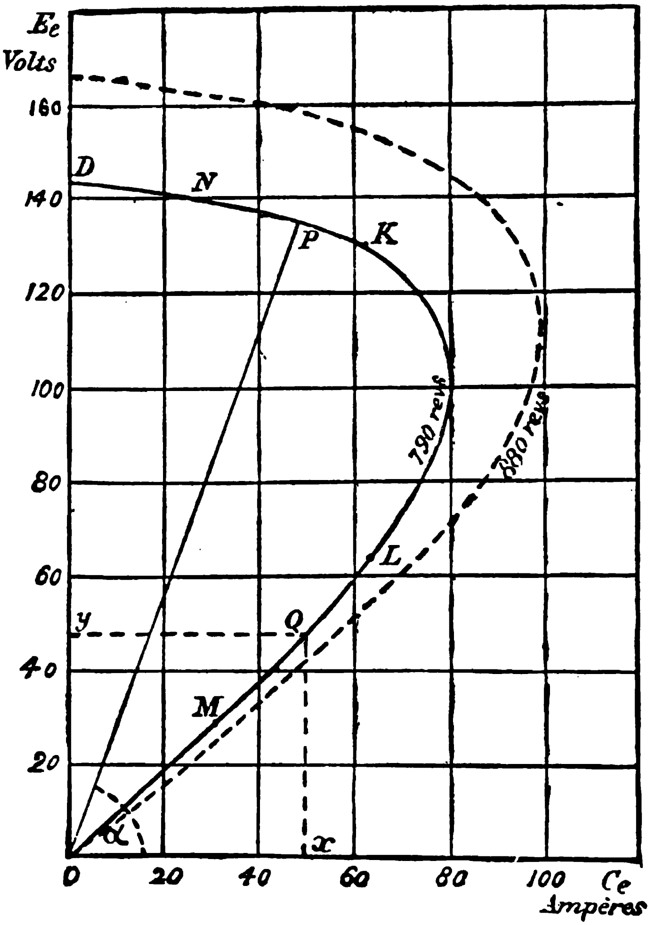

FIG. 143.—Characteristic curve of shunt-wound dynamo.

of fig. 144 show the curves of fig. 139 thus converted on the assumption that in our 9″ × 12″ dynamo the number of armature inductors is 234, the number of shunt turns 4,012, and their resistance 37·1 ohms, and that the constant speed of rotation is to be 790 revolutions per minute. Curve 1 connects E_a or E_b and E_x when $C_a=0$. Curve 2 connects E_a and E_x when C_a has the value A_1 (=33 ampères), and curve 4 connects E_a and E_x when $C_a=A_2$ (=66 ampères). From curve 2 is obtained the dotted curve 3, connecting E_b and E_x when $C_a=A_1$ ampères, by deducting from the ordinates of curve 2 the loss of volts over the armature resistance when warm ; in fact, the vertical distance between curves 2 and

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[1] See *Electrician*, Dec. 20, 1895

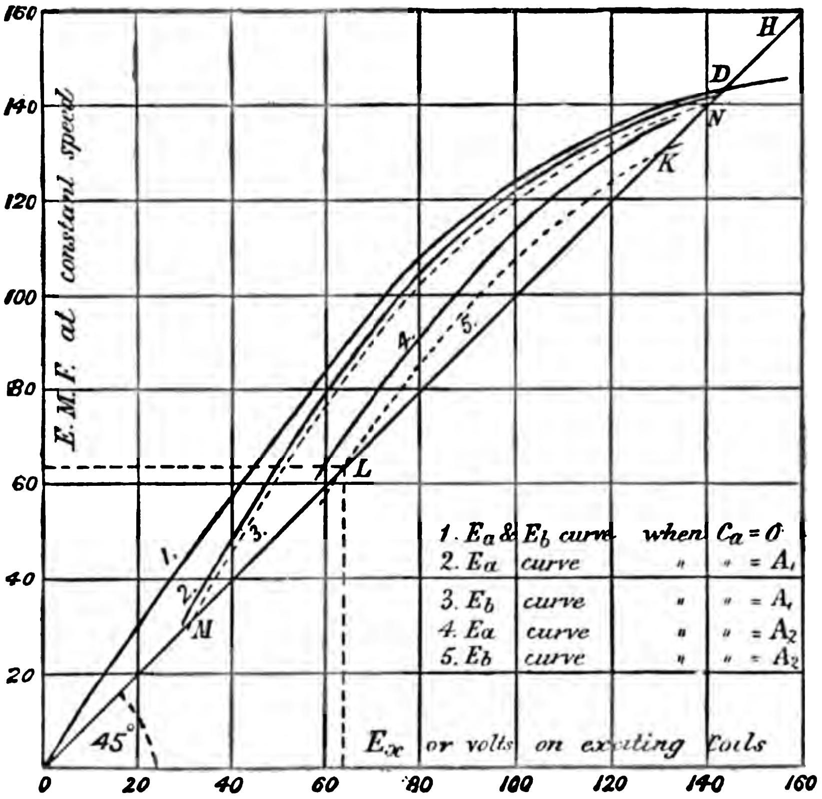

FIG. 144.

3 is throughout equal to $A_1 \times R_a$ ($\cdot 09 \times 33 = 3$) volts. Similarly, from curve 4 is derived curve 5, connecting E_b and E_x when $C_a = A_2$, or 66 ampères. Now, since in a shunt-wound machine E_b is the same as E_x, those points on curve 5 (such as L or K) for which the ordinate E_b=the abscissa E_x are the only values of voltage which the machine would give when run at the fixed speed with A_2 ampères flowing through the armature; and in general those points on any curve connecting E_b and E_x, for which ordinate and abscissa are equal, give the only possible values of the terminal E.M.F. for the particular armature current. Deducting the shunt current from the total armature current, we obtain the external current; thus at point K, $C_s = \frac{130 \text{ volts}}{37 \cdot 1 \text{ ohms}} = 3 \cdot 5$, C_a=nearly 66 ampères, and therefore $C_e = 62 \cdot 5$ and $E_e = 130$ are simultaneous values of external current and terminal voltage. All such points are necessarily passed through by the straight line O H, which is drawn at an angle of 45° from the axis of abscissæ, as, *e.g.*, K and L, N and M, D and O; and after deduction for the shunt current, these may be plotted as part of the external characteristic, as shown by the corresponding letters in the full-line curve of fig. 143.

It will be seen from fig. 143 that for the shunt-wound machine running at any one fixed speed there is a maximum value for the external current, beyond which it is impossible to go. Owing to the heat which would be generated in the armature winding by a long-continued passage of this maximum current, it may be, and usually is, impossible to work the machine at the point of maximum current: apart, however, from the question of the heat damaging the winding, it is for another reason inadvisable to work close to this point. It will be seen from fig. 143 that the characteristic curve after rounding the point of maximum current descends very rapidly and almost in a straight line to the origin. Thus in the diagram, the part O Q is practically a straight line, and this implies that for an external resistance, $\frac{Qx}{Ox}$, the terminal E.M.F. and the external current may have any value between nought and O*y* volts and nought and O*x* ampères; the machine may therefore give widely different voltages at different times, although running at exactly the same speed on the same external resistance. The magnetism of the field is, in fact, unstable, owing to the voltage on the shunt not being sufficient to duly magnetise the iron ; hence, if, when working near the point of maximum current, the resistance of the external current becomes lowered to a value considerably below that which corresponds to the maximum current, the machine loses its magnetism altogether, and the voltage runs down to zero. The practical importance of this is that if a shunt-wound machine be accidentally short-circuited when at work, or, in other words, if R_e is reduced practically to $= 0$, the armature winding is not burnt up by the continued passage of an abnormally large current; the machine, on the other hand, becomes demagnetised, and gives no current, although the driving engine may be running at a speed higher than the normal. Neither will a shunt-wound machine excite if an external circuit of abnormally low resistance be closed, so that it is unharmed if an accidental misconnection of the leads has resulted in its terminals being short-circuited. For every value of the external current except the maximum there are two values of the external E.M.F., and which of the two E.M.F.'s is obtained depends entirely upon the resistance of the external circuit : this latter is equal to $\frac{E_e}{C_e} = \tan \alpha$, or the slope of the line to any point, P, on the curve when E_e and C_e are plotted to the same scale, as usual (fig. 143). The slope of the full-line curve between L and O marks the critical resistance of the external circuit for a speed of 790 revolutions per minute; if it be reduced below this value, the machine will fail to excite, or its magnetism will at least be unstable.

For each constant speed the same shunt-wound dynamo gives a different external characteristic. Thus, in fig. 143 the dotted curve shows the external characteristic of the same dynamo at a constant speed of 880 revolutions per minute, instead of 790, and it will be seen that at the new speed the output may be raised to 145 volts and nearly 80 ampères, provided that considerations of heating and sparking do not forbid it. In short, the higher the speed, the less is the slope of the descending branch of the curve; or, in other words, the smaller is the critical resistance which the external circuit may have, without the dynamo losing its magnetism.

Instability of the magnetism requires to be carefully guarded against in the design of self-exciting dynamos. It will be seen from figs. 143 and 144 that the unstable portion of the curve between L and O is obtained if the total induction for which the machine is designed falls on the initial straight portions of the curves in fig. 139, where they descend rapidly towards the axis of abscissæ; and, in general, if the working number of lines falls low down on the curve of magnetisation, the magnetism of the machine as a self-exciting dynamo will be unstable. Even if the field hold its magnetism, a very slight variation of the speed will cause a large variation in the E.M.F.—a result which is always undesirable, and especially so if the machine be feeding lamps directly. Thus, in the case of a shunt-wound machine, suppose that its speed is raised slightly; the increased E.M.F. causes an increase in the current through the shunt, which will again increase the induction and the E.M.F., and the lower down the curve of magnetisation that the machine is worked, the greater is the effect of any variation in the shunt current upon the E.M.F., owing to the magnet being less saturated.

A further disastrous consequence of designing a shunt-wound machine to work on the initial straight portion of the magnetisation-curves is that, even when the external circuit is open, it may never become properly excited when run at the normal speed for which it is designed. Just as excitation begins, the magnet reaches a state which satisfies all the conditions of speed, &c., and the process stops abruptly. Hence, if the designed ampère-turns of the shunt do not correspond to an induction well up on the magnetisation curve, the machine may entirely fail to reach its due excitation when started at its normal speed. It is therefore extremely important in designing a machine which is to become excited on its shunt to work fairly high up on the magnetisation curves, and in fig. 139 the best portions of the curves whereon to work fall within the horizontal lines marking four and five million lines respectively (corresponding to a working range from D to K on the characteristic curve of fig. 143). These portions may be said to lie on the bend or knee of the curve, but it must be remembered that the apparent position of this bend depends largely on the relative scale to which lines and ampère-turns are plotted. Another advantage of working high up the curve is that the difference of the ampère-turns of the shunt when hot and when cold then produces but small effect upon the induction and E.M.F. of the machine, since the magnet is well saturated. Also, the vertical distance between the no-current and full-current curves decreases as the total induction is increased, so that the variation in E.M.F. of the machine between full load and no load is small when the magnet is strongly excited. If, however, the working induction be taken too high, a large amount of copper will be required on the field, and if for any reason the actual curve comes below the predetermined curve, it may be practically impossible to rewind the magnet so as to obtain a greater induction.

It will already be apparent that much information can be obtained from the shape of the characteristic curve of the dynamo ; in the case of a shunt-wound dynamo run at a constant speed, we see that its E.M.F. is highest when the external circuit is open, and the only current flowing is that through the shunt. When the external circuit is closed, and the armature current is swelled by the addition of an external current, not only does the loss of volts over the resistance of the armature progressively increase as the armature current increases, but the demagnetising effect of the back armature ampère-turns increases. Hence, not only does the exciting voltage at the brushes of the dynamo fall, but the magnetisation due to a given number of shunt ampère-turns is decreased. Owing, therefore, to the combined effect of the two causes, viz. the increase of the back ampère-turns, X_b, and the loss of volts over R_a, the characteristic gradually falls throughout the entire working range of the dynamo. If, however, the armature be of low resistance, the drop in volts for any current within the working range will be but small, and consequently the shunt-wound dynamo may in such cases be practically regarded as giving a nearly constant voltage when run at a constant speed.

In order to regulate the voltage of a shunt machine, it is usual to insert in series with the magnet winding a rheostat or resistance, the coils of which can be successively thrown into or out of the shunt circuit. Since the exciting current for a given voltage at the brushes is thereby reduced or increased, the voltage of the machine can be lowered or raised by small steps. Less often the speed of rotation is altered, or both methods are used in conjunction.

Shunt-wound dynamos are chiefly used for charging accumulators, and for electrolytic or electro-metallurgical work. For such purposes they are especially suited, since, if the E.M.F. of the dynamo becomes less than the E.M.F. of the cells, or electrolytic baths, the direction of the current round the shunt remains unchanged, although the current through the armature will be reversed. The result is that the machine runs as a motor in the same direction as it did when acting as a generator, and therefore opposes a back E.M.F. to the discharge from the cells : this back E.M.F. prevents the flow of a large current which might burn up the armature and also deteriorate the cells.

Large continuous-current dynamos for central-station work, which supply current at low pressure to a network of mains, are also usually shunt-wound. The voltage at the station end of the feeders to the network is in such cases conveniently regulated by means of rheostats. For the direct lighting of incandescent lamps in smaller installations, shunt-wound dynamos are not so suitable, owing to the necessity for constant attendance in order to regulate their E.M.F. according to the load, if a constant voltage is to be maintained on the lamps. They are, however, frequently used in conjunction with accumulators, and in such cases, by means of a supplementary resistance in the shunt, they are arranged to charge accumulators during the day up to about 130 volts, and at night to work in parallel with the battery, lighting incandescent lamps at 100 volts, the speed under the two conditions being maintained at nearly the same value.

Arc lamps are in many cases run in parallel from shunt machines, but in each parallel a steadying resistance is usually required. More often, however, arc lamps are connected in series, and fed from a series-wound dynamo, giving, say, 10 ampères and any E.M.F. up to about 3,000 volts. By this arrangement, not only is any necessity for steadying resistances avoided, but the weight of copper in the line is reduced, since the higher the voltage, the less the weight of copper required for a given percentage loss in the line. At high voltages, the required diameter of wire for a shunt-winding becomes very small, and fine wire is not only expensive in itself, but the time and trouble of winding the great length of it which is required become very considerable.

Hence high-voltage machines are usually series-wound, and the maximum E.M.F. for a machine which has all or part of its excitation provided by a shunt-circuit may be set at about 500 volts.

The determination of the field-winding for a series dynamo follows the same lines as in the case of a shunt machine, save that now the magnetising current and number of turns are fixed, and the necessary size of wire has to be determined from the permissible loss of watts in the field. Thus, if it be desired to convert our 9″ × 12″ dynamo into a series-wound machine without alteration of the armature, then, while the full armature current becomes available externally, the E.M.F. at the terminals will be less than the E.M.F. at the brushes by the loss of volts over the series-winding, i.e. approximately by about 5½ per cent. ; the output therefore may be fixed at, say, 125 volts and 65 ampères, at a speed of 800 revolutions per minute. The ampère-turns required on the field in order to give 4,400,000 lines through the armature and an internal E.M.F. of 137·3 volts will be 14,000, as before. The allowable loss of watts over the field-winding being assumed at about 415, the voltage between the ends of the series-winding is $\frac{415}{65}=6{\cdot}4$, and the resistance of the 'main,' as the series coils are often called, will be when warm $R_m=\frac{6{\cdot}4}{65}={\cdot}0985$. The necessary area of wire is by equation 32 (p. 320)

$$=\frac{{\cdot}0081\times X\times l_1\times k}{1000\times E_s}=\frac{{\cdot}0081\times 14000\times 3{\cdot}25\times 1{\cdot}13}{1000\times 6{\cdot}4}$$

$$={\cdot}065 \text{ sq. in.}$$

We further have $X=14000=C_e T_m$, whence $T_m=\frac{14000}{65}=216$ nearly.

As a round wire, the necessary diameter will be so large as to render it troublesome to wind ; hence it will be better to adopt a copper strip of rectangular section, and we are at liberty to select such a width and thickness as will give a whole number of layers, and will render it easy to wind on flatwise. Such dimensions will be ·325″ × ·200″ double-cotton-covered to ·345″ × ·220″ : four layers, each of 27 turns, will give 108 turns on each magnet bobbin, or 216 in all. Owing to the rectangular section of the wire, less space is wasted than if it had been round, and the depth of the series-winding will not be so great as that of the shunt-winding of p. 330; hence, if greater accuracy were required, the values assigned to l_s and k should be slightly modified, but for ordinary work the above result will hardly require further correction. Having regard to the close packing of the rectangular wire, and the consequently reduced cooling surface, a somewhat smaller expenditure of watts in the field was in the first instance assumed for the series-winding. The actual weight of copper on the field in the two cases is practically the same; it is, however, a little more in the series than in the shunt machine, and due to this the efficiency of the former is slightly the greater.

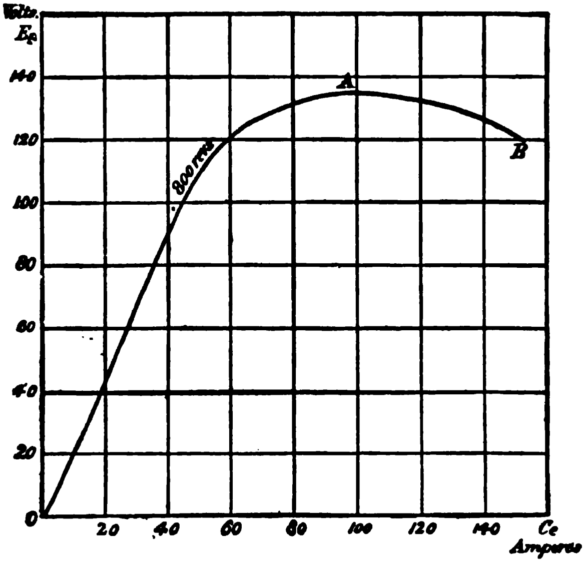

FIG. 145.—Characteristic curve of series-wound dynamo.

The external characteristic of the above dynamo when series-wound for a speed of 800 revolutions per minute is shown in fig. 145. It may be derived from the magnetisation-curves of fig. 139 in the following manner. Taking any current, c_1, multiply it by

T_m, the number of turns in the series-winding, and then find on the magnetisation-curve for the particular current C_1 the Z_a corresponding to the exciting power, C_1T_m. Thence $E_a = \frac{Z_a \times \tau \times N}{60 \times 10^8}$ can be determined; the loss of volts over the resistance of the armature and series winding $= C_1 \times (R_a + R_m)$, so that E_e corresponding to a value of $C_e = C_1$ is $E_a - C_1 \ (R_a + R_m)$, and the corresponding point on the external characteristic can be plotted. Next take another value for C_e, and in the same way find the corresponding value of E_e, and so on until sufficient points have been obtained to draw in the curve. Of course, if we only have three or four magnetisation-curves, it will be necessary to interpolate or add other curves in order to determine the characteristic throughout a large range of current. Thus the curve for $\frac{3}{4}$ths of full current will fall approximately midway between the curves for half and full current, but slightly nearer to the former; the back ampère-turns of the armature are less at three-quarters than at full load, owing to the decreased current, and the proportion of leakage is thereby decreased, but, more than this, the lead of the brushes may be slightly decreased, and therefore, on both grounds, the effect on the field is to cause the $\frac{3}{4}$ curve to fall nearer to the half- than to the full-current curve.

The shape of the external characteristic for a series-wound dynamo is widely different from that for a shunt machine. Since the external current is also the magnetising current, it resembles in the main a curve of magnetisation. It will be seen, however, that after a certain point, A (fig. 145), is reached, the terminal E.M.F. decreases as the external current is increased. This fall of the external characteristic is due to the loss of volts over the resistance of the armature and series winding, and to the demagnetising effect of the back ampère-turns on the armature. The former loss is directly proportional to the current, while the effect of the back ampère-turns increases somewhat faster than the current. On the other hand, when the magnet is approaching saturation, if the external current is increased, the increase in the total induction is by no means proportional to the increase in the exciting power of the field ampère-turns. Hence the induced E.M.F. rises less and less rapidly as the current is increased, and after a certain point the increase in the induced E.M.F. does not so much as compensate for the increased loss of volts over the resistances of the machine; the external characteristic curve therefore attains a maximum height, and thence bends gradually downwards.

The external characteristic at any other speed is obtained by simply altering the height of the ordinates E_a (p. 341) being altered in proportion to the alteration of the constant speed.

Series-wound dynamos are chiefly used for the electric transmission of large powers over considerable distances,[1] electric traction being included under this head; and also, as already mentioned (p. 338), for high-potential arc-lighting. For this latter purpose the drooping external characteristic is a distinct advantage, and the series-wound arc-lighter is frequently worked on the part of the curve marked A B in fig. 145. For the proper working of arc lamps a nearly constant current is required, which implies that the E.M.F. on the series of lamps must be varied in proportion to the resistance which at any moment they present. Now if the resistance of any one or more lamps be decreased owing to the carbons being fed together, or if a lamp be entirely cut out of circuit, the momentary increase in the external current is, in the case of a series-wound dynamo working on a falling characteristic, accompanied by a decrease in the terminal volts: this helps to bring the current back to its normal strength, and although it may not be sufficient to make the machine entirely self-regulating for constant current at varying potentials, it tends in the right direction, and leaves less for the automatic constant-current regulator to do.

Series dynamos are unsuitable for charging accumulators, since if by any chance the E.M.F. of the machine be allowed to fall below the E.M.F. of the cells, the latter send a reversed current through the dynamo: this reverses the field-magnetism, and consequently the direction of the machine's E.M.F. The result is that both dynamo and cells act in series round a circuit of almost negligible resistance, and generate a current which increases until either a safety fuse is melted or armature and cells are damaged.

While a series-wound dynamo may be made to yield an approximately constant current at varying pressure by working it on the drooping portion of its characteristic, it has been shown that a shunt-wound dynamo with armature of low resistance will give an approximately constant voltage over a considerable range of current. This latter requirement, which is the essential of direct incandescent lighting, is, however, much more nearly fulfilled by the 'compound-wound' dynamo (fig. 146), to a consideration of which we now pass. If a dynamo be shunt-wound to give a certain voltage at no load, and the magnet be in addition wound with a certain number of 'main' turns connected in series with the external circuit, then, as the load is increased the current flowing through the series turns will progressively increase the exciting power. Now this increase in the exciting power may be made, not only to balance the increase in the back ampère-turns, as more load is thrown on, but further to increase the total induction through the armature. As a consequence the induced E.M.F. is increased, and when properly proportioned this increase may be made to exceed the loss of voltage over the series turns, and in addition to compensate for the increased loss of volts over the armature resistance. Thus, while the armature E.M.F. rises with the load, the terminal E.M.F. remains practically constant, how-

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[1] Kapp's '*Electric Transmission of Energy*,' 4th ed. p. 179 ff.

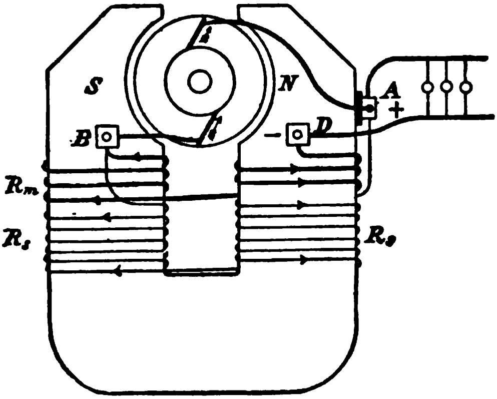
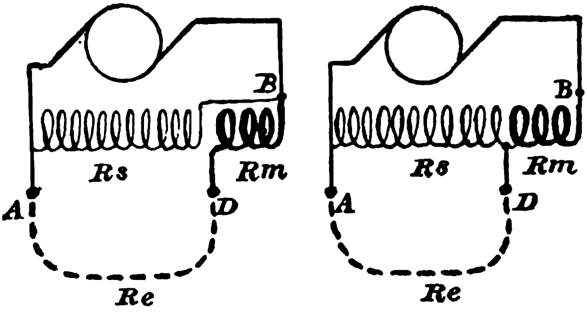


FIG. 146.—Compound-wound dynamo.

ever the current be varied over a very considerable range. The external characteristic is therefore practically a straight line throughout the entire working range of current. In the compound-wound dynamo the shunt may, in fact, be regarded as providing a certain initial magnetisation and voltage : these latter are maintained and reinforced against the combined causes which tend to reduce them by means of the series turns, the exciting power of which varies directly with the external current.

The connections of a compound-wound dynamo may be arranged in two slightly different ways, shown diagrammatically in fig. 147. By the first, which is also shown in fig. 146, and is known as the 'short-shunt' winding, the shunt is placed across the brushes of the machine, and the E.M.F. acting on it is $E_b = E_e + C_e R_m$, the current flowing through the main being simply the external current, C_e. By the second, or 'long-shunt' winding, the shunt is placed across the terminals of the external circuit : the E.M.F. acting on it is then E_e, while the whole armature current $C_a = C_e + C_s$ flows through the main. Of the two methods, the first is the more common, but their practical difference is not very important. As regards the winding of the two sets of magnetising coils on the magnet bobbins, it is purely a matter of convenience whether the main be wound over the shunt or *vicè versâ*.



SHORT SHUNT LONG SHUNT

FIG. 147.

A constant speed and a required constant voltage have been tacitly assumed above for the compound-wound dynamo; but such are not in strictness the ordinary conditions of working, and it must therefore be designed to meet the more usual working conditions. When a dynamo is driven by an engine, and forms a considerable portion of the load on that engine, the greater the output, the slower will be the speed of the engine, even if it be governed for approximately constant speed. The difference of speed between full and no load on an electric-light engine varies; but it should not exceed 5 per cent. of the no-load speed. Further, if the dynamo be belt-driven, there will be extra slip of the belt at full load, which will diminish the speed of the dynamo; the allowance of 5 per cent. variation may, however, be taken to cover this further reduction. Apart from this mechanical lowering of the speed, there is also the electrical loss of volts over the main leads between the dynamo and the lamps, which increases in direct proportion to the current passing. If, therefore, a constant potential is required at the far end of the line (as, for example, in a house with 100-volt incandescent lamps run direct from a dynamo placed at some distance) the dynamo must give a terminal E.M.F. of, say, 105 volts when supplying full current in order to allow for the loss of potential over the leads. When, however, only a few lamps are burning, the dynamo must give little more than 100 volts, and for any external current between zero and the maximum, the external E.M.F. must be between 100 and 105 volts. Hence in such a case the compound-wound dynamo must give a higher E.M.F. at full load than at light loads, and that at a lower speed. The desired external characteristic is thus a straight line, not horizontal, but slightly inclined upwards, or, as it is termed, the dynamo is to be 'over-compounded.'

Let us suppose that the 9″ × 12″ dynamo of p. 295 has to be compounded to fulfil the following conditions: at 650 revolutions per minute, $E_e = 105$ volts when $C_e = 62$ ampères, and at 680 revolutions $E_e = 100$ when $C_e = 0$, i.e. the voltage is to rise by 5 volts while the speed drops about $4\frac{1}{2}$ per cent. as the load is increased from no current to full current.

To give the required voltage at no load, z_a must be 3,770,000, and from the no-current curve of fig. 139 we see that the shunt-winding must then give $X_{s1} = 9{,}620$ ampère-turns. At full load, allowing a loss of nearly 6 volts over the armature resistance (as before), and of 1·8 volts over the series-winding, we find that the induced E.M.F. must be 112·8 volts; the total number of lines to give this E.M.F. must be 4,450,000, requiring a total excitation of a little over 14,000 ampère-turns, say $x=14{,}120$. At full load the exciting voltage on the shunt is 106·8, if the machine has a 'short-shunt winding,' and the ampère-turns due to the shunt are then

$$x_{s2}=x_{s1}\times\frac{E_{b2}}{E_{b1}}=9620\times\frac{106{\cdot}8}{100}=10270.$$

The ampère-turns which the series-winding must produce at full load are therefore $x-x_{s2}=14120-10270=3850$, and the number of series-turns is $T_m=\frac{x_m}{C_e}=\frac{3850}{62}=62$.

Assuming that a loss of about 445 watts in the field is decided upon, we have still to settle the question of how many watts are to be lost in the shunt and how many in the series or main coils in order that the total weight of wire used may be a minimum. That the total weight will vary with the distribution of the loss of energy between the shunt and series coils is evident if we consider the extreme case in which all the watts are lost in the one portion and none in the other; then, while the one has a definite weight, the copper in the other must be infinitely great, so that the total weight required will be infinitely great. It can be shown that the most economical distribution is to make $\frac{w_m}{w_s}=\frac{x_m}{x_s}$, or, which is the same, to make $\frac{w_s}{w}=\frac{x_s}{x}$ and $\frac{w_m}{w}=\frac{x_m}{x}$,[1] where w_s and w_m represent the watts to be lost in the shunt and main respectively. We thus

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[1] Neglecting any difference in the price of the two kinds of copper wire. To allow for differences in the lengths of a turn of the shunt and main, and for differences of temperature, according as the one or the other is wound outside, the above formula may be modified to

$$\frac{w_m}{w_s}=\frac{x_m\sqrt{\frac{l_m^2\times k_m}{l_s^2\times k_s}}}{x_s},\ \text{whence}\ \frac{w_s}{w}=\frac{x_s}{x_s+x_m\sqrt{\frac{l_m^2\times k_m}{l^2\times k_s}}}.$$

arrive at the result that the watts lost in the shunt should be roughly $=445\times\frac{10270}{14120}$, or about 325, and the watts lost over the main about 120.

The mean lengths of a turn of the shunt and series coils will be slightly different according as they form the inner or the outer layers of the coil; and also their temperature coefficients will vary slightly, the outer layers being less hot than the inner. Assuming both shunt and main to be equally divided between the two bobbins, and the main to be wound on the outside above the shunt, the mean length of a shunt turn will be $37\frac{3}{4}''$, and that of a series turn $42''$ or $l_s=3{\cdot}15$ ft., $l_m=3{\cdot}5$ ft., while k_s and k_m may be estimated at 1·14 and 1·1 respectively. We are now in a position to definitely determine the winding. The necessary resistance of 1,000 ft. of the shunt wire is (by equation (31), p. 319),

$$\omega' = \frac{100\times 1000}{9620\times 3{\cdot}15\times 1{\cdot}14} = 2{\cdot}9\,,$$

and the necessary diameter is ·060″. When double-cotton-covered to ·075, there will be 124 turns per layer, and if there are 14 layers on each bobbin, $T_s = 3472$, $R_s = 31{\cdot}6$, or, when hot, 36 ohms; thence $C_{s1} = \frac{100}{36} = 2{\cdot}77$, and $C_{s2} = 2{\cdot}96$. Hence, under the full-load condition, $W_s = 320$ nearly.

At this point it will be advisable to refer back to the data assumed above, so as to verify the correctness of our allowances for the loss of volts over the armature and series winding, and of our estimate of l_s and l_m as affected by the depth of winding. The shunt current being now determined, it will be found that the loss of volts over R_a will be hardly 6 volts. On the other hand, a loss of only 1·8 volts has been allowed for the main, while the permissible loss of watts may be as much as 125, corresponding to 2 volts. We may therefore assign 1·9 volts to the main, and can proceed to determine its necessary area (from eq. 32), viz.

$$\frac{{\cdot}0081\times 3850\times 3{\cdot}5\times 1{\cdot}1}{1000\times 1{\cdot}9} = {\cdot}063 \text{ sq. inch.}$$

If of rectangular section and wound in one layer of 31 turns on each bobbin, the width, which when double-cotton-covered to a thickness of 20 mils will form a complete layer, is ·275″, and the required thickness of the copper is therefore ·230″.

Although a dynamo may compound perfectly at full and no load, it will not do so with equal accuracy at intermediate points. This is due partly to the magnetic curves being rounded between the working points of no and full load, as, *e.g.*, between the limits of 3,750,000 and 4,450,000 lines in fig. 139, and partly to the fact that the back ampère-turns at half current are somewhat less than half the back ampère-turns at full current, and those at quarter current less than half of those at half current, and so on.[1] The effect of these two causes is seen in fig. 148, which shows a portion of the magnetisation-curves of fig. 139 on an enlarged scale. At no load (or practically no load) the working point of our compound-wound dynamo is C, and at full load F; since the length GK gives the increase in the ampère-turns, when the current is increased up to full load, the increase in the ampère-turns at half load will be GH $= \frac{\text{GK}}{2}$, and the corresponding z_a on the half-current curve is HL. But for perfect compounding only HM lines are then required, and the result is that the E.M.F. is rather higher than it should be: this, of course, increases the shunt-current, and the excess of E.M.F. is thereby rendered even greater. The terminal E.M.F. is higher than is required, not only at half load, but at all loads between zero and the maximum. The excess of E.M.F. is roughly proportional at any point to the vertical distance between the full line CLF and the dotted line CMF, from which it will be seen that it is greatest at about half load. In order to minimise this imperfection, a compound machine should be worked as far as possible on the upper part of the magnetisation-curves; the higher the working limits, the flatter and more horizontal do the curves become; and the effects of the unequal spacing of the intermediate curves, and of the increase of the shunt-excitation, are lessened. But in slow-speed machines, as, *e.g.*, an 18-kilowatt dynamo, running at 250 revolutions per minute, the percentage loss of volts over the resistances of the armature and main may amount to about 8 per cent. of the terminal voltage (6 per cent. in the armature, and 2 per cent. in the main); allowing a further loss of 3 per cent. in the leads between the dynamos and the lamps, and also a decrease in speed of the engine at full load of 5 per cent., it will be found that z_a at full load is $1{\cdot}11 \times {\cdot}05 = 1{\cdot}17$ times the no-load z_a. In such a case it is practically necessary

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[1] Due to the shifting of the brushes necessary to avoid sparking.

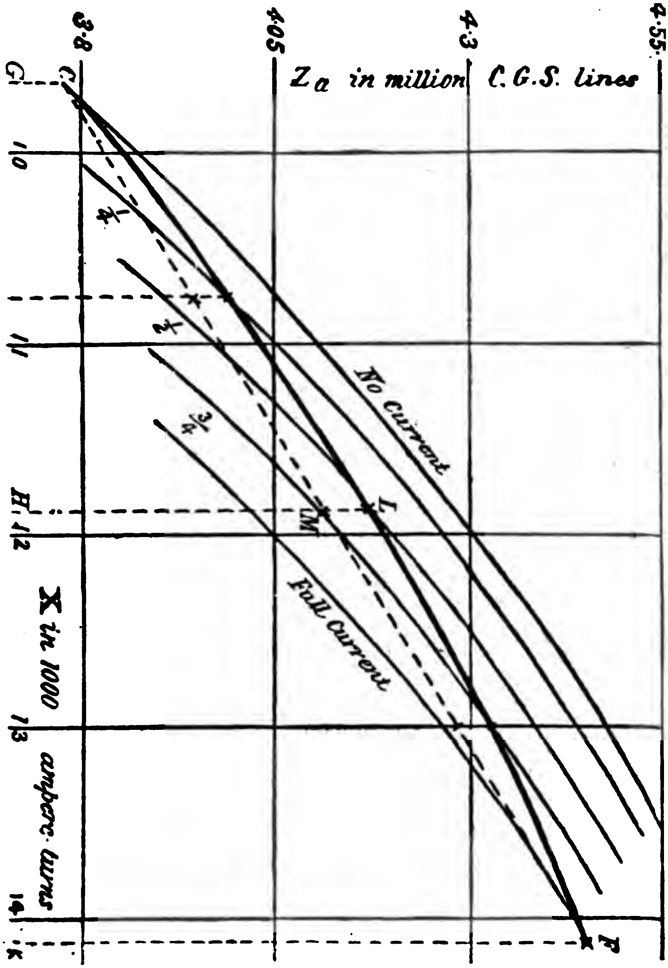

FIG. 148.

to work the machine on the rounded knee of the magnetisation-curve, since to work higher up would require an excessive amount of main wire. It results that slow-speed machines and those which have to compound over a large range of E.M.F. always compound more or less imperfectly, and the terminal E.M.F. for any current between ¼ and ¾ of full load may be from 3 to 4 per cent. too high. In machines running at comparatively high speeds the voltage should not exceed the required amount by more than 3 per cent., even at the point of greatest difference.

Differences of temperature considerably affect the perfection of the compounding of a dynamo: if it be worked on the bending portion of the curve at no load, and if the temperature of the field-coils rises during working 50° or 60° F., the machine, if designed to compound when hot, will give too high a voltage at no load, and, in general, as the temperature of the dynamo changes, the relative value of the shunt- and series-turns is altered. Such effects can be compensated by inserting a small variable resistance in the shunt-circuit, which can be gradually short-circuited as the dynamo warms up during working.

The field-winding of separately-excited machines can be at once calculated from the formulæ of pp. 319, 320. Such machines are never unstable, since the excitation is maintained from an external source. Separate excitation is most used in the case of alternators, and in machines which have to yield a high E.M.F. of over 300 volts, and a variable current.

CHAPTER XVII

SPARKING, AND THE ANGLE OF LEAD

ALL who have had practical experience of the working of dynamos will know that in bi- or multi-polar machines giving a continuous current, the brushes by which the current is collected are so mounted as to permit of their being shifted round the cylindrical surface of the commutator at least through some small angle. They will also be aware that if the position of the tips of the brushes, as they press on the commutator, be not properly adjusted, the result will be 'sparking at the brushes.' The nature of these sparks is similar to the sparking which occurs on breaking circuit at a switch : they are, in fact, a succession of small arcs between the brush and commutator segments caused by the sudden opening or breaking of certain closed paths in which at the time of opening a current is flowing. As regards the detrimental effect of such sparking, a certain difference must be drawn between the cases of closed-coil and open-coil armatures : its presence, however, always implies a waste of energy, and tends to shorten the life of the commutator, so that, even where least injurious, its suppression, so far as possible, is always desirable. The presence or possibility of sparking at the brushes is, in truth, the peculiar bane of continuous-current bi- or multi-polar dynamos, as contrasted with alternators ; the current of the latter may require to be collected by brushes or rubbing-contacts such as have been shown in many previous diagrams ; but the nicety of adjustment required by the brushes of bi- or multi-polar continuous-current dynamos, if sparking is to be minimised or entirely avoided, is a disagreeable characteristic of their whole class, and is entirely due to the presence of the commutator as opposed to the simple collecting rings of the alternator. The requirement that the brushes of the continuous-current dynamo should be properly adjusted by no means expresses the full extent of the disadvantage; more than this, the exact position which the tips of the brushes should have on the commutator depends on the amount of the current or load on the armature, and hence their setting further requires to be varied to meet changes of the load. If the load on a dynamo were never altered, the position of the brushes might be accurately adjusted once and for all, and in that position they might be fixed; but the conditions of ordinary working are by no means those of constant load, and consequently it must be possible to shift the position of the brush-tips.

Leaving for the present the consideration of sparking in dynamos with open-coil armatures, let us first deal with the case of closed-coil armatures. In all such machines, if the position of the brushes be incorrect, a row of sparks will appear, leaping across between the commutator and the tips of the brushes. These sparks may be small, bluish-white in colour, and comparatively harmless, or if the inexactness of the adjustment be considerable, they may be of a reddish colour and extremely violent. But in either case, if allowed to continue, they will sooner or later pit the surface of the commutator segments, destroy their smoothness and evenness, and heat the brushes. Once started, the mischief grows apace: the commutator becomes untrue and worn into deep and rugged grooves; increased sparking is caused by the 'jumping' of the brushes as they pass from segment to segment, and perhaps the tips of the brushes become partially fused: thus the commutator is gradually eaten away until its state is past all remedy. To check the evil, it is necessary to continually trim the brush-tips and 'true up' the surface of the commutator by turning it in a lathe, and these often-repeated processes result in a greatly reduced life of both commutator and brushes.

Sparking in closed-coil armatures is therefore an unmitigated evil, to be overcome almost at any cost, and the possibility of moving the brushes so that the position of their tips may be adjusted to suit the load on the armature becomes a first necessity. It must further be possible to do this easily and steadily while the machine is running and without in any way interrupting the passage of the current. In the 2-pole dynamo this is effected by mounting the brushes on two arms projecting horizontally from a cast-iron 'rocking bar' or lever. This latter is made in two pieces, bolted together so as to fit into a groove turned in the surface of the bearing next to the commutator; it is then locked in position either by a set screw or more usually by a pinching screw which clamps the two halves together and causes them to closely embrace the turned portion of the bearing. Through holes at opposite ends of the rocking lever there project two gun-metal spindles, entirely insulated from the iron by means of ebonite or boxwood bushes, and rigidly held in position by means of nuts: the brush boxes which hold the brushes are threaded on the spindles and firmly screwed in place (fig. 149). Each brush box is fitted with a spring, by which the brushes are kept pressed down on the commutator, and also with a 'hold-off' catch, by which the brushes are held when their tips are raised off the commutator surface. When the pinching screw of the rocker is slightly loosened, the whole can be swung round concentrically with the cylindrical commutator by means of a handle or handles on the lever. In large multipolar machines, using as many sets of brushes as there are poles, the rocker takes the more complicated shape of a star frame, through the projecting rays of which the insulated brush spindles pass, and it is then usual for the shifting to be effected mechanically by means of a worm gearing into worm-wheel teeth on a sector of the star. (Cp. fig. 167.)

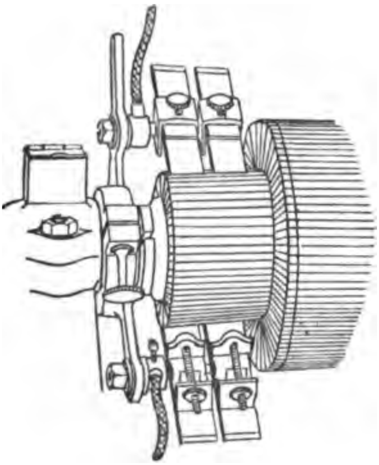

FIG. 149.—Brushes and rocking-bar.

The exact nature of the process by which a section of the winding is 'short-circuited' in a closed-coil armature has hitherto only been generally described. It has been shown that the brushes must be placed so that they sum up the E.M.F.'s generated in the sections forming one of the two or more parallels into which the winding is divided, and at the same time so that they short-circuit each separate section when it is passing approximately through the neutral gap between the poles, and has therefore little or no E.M.F. generated in it. Hence, if the field be bipolar, their position will be at opposite ends of a diameter, corresponding roughly with a position of the short-circuited coils between the poles or on a line at right angles to the general direction of the field. This preliminary description now requires to be further amplified.

If the reader refers to diagrams fig. 27 or fig. 107, it will be evident that in any closed-coil armature, whether ring- or drum-wound, in a bipolar or a multipolar field, the exact position relatively to the poles which the loops or coils of the winding occupy at the time when they are short-circuited depends upon the position of the tips of the brushes as they rest upon the commutator, and that it is this position of the short-circuited coil which is in effect altered by shifting the brushes backwards or forwards round the circle of the commutator segments. It may here not be out of place to point out that the relative position of the line of the brushes on the commutator and the line passing through the short-circuited coils on the armature core depends on the method by which the coils are connected to the commutator segments; if the lugs connecting these are straight, the two will coincide in a ring machine, and so also in a drum machine, unless the connection be made with the end connectors nearly at their centre. Such coincidence, however, although usual, is not necessary, and in some machines the commutator lugs are twisted round in order to bring the brush line into a more convenient position. The line of commutation must therefore always be understood to refer to the actual position of the short-circuited coils on the armature core.

Considering any one section of the armature winding (whether a single loop, or a coil of many loops), terminated by connection to a commutator segment at either end, let us call that segment which first enters under the edge of the brush the 'leading' segment of the coil, and, similarly, that edge or corner of a pole-piece under which a coil first enters after passing through the gap between two pole-pieces the 'leading' edge, these being opposed respectively to the 'trailing' segment and the 'trailing' edge or corner.[1] First, suppose in fig. 97 that the brushes are shifted slightly backwards against the direction of rotation ; then, in the ring armature, each section of the winding, when short-circuited, say by the upper brush, is brought into a position where it is still just moving in the field under the N. pole-piece, and the passage of the coil through this field, even though it be weak, will generate an E.M.F. in the same direction as that of the other coils under the N. pole-piece. Similarly, the coil short-circuited by the lower brush will be moving through the field under the S. pole-piece. So also in the drum armature, the short-circuited loops will each be moving through a field, producing an E.M.F. in the same direction round them as that which was induced in them immediately prior to their passage under the brush. Now the time during which a coil is short-circuited will depend upon the width of the brush and the speed of rotation, and is usually but a small fraction of a second. Thus, with a 50-part commutator and brushes having a normal width, equal to the width of one and a half segments plus one and a half strips of insulation, the time of short-circuiting is less than ·0015 second, with a speed of 1,200 revolutions per minute. But during the brief time for which the short-circuit lasts, the E.M.F. generated within the limits of the short-circuited section is acting round a single coil of very low resistance, the ends of which are closed by the practically negligible resistance of the brush contact surface : hence, even though the E.M.F. acts only for a short time, and be itself small, amounting, perhaps, to no more than one volt, or even less, it will very quickly raise the current which the coil is carrying to a value perhaps several hundreds of ampères in excess of the normal current carried by the other coils between the brushes. Immediately afterwards, as rotation is continued, the leading segment of the coil emerges from under the leading edge of the brush, and the coil should then carry the normal current flowing through the

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[1] By some writers these terms are used in an exactly opposite sense, but the above use appears preferable to the authors.

other coils (*e.g.* in a bipolar dynamo, half of the total armature current), but in a direction the reverse of that which it formerly had. Now, the large current set up during the short-circuit cannot die away instantaneously owing to its being interlinked with a magnetic system of lines. The current-turns of the coil since they surround a portion of the magnetic circuit react on the field, so that the number or distribution of the lines of the resultant field is different from what it would be if the short-circuited coil were absent. When, by rotation, the leading edge of the brush breaks contact with the leading segment of the coil, and the high resistance of the air- or mica-gap is interposed, the short-circuit current must fall in value, and the cutting of the coil by the field as it recoils towards its former distribution generates an E.M.F., which tends to keep up the old current. So, also, with the normal current in the rest of the armature. Hence the opposing currents of the single section, and of that half of the winding which it is about to join, both combine to maintain the continuity of the circuits which they were previously traversing; this they do by means of sparks passing between the trailing edge of the segment which has just emerged from under the brush and the tip of the brush itself. The result, therefore, of opening short-circuited sections of the winding while an E.M.F. is still induced in them, similar in direction to that of the coils which they are leaving on the trailing side of the brush, is that destructive sparking ensues; in other words, the short-circuit must not end while the coil is still moving through the field due to the trailing corners of the poles.

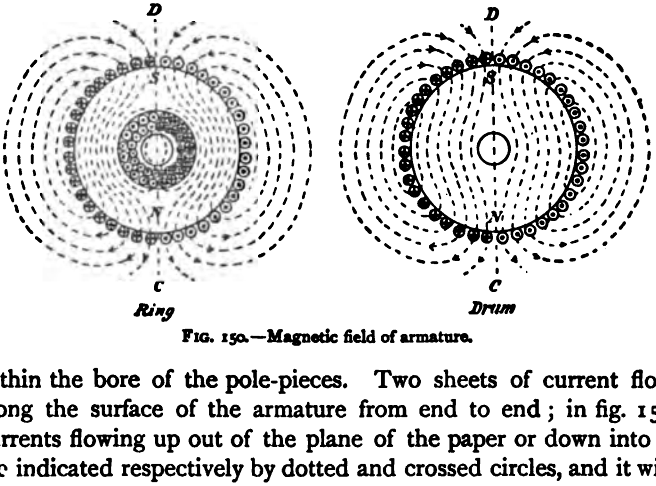
Now the lines of the field entering into or leaving the armature core shade off in denseness more or less abruptly as we pass out from beneath a pole-piece into the gap which separates it from a neighbouring pole-piece; where the lines just dip into the core and immediately leave it, is the neutral line on passing which the direction of the E.M.F. in the inductor changes. Thus, if we move the brushes forwards in the direction of rotation and so shift the diameter of commutation, we can bring the coil so that it passes across the neutral line during the time of commutation; the short-circuit is then not opened until there is a reversing E.M.F. acting on the coil and producing a current in it in the same direction as that which the coil will be called upon to carry as it emerges from under the brush. Our case is illustrated by fig. 97, where the coil short-circuited *e.g.* by the upper brush is slightly in advance of the vertical diameter, and is just moving in the fringe of lines under the S. pole-piece when short-circuit ends. Although its E.M.F. is now in the required new direction, the passage of the coil through this reversing field may not raise its current to equality with the normal current; yet any current flowing in the short-circuited coil will mean an equal amount withdrawn from the leading segment and diverted into the trailing segment. Only the surplus of the normal current will therefore be forced at the end of the period of commutation to maintain its former path to the brush by leaping from the rear edge of the leading segment to the brush-tip in a rush of sparks.

But now let the brushes be so shifted that at the end of short-circuit the E.M.F., then generated in the coil, exactly corresponds to the voltage required to drive the normal current through it. The opening of the short-circuit will then find the coil carrying the exact current required, the whole of it having been withdrawn from the leading segment. The commutation will thus be effected without any violent change, and consequently without any sparking. A reversing field is therefore required at that part of the air-gap where commutation ends, and this field must be just so strong that the E.M.F., due to movement through it, yields the normal current through the short-circuited coil. Thus in an armature with 100 sections, over which 5 volts are expended, an effective voltage of $\frac{1}{10}$ of a volt must be acting round the short-circuited coil as it leaves the brush. If the reversing field be stronger or weaker than this, in either case, although the current may be reversed into the right direction, there will be sparking; in the first case, the excess of current generated in the short-circuited coil maintains its closed path by sparks, and in the second case, when the current set up is too small, the current from the other coils causes sparking. In all cases of incorrect adjustment of the diameter of commutation, if the effect of the sparking on the commutator be carefully examined, it will be found that it is the trailing edges of the segments which are first pitted and worn by the sparks, and this shows that it is caused by the normal or the short-circuit current maintaining its path by small arcs between that edge of a leading segment which leaves the brush last and the brush itself. Thus, to sum up and emphasise these results, the passage of a section of the winding from one half of the armature into the other half can only be effected without sparking if its original current is stopped, and a reversed current is started during the period of short-circuit. In order to effect this, the brushes must be set so that at the end of the short-circuit the coil is moving through a field which produces an E.M.F. opposed in direction to the current flowing in it at the beginning of the short circuit; *i.e.* commutation must end in advance of the neutral line when the short-circuited coils are within the influence of the field issuing from or entering into the *leading* pole corners. Further, the reversed current must be brought up to a value very approximately equal to that of the coils on the leading side of the brush, within the time that elapses between entrance of the trailing segment under the brush and emergence of the leading segment on the other side. In practice, the proper position of the brushes is found by shifting the rocking bar slightly backwards and forwards until a position is observable on either side of which the sparking becomes greater.

The reversing E.M.F. and the current thereby generated will depend on the strength of the field in which the coil is then moving, and on the speed of movement through it. It is therefore apparent that with a given speed, armature winding and distribution of field, the amount by which the short-circuited coils must advance beyond the neutral line will require to be varied when the strength of the armature current is varied. As the total current passed through the armature is increased, the new current which has to be started becomes greater; a stronger reversing E.M.F. is therefore required, and the angular advance of the brushes beyond the neutral line must be progressively increased to keep pace with the increased current. A shifting of the brushes forwards in the direction of rotation, or, as it is termed, 'giving the brushes a forward lead,' is therefore necessary in a dynamo when the armature current is increased, even if the neutral line remains fixed.

But this is by no means all; the very fact of increasing the armature current, even if it does not affect the total number of lines passing from pole to pole, entirely alters the distribution of the field. The neutral line is itself shifted forwards, or away from the vertical line of symmetry between the two pole-pieces in the direction of rotation, and, therefore, the forward lead of the brushes reckoned from this symmetrical line is increased. To understand this new problem it is necessary to consider the reaction of the armature current on the field.[1]

Taking the case of a ring- or drum-wound armature in a 2-pole field with currents flowing in its wires, we have to ask what is the magnetic effect of these currents when the armature is placed within the bore of the pole-pieces. Two sheets of current flow along the surface of the armature from end to end; in fig. 150 currents flowing up out of the plane of the paper or down into it arc indicated respectively by dotted and crossed circles, and it will be seen that the direction of the currents in the inductors changes



Ring *Drum*

FIG. 150.—Magnetic field of armature.

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[1] The general principles on which the question of armature reaction is to be solved were first published by Drs. J. and E. Hopkinson in their oft-quoted paper reprinted from *Phil. Trans.* 1886, in *Original Papers on Dynamo Machinery*, pp. 103–111. These principles have been further developed by Messrs. Swinburne and Esson in their two papers, entitled respectively 'The Theory of Armature Reaction in Dynamos and Motors' and 'Some Points in Dynamo and Motor Design,' published in the *Journal Inst. Elec. Eng.* vol. xix. part 85. The authors here acknowledge their general debt to these several papers: together with the discussions that arose therefrom, they contain a large mass of theory and facts bearing on many of the obscurer points in the design of dynamos.

at the diameter of commutation (D C) corresponding to the position of the brushes. The general effect of the two current-sheets is to convert the iron armature into a cylindrical electro-magnet with two polar surfaces at opposite ends of the diameter of commutation, and stretching along the entire length of the core. If the armature be ring-wound, with the turns of wire passing round and inside the core, the analogy to a pair of magnets of semicircular shape, each wound with a magnetising coil and placed so that their like poles abut on each other to form a common N. and a common S. pole, is at once apparent. If the armature be drum-wound, this effect is not immediately so apparent, but we may for simplicity neglect the actual end-connections of the wires and imagine each inductor connected across, in a plane at right angles to the diameter of commutation, to a corresponding inductor on the opposite side of this diameter. A number of loops are thus obtained, each carrying a current equal to half the total armature current, and magnetising the core in a general vertical direction. Hence, whether the armature be ring- or drum-wound, lines of induction will flow through the two halves of the core on either side of the diameter of commutation, issue forth from the lower surface and circle round to enter again at the upper surface; or, as it is otherwise expressed, poles are formed on the armature at opposite ends of the diameter of commutation.[1]

Apart from the field-magnet these lines of induction will be comparatively few in number (as shown in fig. 150), but when the armature is surrounded by the iron pole-pieces, their length of path in the air is enormously reduced, and a strong flow will arise, passing through the two pole-pieces and traversing the short air-gaps of the dynamo. Fig. 151 serves to show the course of the increased number of lines due to the currents in the inductor-loops when the armature is placed within the embrace of the pole-pieces, the magnet being otherwise entirely unmagnetised. When the armature is rotated, the inductors pass successively from one half of the armature into the other, but the current-sheets remain fixed in position on either side of the diameter of commutation, and, therefore, the field of lines which they produce will likewise remain stationary and unchanging. The magneto-motive force of the armature ampère-turns rises as we pass from a central horizontal line towards either the upper or lower limits of the armature core. Taking any angle, *k o l*, the magneto-motive force between the limits *k* and *l* is proportional to the number of ampère-turns which they include: thus, if τ = the total number of inductors evenly spaced round the armature, and the current flowing in each be $c = \frac{C_a}{2}$, the magneto-motive forces of the ampère-turns summed up between $k, l = 1{\cdot}256 c\tau \frac{\theta}{360^\circ} = 1{\cdot}256 \frac{C_a}{2} . \tau . \frac{\theta}{360^\circ}$, where θ = the angle *k o l* expressed in degrees. The greatest magneto-motive force is thus obtained between the extreme top and bottom of the core, and is there equal to $1{\cdot}256 \frac{C_a}{2} . \frac{\tau}{2}$. Now, provided that the area across the narrowest part of the pole-pieces be considerable, and the iron be therefore free from saturation, either across the neck or at the pole-corners, almost the whole of the magneto-motive force may be regarded as expended in producing the induction in the two air-gaps traversed by each set of lines on either side of the armature. The lines will be densest towards the two ends of each air-gap, where they enter into or issue from the pole-tips, and will thence gradually shade off to zero at the central horizontal line.

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[1] In the case of the ring armature some lines will cross the interior of the ring, passing through the shaft and supporting hub (or spider) as indicated in fig. 150.

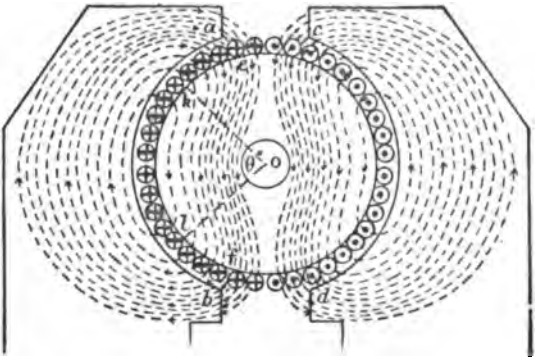

FIG. 151.—Magnetic field of armature placed within pole-pieces.

Let us suppose that the sheets of armature current shown in fig. 151 are due to rotation of the armature as a dynamo, say, in a counter-clockwise direction; then in order to produce armature currents in the directions shown there must be a N. pole on the right, and a S. pole on the left hand, i.e. such a field as that shown in fig. 44. If we now compare together the general direction of the two sets of lines, viz. those of fig. 44, that would be due to the field-magnet by itself, and those of fig. 151, due to the armature ampère-turns alone, it will be found that the direction of the lines due solely to the armature current is immediately opposed to the direction of the field-magnet lines in the portion of the air-gap under each of the two leading pole-tips (*a*, *d*) while they are in the same direction under the two trailing pole-tips (*b*, *c*). But when the dynamo is at work and supplying current, the magneto-motive forces of both the field ampère-turns and the armature ampère-turns are simultaneously present, and it remains to ask, What is the actual distribution of the field as resulting from the combined effect of the two acting together? In nature there is only one resultant distribution of field produced which satisfies all the conditions; this resultant field may be arrived at by an imaginary superposition of the two sets of lines, the one on the other, followed by a process of compounding the two vectorially, with due regard to their directions, sense, and strength; or we may adopt the simpler course of dealing solely with the magneto-motive forces acting along any closed path. The diagram of the lines separately due to the armature will then serve to bring before us clearly the general direction of the magneto-motive forces of the armature ampère-turns. The most important closed path to be considered is that approximately marked by the letters *a*, *e*, *f*, *b*, in fig. 151. The diameter of commutation is for the present assumed to coincide with the vertical line of symmetry between the two pole-pieces. Since the points *e* and *f* are taken nearly at the upper and lower extremes of the core, the magneto-motive force of the armature ampère-turns between them may be regarded as approximately equal to the maximum value, i.e. $1{\cdot}256 \frac{C_a}{2} . \frac{\tau}{2}$; and on our previous assumption that the iron of the armature and pole-pieces is very far from saturation, or infinitely permeable, the whole of this armature magneto-motive force may be regarded as expended over the two air-gaps at the top and bottom, half over the portion of the path *a e*, and half over the portion *f b*. Now over these two air-gaps the direction of the armature magneto-motive force exactly coincides with the direction of the magneto-motive force impressed upon the air-gap as a whole by the field ampère-turns. In each of these two portions of the closed path it will therefore suffice to simply add together the magneto-motive forces acting there; due regard, however, must be paid to their sense or algebraic sign, as opposed to their mere direction, for, although along *f b* the magneto-motive force of the armature ampère-turns exactly coincides in both direction and sense with the magneto-motive force due to the field-winding, along the upper portion of the air-gap *a e* the sense in which the former acts is from *a* to *e*, or directly opposed to the sense of the latter, which is from *e* to *a*. Let X_g=that part of the field-magnet excitation which is expended over the two air-gaps on either side of the armature, each having a length of l_g cm.; then the magneto-motive force impressed by the field-winding on each air-gap between the surfaces of a pole-piece and the core is $1{\cdot}256\frac{X_g}{2}$. We are now in a position to determine the density of the lines, or the induction, over any small portion of the air-gap. Starting close under the leading pole-corner, *a*, the magneto-motive force acting over the air-gap *a e* is the difference between that due to the field ampère-turns and that due to the armature ampère-turns, or $1{\cdot}256\left(\frac{X_g}{2}-\frac{C_a}{2}\cdot\frac{\tau}{4}\right)$; and this when divided by its length, or l_g, will give us the induction in the air-gap at the leading pole-tip, or

$$B'_g=\frac{1{\cdot}256\left(\frac{X_g}{2}-\frac{C_a}{2}\cdot\frac{\tau}{4}\right)}{l_g} \quad . \quad . \quad . \quad . \quad . \quad (36)$$

But at the trailing pole-corner, *b*, the magneto-motive force of the armature ampère-turns assists that of the field-winding; consequently the resultant magneto-motive force acting across *f b* is the sum of the two, or $1{\cdot}256\left(\frac{X_g}{2}+\frac{C_a}{2}\cdot\frac{\tau}{4}\right)$, and the induction there is as much increased as it was diminished at the leading pole-corner, i.e.

$$B''_g = \frac{1{\cdot}256\left(\frac{X_g}{2} + \frac{C_a}{2} \cdot \frac{\tau}{4}\right)}{l_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Any two other points symmetrically placed above and below the horizontal line may be taken ; the smaller the angle they include, the less will be the difference in the magneto-motive forces acting over the air-gap, until finally at the centre of the pole-piece, $\frac{X_g}{2}$ alone acts, and the induction there has its average or normal value

$$B_g = \frac{1{\cdot}256X_g}{2l_g}.$$

On the other side of the armature, the distribution of the field is altered in exactly the same way, but now the leading pole-corner, where the induction is below the average, is at *d*, and the trailing pole-tip, where the induction is above the average, is at *c*. It should be observed that the length of the armature does not enter at all into the question of the *induction* of the air-gap, but only affects the total number of lines which traverse any strip of the air-gap taken parallel to the length of the core.

The general result of the reaction of the armature currents on the field is shown in fig. 152, from which it will be seen that the field is, as it were, displaced and twisted round in the direction of rotation. The distribution of the field or the induction in the air-gap on either side of the armature, instead of being uniform over the greater part of the bored face of the pole-piece, varies continuously from a minimum value at *a e* to a maximum at *b f* ; the densest part of the field is shifted up to and beyond the trailing pole-tip, while the field under the leading pole-corner is weakened. The distortion of the resultant field now corresponds to the inductance of the armature current-turns. If the armature current be reduced in strength, the lines would straighten themselves and cut the inductors, giving a forward E.M.F., and tending to keep up the previous value of the current. In so doing the stored energy of the armature current-turns, due to their reaction on the magnetic field, would reappear. As we are dealing with the *resultant* field and its distribution at any moment, the armature loops must be regarded as entirely destitute of any inductance, whether self or mutual, the rate at which they are cut by the lines of the resultant field giving at once their effective voltage, to which the current value corresponds.

So far we have assumed the diameter of commutation to coincide with the vertical line of symmetry, or that the brushes are given no forward lead beyond this line. Under these conditions it is now evident that the average induction in the air-gap, or, taking into account the length of the armature, the total number of lines entering into it on the one side and leaving on the other is entirely unaffected by the current-turns of the armature, since the induction is as much strengthened over the one half of each polar face as it is weakened over the other half. The distribution of the field has, however, been altered, and the greater the current passed through a given armature, the greater will be the forward displacement or twisting round of the field. Any such displacement, however, implies an equal displacement of the neutral line, as is evident from fig. 152, where the line *n n* is seen to be shifted forwards in advance of the vertical line of symmetry between the poles. But it has been shown that at

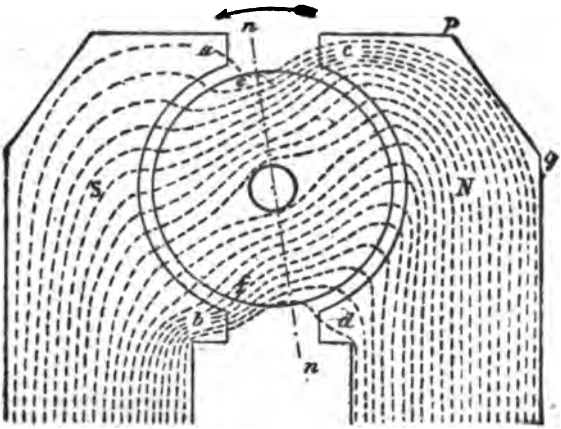

Fig. 152.—Magnetic field of dynamo reacted on by armature.

the end of commutation the coils must be slightly in advance of the neutral line; theoretically, therefore, even with a small armature current, the field will be slightly displaced and the brushes must have a very small lead. When, however, the armature current is very small, the displacement due to its reaction is negligible. Further, a very weak reversing field is sufficient; hence the diameter of commutation for no sparking at light loads and the neutral line will so nearly coincide with the vertical line of symmetry that all three may be regarded as identical. But when the armature current is increased, the brushes must be shifted forwards, so that the short-circuited coils may overtake and pass the displaced neutral line before commutation ends.

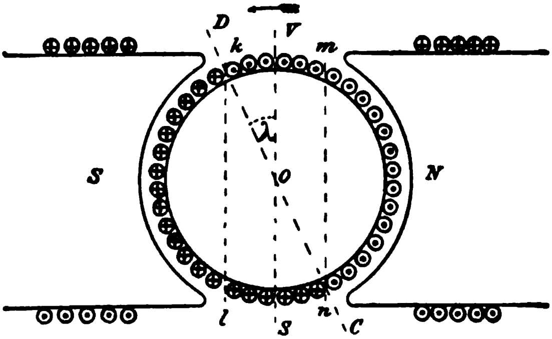

FIG. 153.—Ampère-turns of field and armature.

The lead of the brush-tips (reckoned from the no-lead line of symmetry) thus becomes very much more than the mere lead beyond the neutral line with which we first dealt; besides this, it comprises the angle through which the neutral line has itself been shifted by reason of the reaction of the armature ampère-turns on the field. The result of thus advancing the diameter of commutation forwards beyond the vertical line is to introduce an entirely new effect. So long as it was vertical, the armature ampère-turns all tended to produce a cross-magnetisation, the general direction of which in the armature was at right angles to the direction of magnetisation due to the field alone. But now, when the brushes are given a forward lead, if the directions of the currents in the inductors on the armature be compared with the directions of the currents in the magnetising coils of the field, as shown by dotted and crossed circles in fig. 153, it will be seen that in all the inductors lying between the two vertical lines *k l*, *m n*, the direction of the current is absolutely opposed to the direction of the current in those field-magnet wires which are on the same side of the lines of the field. It will assist us to treat the problem if we now regard the inductors as connected together across the ends of the armature in two sets of turns, one forming a coil in a vertical, and the other a coil in a horizontal, plane.[1] These are seen in fig. 154, the imaginary end-connections of the vertical coil being shown by full lines, and those of the horizontal coil by dotted lines. Thus, inductors 1 and 2 are equivalent to one turn or loop, round which a current of $\frac{C_a}{2}$ ampères flows, and similarly inductors 3 and 4 are equivalent to another turn, but in a plane at right angles to the former. The entire number of armature ampère-turns is divided into two sets, and we are justified in so dividing them, since the effects of the two are totally distinct. The two vertical lines of fig. 153 are fixed in position by the total angle of lead, the angle *k o m* or *l o n* being equal to twice the angle of lead λ, reckoned from the vertical diameter or line of symmetry. The belt of ampère-turns included between the lines *k l*, *m n* are in direct opposition to the ampère-turns of the field, and tend to demagnetise the armature and field; they are therefore known as the 'back ampère-turns' of the armature. On the other hand, the belt of ampère-turns lying horizontally between the points *k l* or *m n* have the same effect as was previously considered when the diameter of commutation was vertical; that is, they have a cross-magnetising tendency, which distorts the field round in the direction of rotation, weakening it at the leading pole-corners and

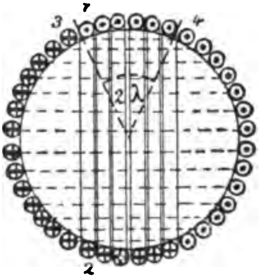

FIG. 154.—Back and cross ampère-turns of armature.

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[1] This method of treating the subject was first published in 1887 by Mr. Swinburne, *Journal Soc. Tel. Eng.* vol. xv. No. 64, p. 542.

strengthening it at the trailing pole-corners; they are therefore known as the 'cross ampère-turns' of the armature. Every conductor not situated on the neutral line or at right angles to it has both a cross and a direct magnetising tendency, but of those situated under one pole-piece the forward tendency of one half is balanced by the back effect of the other half, leaving them pure cross loops, while of those between the pole-tips the cross tendency of the one half is balanced by the other half, which tend to lessen the distortion, so that only their back effect is summed up.

In order to examine the question of commutation more closely, we must consider separately the magnetic effect of the short-circuited loops and the magnetic effect of the remaining armature loops, both being affected by rotation. Let us call the undistorted system of lines due to the magnet-winding field (1), the displaced field due to the composition with field (1) of the effect of all the armature loops except those which are being short-circuited during commutation field (2), and finally the actual resultant lines due to the composition with field (2) of the short-circuited loops, field (3). Taking the short-circuited loops by themselves, we find that during the process of commutation they are converted from back into forward ampère-turns. From this there should result an increase in the total number of lines, and a consequent storage of energy in the magnetic field. Further, at the beginning of commutation, when they are behind the neutral line of the field (2), the short-circuited turns lessen the distortion so that the resultant neutral line of the whole system (field 3) is behind the neutral line of field (2). As the old current falls in the short-circuited loop, the neutral line of field (3) advances towards the neutral line of field (2) until when the current is zero, the two coincide. As the current reverses, the short-circuit loops increase the distortion, so that the neutral line of field (3) is ahead of that of field (2). Such increased distortion again implies a storage of energy. Thus the process of commutation regarded entirely by itself should be a continuous process of storing energy. It is, however, evident that in a dynamo at steady work there is no continuous storage of energy, and hence it must be that an equal amount of energy would otherwise be liberated if the short-circuit loops be supposed absent. And this is, in fact, found to be the case, if we consider field (2) alone, and examine the effect of rotation upon it. As the band of back ampère-turns moves forward through the arc corresponding to the angular width of the brush, they are increased in number, and the field should, therefore, become weaker—a process which must imply a liberation of energy. Further, by the same movement, the cross ampère-turns as they advance under the pole are lessened in number so that the distortion should be reduced, again implying a liberation of energy.

Any alteration in the total number of lines of the field, whether due to the short-circuit loops as explained above, or to the remaining normal-current loops, cannot be very large, as it would be damped by eddy-currents in the iron of the magnet and by the closed circuit of the field winding. Whatever it is, the magnetic effect of the rotation of the short-circuit loops is opposed to the magnetic effect of the rotation of the remaining armature loops, and if the commutation be sparkless, the two effects are balanced. But even though the total number of lines remain unaltered, their distribution may vary. Now it has been shown that while the neutral line of field (2) continuously recedes during the short period of commutation, the neutral line of field (3) continuously advances with respect to that of field (2). The recession and the advance might be exactly balanced, so that the actual neutral line of resultant field (3) remains fixed in space, and no energy is either stored or liberated. This is, in fact, the case of a perfectly sparkless commutation. At the end of such commutation, the current loops are in every particular precisely the same, and similarly distributed as at the beginning. But if the reversal of current is carried too far, energy has been stored and is liberated by the spark when the short-circuit is opened. If the current is not reversed at all or is not reversed enough, energy is liberated in the system, and then the deficit has to be abruptly made good, so as to restore the balance, the necessary accompaniment of the sudden storage being a spark which is entirely waste energy.

The back ampère-turns of the armature are equal to the product of half the total armature current into the number of inductors inclosed within twice the angle of lead, or if λ be expressed in degrees—

$$X_b = \frac{C_a}{2} \cdot \tau \cdot \frac{2\lambda}{360^\circ} = C_a \cdot \tau \cdot \frac{\lambda}{360^\circ} \quad . \quad . \quad . \quad . \quad (38)$$

Similarly, if $\theta =$ the angle *k o l* or *m o n*, also expressed in degrees, i.e. $= 180^\circ - 2\lambda$, the cross ampère-turns of the armature are equal to

$$\frac{C_a}{2} \cdot \tau \cdot \frac{\theta}{360^\circ} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

The relative proportion of these two sets of ampère-turns will of course depend entirely upon the angle of lead.

The magneto-motive force of the cross ampère-turns as summed up between the points *k* and *l* or *m* and *n* is on the principle laid down on p. 361

$$= 1{\cdot}256 \frac{C_a}{2} \cdot \tau \cdot \frac{\theta}{360^\circ},$$

and on our previous assumptions this force may be regarded practically as expended over the two air-gaps of the cross-circuit. Hence by the reasoning of p. 363 it follows that, with a given position of the brushes, the induction within the air-gap at opposite ends of the diameter of commutation or at the leading pole tips, where each section of the winding is short-circuited, may very approximately be represented by

$$B'_g = \frac{1{\cdot}256\left(\frac{X_g}{2} - \frac{C_a}{2} \cdot \frac{\tau}{2} \cdot \frac{\theta}{360^\circ}\right)}{l_g} = \frac{1{\cdot}256\left(X_g - \frac{C_a}{2} \cdot \tau \cdot \frac{\theta}{360^\circ}\right)}{2 l_g} . (40)$$

Evidently, if the collection of current is to be sparkless, this induction must have some positive value in the right direction to reverse the current. But from our last equation for B'_g, it appears that if with a given setting of the brushes the armature current of the dynamo be increased to such a value that $\frac{C_a}{2} \cdot \tau \cdot \frac{\theta}{360^\circ} = X_g$, the magnetising forces of the field and the cross armature-turns are exactly balanced along the diameter of commutation, and there is no field to reverse the current in the short-circuited coil. If it be still further increased, the direction of the field at the points on the armature surface corresponding to the position of the brushes on the commutator is actually reversed ; in both cases destructive sparking will ensue, and the machine become unworkable. The remedy must therefore lie in shifting the brushes still farther forward into a part of the field where the induction is in the right direction, and is strong enough to reverse the increased current in the short-circuited coils. But the very fact of giving the brushes more lead so that their tips may pass the neutral line which has been displaced out of its symmetrical position by an increased armature current increases the back ampère-turns, which tend to weaken the strength of field. In series- or compound-wound machines this weakening or demagnetising tendency is partially or entirely neutralised by the corresponding increase of the magnetising turns of the field-winding ; for the increased exciting power of those turns which are in series with the major part of the armature current will tend to maintain, and may even increase, the total strength of the field. But in simple shunt-wound dynamos it may be asked how it is that a position can be found where commutation can take place without sparking ; since, as the brushes are shifted forwards to overtake the neutral line, the field is continuously weakened. The answer lies in the fact that, whether the machine be shunt- or series- or compound-wound, in all alike, as the brushes are shifted forwards, armature ampère-turns are progressively shifted from the belt of cross ampère-turns into the belt of back ampère-turns, and thence, though the total strength of field may be weaker and its displacement more marked, yet the difference in the strength of field under the leading and trailing pole-corners becomes less ; as the lead is increased, and the angle θ becomes less, the cross magneto-motive force, viz. $1{\cdot}256 \frac{C_a}{2} \cdot \frac{\tau}{2} \cdot \frac{\theta}{360^\circ}$, which acts differentially at the upper and lower limits of the field, weakening B'_g and strengthening B''_g, is reduced. It is still true that the lines are crowded up towards one end of the polar surface and air-gap, but their density over the air-gap or the induction has more nearly one uniform value. Hence under ordinary conditions if the brushes are shifted far enough forwards a reversing induction of the requisite strength should be reached.

If, however, an abnormally large current be taken through the armature of any machine, the reaction of the armature ampère-turns on the field will become excessive, a very large angle of lead must be given to the brushes, and the number of demagnetising ampère-turns, x_b, is therefore correspondingly great; even then the cross ampère-turns may remain so powerful that the short-circuited coils are not brought within the influence of a reversing field. As a consequence, no position can be found for the brushes where violent sparking does not occur. We are thus met with a condition which, entirely apart from any question of heating, limits the maximum current that can be passed through the armature, and therefore for a given speed of rotation and voltage limits the output of the machine. The armature ampère-turns at full load must not be so large that their reaction on the field renders sparking unavoidable; a sensible field is required to reverse the current, and the magneto-motive force impressed on the air-gap by the field-winding must be in excess of the magneto-motive force of the cross-ampère-turns at the ends of the diameter of commutation; without as yet laying down a definite angle of lead, it is evident that, after allowing for even a large angular advance, so far from allowing the cross ampère-turns $\frac{C_a}{2} . \tau . \frac{\theta}{360^\circ}$ to nearly balance x_g, they must not be allowed to approach to anything like equality. Practice has shown that if the dynamo is to work satisfactorily, and to allow of a slight temporary increase of current above its designed capacity, the value $\frac{C_a}{2} . \tau . \frac{\theta}{360^\circ}$ must not be greater than about $\frac{X_g}{1·5}$ in a drum machine, or $\frac{X_g}{2}$ in a ring machine. Under these conditions, which are fulfilled in any ordinary dynamo, the short-circuited coils can at full load be brought into a position where no sparking ensues, by shifting the brushes forwards; a process which effects the desired end, since it progressively shifts ampère-turns out of the belt of cross armature-turns into the belt of back armature-turns, and brings the short-circuited coil into a denser field.

As will be more fully described later, in machines of good design it is seldom requisite to shift the brushes so far forward at full load as to bring the short-circuited coils far under the leading pole-tips; the full-load position is, however, often close to the leading pole-tips, although more generally it is just behind them and within the fringe of lines immediately extending slantwise from them. It must be remembered that the actual strength of field required to reverse even a large current in the short-circuited coil is not very great, and under ordinary non-sparking conditions the induction at that part of the air-gap where commutation ends probably seldom amounts to more than about a quarter[1] of the average value $B_g = \frac{Z_a}{\text{area of air-gap}}$, and will usually be less. At first sight this may seem to contradict our previous requirement that at most the cross ampère-turns must not be more than $\frac{X_g}{1 \cdot 5}$, since this rule would appear to give an induction B'_g, equal to at least one-third the average value, B_g. The apparent contradiction is, however, explained by the fact that when commutation occurs outside the pole-tips, the lines there have a sloping direction, by reason of which the denominator in our equation for B'_g should have some value, l'_g, greater than the normal l_g.

Thus in any bi-polar armature, when a considerable current is passed through it and the magnetic effect of this is taken into account, the general distribution of the lines is as shown in fig. 152. Owing to the cross ampère-turns of the armature the diameter of commutation is in advance of the line of symmetry. The effect of the lead of the brushes is to convert a portion of the armature-turns into back ampère-turns, directly weakening the total strength of the field, unless they be counterbalanced by an increase in the exciting power of the field-magnet coils. The fringe of lines at the leading pole-tips should shade off fairly gradually; if the variation of the induction be abrupt, a small error in the position of the brushes will mean a large error in the strength of reversing field, the brushes will require excessive care in their adjustment, and the machine will be sensitive to any changes of the load. Hence, even apart from the question of leakage across from one pole-edge to the opposite edge, it is not practicable to bring the pole-pieces very close together; the direct distance across from one pole-tip to the other should bear a certain relation to the length of the air-gap, and a common proportion to find is that the former is about nine times l_g. In the 2-pole dynamo the angle subtended by the bored face of the pole-piece usually falls within the limits of 120° and 140°, a common polar angle being 130°, and after allowing for the fringe of lines extending beyond the actual edges of the pole-piece, this may be taken as resulting in an effective field-angle of about 140°.

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[1] For an approximate calculation of the induction requisite to reverse the full-load current in an ordinary dynamo, *vide* 'The Theory of Armature Reactions in Dynamos and Motors' (Swinburne), *Journal Inst. Elec. Eng.* vol. xix. No. 85, p. 99; see also No. 93, vol. xx. pp. 299 and 302.

The question of saturation in the pole-pieces as resulting from the armature reaction on the field depends entirely upon their shape. So far the pole-pieces have been assumed to require little or no magnetising force at any part. If, however, the pole-corners are rapidly thinned off to a fine edge, the magneto-motive force of the armature cross ampère-turns will not be expended entirely over the air-gap, but some part of it will be spent over the trailing pole-corners. The only direct result of this will be to reduce the distortion, without in any way affecting the total number of lines,[1] and so far it is not detrimental. There may, however, be indirect results, due to the two pole-pieces not being affected in precisely the same way, and any difference in the distribution of the field over the two air-gaps, while hardly affecting the drum armature, will be prejudicial in a ring (see p. 387).

Thus, if the upper corner of the pole-pieces as at *pq* (fig. 152) be cut off too sharply, the induction at the trailing-corner *c* will be throttled, although there will be no corresponding throttling at the other trailing pole-corner, *b*. In such a case, therefore, the values of the induction at different points along the one polar arc from *c* to *d* will not correspond exactly with the values at diametrically opposite points of the air-gap under the other pole-piece, and the resultant distribution of the field will not be symmetrical in accordance with our definition of p. 281.

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[1] *Vide* an interesting paper on 'The Relation of the Air-gap and the Shape of the Poles to the Performance of Dynamo-electric Machinery' (Ryan,, reprinted in *Electrician*, October 9, 1891.

Again, it will be seen, from fig. 152, that in the pole-piece on one or other side of the armature (in the diagram on the right side) the greater part of the field is forced to continue its path upwards through the pole-piece across the neck at N : a contracted area of iron across this part may thus become saturated and throttle the lines, while on the other side no such throttling will take place ; again, therefore, the distribution of the field becomes unsymmetrical on the two sides. Prevention of the displacement of the field by a contracted area of iron across the neck would be in itself an advantage, but in single magnet-fields it cannot be secured without producing a want of symmetry. Attention has already been drawn (p. 277) to the necessity of providing sufficient area across the narrow necks of the pole-pieces, but now it further appears that on the one side more than half of the lines will have to pass through that area. In double-horseshoe fields we are not met by the same difficulty, and it becomes possible to reduce the area of iron across the neck *c* on the line *a b* in figs. 125 and 126, or even to split the fields along that line; the distortion of the field by the cross ampère-turns is thereby lessened, and hence the double horseshoe, apart from other reasons, can be made to yield a more symmetrical field than the single magnet.[1]

The exact lead which the brushes will require for a given armature current it is almost impossible to determine by calculation with complete certainty, since to accurately map out the distribution of the field in the interpolar space, and immediately under the leading pole-tips, would be a most complex problem ; yet until the lead is known, the relative values of the cross and back ampère-turns are also unknown. Now the quantity x_b has already appeared in Chap. XV. as the number of ampère-turns that must be added to the field ampère-turns in order to counteract the demagnetising tendency of the armature current when a lead has been given to the brushes. When, therefore, in designing a dynamo, it is required to know x_b in order to determine the field-winding, it will be necessary to fall back on the evidence furnished by similar machines already built and tested. A maximum limit, which is usually observed in machines of good design, has already been implicitly laid down. If the short-circuited coils at full load are brought nearly up to the pole-tips, and the normal polar angle is taken at about 130°, the greatest angle of lead will be something less than $\frac{180^\circ - 140^\circ}{2} = 20^\circ$. This, however, implies that the armature is loaded to the maximum number of ampère-turns permitted by considerations of sparking; the angle of lead at full load is usually not so great, and, although no definite rule can be laid down, an average value may be set as falling within the limits of 8° and 13°. After a machine has been tested, the actual back ampère-turns as determined by the known angle of lead may be compared with the number that appears to be required in order that the actual exciting power on the magnet may agree with the calculated; if there is any discrepancy between the back ampère-turns apparently required and the actual number, the reason can only be sought in the somewhat vague nature of the data which the designer has before him as to the exact lengths of paths or quality of the iron. It has been contended that the increased reluctance of the trailing pole-corners due to their becoming saturated by distortion may result in an absolute reduction in the total number of lines passing through the armature;[1] but such an effect can only be due to the back ampère-turns. If the data employed in the calculations of Chap. XV. were strictly accurate, the lengths and state of saturation being estimated and varied to suit the circumstances of each case, no discrepancy should be found between the back ampère-turns of the dynamo, as deduced from the lead of the brushes, and the actual performance of the machine.[2] As, however, such accuracy would be almost impossible of attainment, the error is on the safe side if in designing we assume the lead to be slightly greater than it will probably be; or, in other words, if we allow a margin in the value assigned to the angle λ in the expression $x_b = c_a \cdot \tau \cdot \frac{\lambda}{360^\circ}$. Any eddy-currents in the armature or pole-pieces exert a demagnetising effect on the field, exactly analogous to that of the actual back ampère-turns; if present in the inductors themselves, they further directly increase the angle of lead required, and may considerably add to the difficulty of avoiding sparking.

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[1] *Vide* Swinburne, *Journal Inst. Elec. Eng.* vol. xix. No. 85, pp. 95-97.

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[1] This view is taken in the above-quoted paper of Prof. Ryan, and was adopted by the authors in the first edition of the present book: they now, however, believe that it is untenable, the only possible effect of cross ampère-turns proper being distortion, and that it is not borne out by Prof. Ryan's curves.

[2] Since the above was written, Dr. Hopkinson has fully established the correctness of his original equations for the back and cross ampère-turns in a further paper reprinted in *Original Papers on Dynamo Machinery*, pp. 134–147.

Not only does the reaction of the armature current increase the necessary weight of copper on the field, but, in the case of compound-wound machines, as explained in Chap. XVI., it renders the regulation for constant potential at varying loads less perfect than it would otherwise be. On the other hand, as mentioned in the same chapter, in the case of series-wound machines for arc lighting, it may be taken advantage of to assist the self-regulation for constant current.

In order to obviate the necessity for shifting the brushes as the load varies, small extra pole-pieces have been employed, these being so placed as to supply a field in which to reverse the armature sections. They are excited by magnetising coils in series with the armature, and are provided with sufficient ampère-turns to slightly more than counterbalance the effect of the armature ampère-turns. When such 'reversing' pole-pieces are used, the brushes do not require to be moved to meet alterations in the load, and a further advantage is that there are no back ampère-turns. Owing, however, to practical difficulties in its application, this device has not as yet met with much favour.[1]

When designing a dynamo the approximate angle of lead at full load must be known not only in order to estimate the value of x_b, but also in order to determine the cross ampère-turns of the armature as bearing on the question of sparking. It further remains for us to consider the closely related question of why the diameter of commutation in machines of good design is seldom brought under the leading pole-corners unless the maximum rated armature current be exceeded.

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[1] *Vide* Swinburne, *Journal Inst. Elec. Eng.* vol. xix. No. 85, pp. 105–107; also *ibid.* No. 86, p. 227, and vol. xx. No. 93, p. 300. See also Sayers' Patents, Nos. 16,572, 1891, and 10,298, 1893.

When the armature current is increased from a very small value, and a lead has to be given to the brushes, the first result of shifting the diameter of commutation forwards away from the vertical line of symmetry is to bring the short-circuited coil within the more clearly marked fringe of lines issuing out of or entering into the leading pole-tips. As the armature current is further increased, this fringe of lines is gradually driven inwards under the leading pole-tips by the increasing cross ampère-turns of the armature, and a further forward lead is necessitated, until with a still larger armature current the advance of the brushes to avoid sparking has brought the short-circuited coil close up under the pole-tips. The current might still be increased, and the brushes might be shifted forwards until the diameter of commutation falls some distance under the pole-pieces; the armature ampère-turns might even then not be so great as to entirely prevent the requirements of p. 372 from being fulfilled, and hence theoretically there should be a position where no sparking would occur. Yet in practice such a position could with difficulty be found, save under somewhat abnormal conditions and in dynamos of special design and peculiarly proportioned.

The variation in the density of the field in the air-gap as we pass from the neutral line up to and under the pole-corners is always more or less abrupt, but when the diameter of commutation is almost or entirely under the leading pole-corners, the change of induction for small movements away from the neutral line, especially on the leading side, becomes very marked—much more so than when little lead is required, and the coils are short-circuited in the middle of the interpolar gap. If, therefore, the conditions for non-sparking require the diameter of commutation to be brought well under the leading pole-corners, a very small difference in the position of the short-circuited coils relatively to the poles means a large difference in the reversing E.M.F. set up in them. Now the inductors of a dynamo are seldom exactly evenly spaced round the circumference of the armature, while it is to be presumed that the commutator sections are all of equal width, and arranged with almost perfect symmetry. Hence, even if the greater number of the sections of the winding are short-circuited in a field of the right induction, the remainder will not occupy exactly the same position when they are short-circuited. As a consequence, therefore, of the diameter of commutation being well under the polar surfaces, any want of regularity in the spacing of the coils will cause sparking, and even if confined to a few coils will render it difficult to work the machine satisfactorily. Apart, however, from any other reasons, a dynamo in which the diameter of commutation at full load is under the pole-pieces, even if the brushes can be adjusted so as to be sparkless, is very sensitive to small changes of load, since a very slight alteration in the armature current materially alters the distribution of the field near the pole-corners. Hence it is decidedly inadvisable to load an armature with so much current that the diameter of commutation is brought under the leading pole-tips.

It may be assumed, therefore, that in machines of good design, having armatures well loaded with ampère-turns, the diameter of commutation at full load is near the leading pole-tips, but if anything is slightly behind them, and on this assumption it is possible to push our conclusions to a further and most important result.[1] So far, the armature cross ampère-turns have been expressed in terms of θ, the angle as fixed by the diameter of commutation (fig. 153), and it has been stated generally that these cross ampère-turns should not exceed $\frac{X_\theta}{1 \cdot 5}$ in drum- or $\frac{X_\theta}{2}$ in ring-machines. Now if the diameter of commutation be just behind the polar-tips, we may for θ substitute ϕ, the arc of embrace of the pole-piece, plus a small margin of allowance for the fairly dense fringe of lines immediately outside the actual polar angle. When designing a dynamo the average density which the lines must have over the whole area of the air-gap is known, and also the length of the gap; hence $X_g = \cdot 8B_g \, . \, 2l_g$ is known at the outset, and substituting this in the equations of pp. 370 ff. we obtain as the maximum permissible number of cross ampère-turns on a bipolar drum armature

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[1] The consequences which follow from the above assumption were first pointed out by Mr. Swinburne in his paper 'On the Theory of Armature Reactions in Dynamos and Motors,' *Journal Inst. Elec. Eng.* vol. xix. part 85, pp. 97–103, and have been further elaborated by Mr. Esson, *Journal Inst. Elec. Eng.* vol. xx. part 93, 'On the Design of Multipolar Dynamos.' To these two papers the authors owe much of that which follows.

$$\frac{C_a}{2} \, . \, \tau \, . \, \frac{\phi}{360^\circ} = \frac{\cdot 8B_g \, . \, 2l_g}{1\cdot 5};$$

or for a ring armature

$$\frac{C_a}{2} \, . \, \tau \, . \, \frac{\phi}{360^\circ} = \frac{\cdot 8B_g \, . \, 2l_g}{2}$$

Thence the maximum number of ampère-turns that the bipolar armature can carry without great liability to sparking is immediately deducible, viz. for drum armatures

$$\frac{C_a}{2} \, . \, \frac{\tau}{2} = \frac{\cdot 8B_g \, . \, 2l_g \times 360^\circ}{3 \times \phi}$$

$$= \frac{96B_g \, . \, 2l_g}{\phi};$$

or for ring armatures

$$\frac{C_a}{2} \, . \, \frac{\tau}{2} = \frac{\cdot 8B_g \, . \, 2l_g \times 360^\circ}{4 \times \phi}$$

$$= \frac{72B_g \, . \, 2l_g}{\phi}.$$

It will be observed that the magnetising ampère-turns of the armature are in a bipolar dynamo equal to half the total armature current multiplied by half the total number of inductors, or, as it may otherwise be expressed, to the single current-sheet formed by the ampères flowing in half the inductors, i.e. in the semicircle corresponding to one pole. Let these magnetising ampère-turns be expressed by A_m, and in any dynamo, whether bi- or multi-polar, let us call the product of the current c flowing in each inductor and the total number of inductors τ, the 'ampère-stream' enveloping the armature, i.e. $A_s = c\tau$, the actual direction of the two or more sheets of current into which the ampère-stream is divided by the brushes being disregarded.[1] Then in the bipolar dynamo the magnetising ampère-turns A_m are equal to half the ampère-stream; consequently the maximum permissible value for the armature ampère-stream will be $A_s = \frac{192B_g \cdot 2l_g}{\phi}$ for drum or $\frac{144B_g \cdot 2l_g}{\phi}$ for ring armatures. We have thus arrived at an expression for the maximum load that a given armature can carry as determined solely by considerations of sparking; all questions of heating are for the present held over, but if the machine is not to be very sensitive to changes of load, and if it is to allow of a slight temporary increase of the armature current above its rated output without sparking, then the product $c\tau$ or A_s must not exceed the values given by the above equations. The rules thus laid down must of course be regarded as more or less elastic, but they will be found of very great service in designing as indicating the nature of the sparking problem and the limits beyond which it is inadvisable to load an armature. Economy in manufacture would dictate the loading of the armature with as large an ampère-stream as possible, since out of the whole cost of the machine, from one third to one half must be credited to the armature, and to increase the output of a given dynamo nothing is easier than to increase the current taken out of the armature. It is now, however, seen that, apart from any question of heating, the output is directly limited by the separate consideration of sparklessness.

It will further be seen that the maximum ampère-stream that can be carried is entirely independent of the length of the armature; it varies directly as the average induction and length of the air-gap, and inversely as the polar angle. If a large ampère-stream is to be carried with a given density of lines and length of air-gap, the polar angle must be reduced; this, however, as a general rule, does not vary much on either side of 130°. Adopting an average value for ϕ, the maximum ampère-stream is thus limited by the values of B_g and l_g. An increase in either or both directly affects the ampère-turns required from the field-winding : it is inadvisable to increase B_g much above 6,000, and unless it be otherwise necessary, l_g need not be more than is just sufficient to admit the height of the insulated inductors, together with the necessary clearance for rotation. In bipolar dynamos the radial depth of iron in the armature core may be taken as proportional to its external diameter ; if, therefore, the induction in cores of different diameters be kept constant, the total number of lines flowing through an armature of given length is proportional to the diameter. Assuming a constant field angle of, say, 140°, the length of the arc of the field is also proportional to the diameter, and therefore the area of the air-gap is altered in the same proportion as the total number of lines, z_a ; that is, the induction in the air-gap B_g remains constant. Now, in bipolar dynamos the ampère-streams carried by armatures of different diameters will be found in practice to vary roughly as the diameter ; hence, if the same non-sparking conditions are to be maintained in the several armatures the length of the air-gap l_g must also be proportional to the diameter.

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[1] The convenience of considering this product will be found later to lie in its ready applicability to the case of bi- and multi-polar machines alike, however they may be wound: this advantage was first brought into prominence by Esson, *Journal Inst. Elec. Eng.* (*loc. cit.*), where, however, the name assigned to it is the 'volume of current,' instead of the 'ampère-stream' of the armature.

Assuming that in a drum armature the radial depth of the core is about ·28 of the diameter (D) in cm., and that the induction in the armature, B_a, is 14,500, the induction in the air-gap is

$$B_g = \frac{\cdot 56D \,.\, B_a \times L \times \cdot 9}{\pi D \,.\, \frac{140^\circ}{360^\circ} \times L} = 6{,}000 \text{ nearly,}$$

which is a fair average value. Further, assuming that $l_g = D \times \cdot 0365$, we obtain for bipolar drum armatures

$$A_s = \frac{192 \times 6{,}000 \times \cdot 073D}{140^\circ} = 600D \quad . \quad . \quad . \quad . \quad (41)$$

Similarly, if we assume that in ring armatures the radial depth of core is ·2D and $B_a = 18{,}000$, we obtain $B_g = 5{,}300$, and thence with the same relation between the length of air-gap and the diameter—

$$A_s = \frac{144 \times 5{,}300 \times {\cdot}073\text{D}}{140^\circ} = 400\text{D} \quad . \quad . \quad . \quad . \qquad (42)$$

for bipolar ring armatures.[1]

It is thus clear that the maximum ampère-streams carried by bipolar armatures may be increased in direct proportion to an increase in the diameter of the core, but only so long as l_g is at the same time increased in the same proportion.

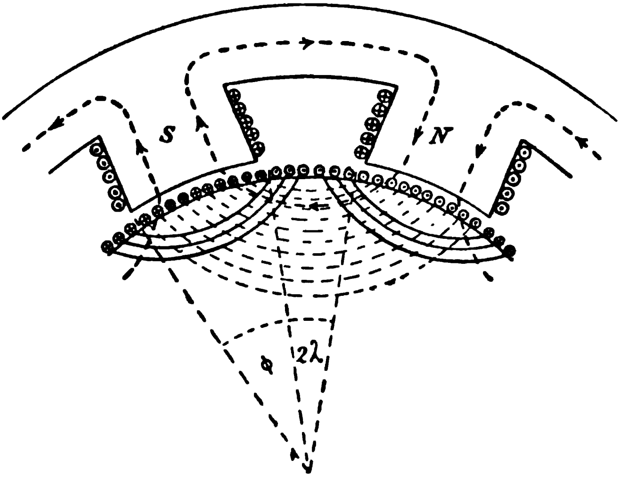

Fig. 155.—Back and cross ampère-turns of multipolar armature.

So far we have spoken only of bipolar dynamos, but the reader will have little difficulty in transferring the same reasoning and its conclusions to multipolar machines. Fig. 155 shows a portion of a multipolar dynamo, the armature inductors being shown connected together to form the two sets of ampère-turns, cross-magnetising and demagnetising. Evidently the cross-magnetising ampère-turns producing a twisting of the field under each pole are equal to $c \times \tau \frac{\phi}{360^\circ}$, and the demagnetising ampère-turns acting on each magnetic circuit are equal to $c \times \tau \,.\, \frac{2\lambda}{360^\circ}$; the two together form the magnetising ampère-turns of the armature $A_m = \frac{c \times \tau}{P}$. In the case of the bipolar dynamo the magnetising ampère-turns were equal to the current sheet corresponding to one pole, and the same is now seen to be the case in multipolar armatures, whatever may be the method by which its inductors are connected together. Thus the 4-pole armature is divisible into four current-sheets, the direction of adjacent sheets being opposite, and similarly the 6-pole armature is enveloped by six current-sheets; in all cases the value of one current-sheet is the magnetising ampère-turns on the armature. If the winding be divided into as many parallels as there are poles, $c = \frac{C_a}{P}$, and the magnetising ampère-turns are $A_m = \frac{C_a \times \tau}{P^2}$; if the armature be series-wound in a multipolar field, $A_m = \frac{C_a \times \tau}{2P}$, but in either case $A_m = \frac{A_s}{P}$. The maximum permissible number of cross ampère-turns is again obtained by equating $c \times \tau \frac{\phi}{360^\circ}$ to $\frac{\cdot 8 B_g \,.\, 2 l_g}{1 \cdot 5}$, or $\frac{\cdot 8 B_g \,.\, 2 l_g}{2}$, according as the armature be ring- or drum-wound. Taking the inferior limit

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[1] These same values are given by Esson, *Journal Inst. Elec. Eng.* vol. xx. part 93, p. 270, but are there deduced from somewhat different premisses (p. 275).

$$c \times \tau \frac{\phi}{360^\circ} = A_m \,.\, P \,.\, \frac{\phi}{360^\circ} = \frac{\cdot 8_g \,.\, 2 l_g}{2};$$

whence $$A_m = \frac{\cdot 8 B_g \,.\, 2 l_g \times 360^\circ}{2 P \phi} = \frac{144 B_g \,.\, 2 l_g}{P \phi}.$$

Now if the number of poles, P, be increased, the angle, ϕ, subtended by each must be correspondingly reduced, and if this reduction be in exact proportion to the increase of P, $P\phi$ is constant.

It thence follows that, so long as the length and induction of the air-gaps are maintained at the same values, the magnetising ampère-turns permissible on a given armature are a fixed quantity, and totally independent of the number of poles by which it is surrounded. But $A_s = P \cdot A_m$, and therefore for a given maximum A_m, the value which the ampère-stream may have is directly proportional to the number of poles. If the number of poles be increased from two to four or six, and the angle subtended by each be proportionately reduced, the maximum ampère-stream on a given armature with given induction and length of air-gap may be doubled or trebled. We have thus arrived at an expression for the maximum permissible ampère-stream of any armature which is applicable to any number of poles, and, further, is applicable whether the inductors are connected together into two parallels or as many parallels as there are poles. In any closed-coil armature the cross magnetising ampère-turns acting on each pole are equal to $A_s \frac{\phi}{360^\circ}$, and the demagnetising ampère-turns acting on each magnetic circuit are $A_s \frac{2\lambda}{360^\circ}$. The maximum cross ampère-turns must not exceed $\frac{\cdot 8 B_g \cdot 2l_g}{2}$ in rings, and therefore the maximum permissible ampère-stream in all cases is

$$A_s = \frac{144 B_g \cdot 2l_g}{\phi} \text{ for ring armatures, } \quad . \quad . \quad . \quad (43)$$

or

$$\frac{192 B_g \cdot 2l_g}{\phi} \text{ for drum armatures.}^{1} \quad . \quad . \quad . \quad (44)$$

If with an armature of given diameter and length the poles are increased in number, and ϕ is similarly decreased, then so long as B_g is kept constant, the total number of lines cut by each inductor in one revolution, viz. $Z_a P$, is independent of the number of poles. The output, therefore, can only be increased by increasing the ampères that flow through the inductors. It will now be seen wherein the advantage of a multipolar field for large outputs lies ;

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[1] *Vide* Esson, *Journal Inst. Elec. Eng.* vol. xx. part 93, p. 267.

whereas an increased ampère-stream on a bipolar armature has been shown to necessitate a corresponding increase in the length of the air-gap, in the case of a multipolar field a comparatively small air-gap may be retained even with a large diameter and a large value for the ampère-stream.[1]

The tendency to sparking at the brushes is considerably greater in ring-wound than in drum-wound armatures, and on this account the maximum cross ampère-turns of a ring have in our previous equations been maintained at a lower value as compared with x_g than in the drum. The reasons for this inferiority are closely allied to the fact that a greater angle of lead is required in the ring-wound armature than in a drum of similar dimensions, wound with the same number of active inductors and divided into the same number of sections. The primary cause of this must be sought in the greater resistance and the inductance of ring loops as compared with drum loops, when both comprise the same number of inductors. Unless the armature be of a very large diameter as compared with its length, the resistance of a pair of ring loops is greater than the resistance of the equivalent single loop on the drum armature, and in consequence the voltage and strength of reversing field required for a given current round the ring loops are greater. A further cause lies in the disturbing influence of the internal field of lines produced within the ring by the armature ampère-turns. Fig. 150 shows that the current-turns on the two parallel halves of the ring unite to propel a certain number of lines across the air from the one side to the other of the interior of the ring along the diameter of commutation. Any such lines form an entirely separate self-induced field which does not blend with the main field, but exists in nature within the ring in addition to any reaction of the armature conductors on the main field. The cutting of this internal field by the inner connecting wires of the short-circuited sections will be found to set up in them an E.M.F. opposing the direction of the E.M.F. induced in the external wires, and therefore again a stronger reversing field is required in the ring than in the drum. To a certain extent, the tendency towards greater inductance in the ring method of winding is counteracted by the fact that the armature core has usually less radial depth than in the drum, and is worked at a much higher induction. The armature induction has its highest value at the diameter of commutation, and in a ring is usually about 17,000 or more; hence the short-circuiting of one section produces but a small variation in the number of lines through it, owing to the saturation of the iron core; it is, in fact, inadvisable to work with a lower induction in the ring armature owing to the increased self-induced field which would result. Thus, on the whole, even if the commutator sections in the Gramme ring are double those of the drum, the resistance and inductance of the single ring loop are greater than those of the half drum loop which is on the surface of the armature. Hence a stronger reversing field is required for the ring armature, and the lead of the brushes must be greater. This in itself need not give rise to sparking, but for very slight changes of the load the inaccuracy of the adjustment of the brushes produces a more violent sparking in the ring than in the drum armature.[1]

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[1] Esson, *loc. cit.* pp. 271, 275.

An additional tendency to sparking in the case of single magnet dynamos with ring armatures, is due to a want of symmetry in their fields, produced by the causes described on p. 281. When the field distribution is unsymmetrical on the two sides of the armature, the correct position of the two brushes will not be at opposite ends of a diameter, but one will require more lead than the other: the prejudicial effect of this, in so far as it divides the armature winding into unequal portions, has been described previously, and with an even number of commutator segments it may further produce sparking owing to the brushes not short-circuiting sections simultaneously. Apart, however, from any such reason, if the field be unsymmetrical at full load, a very slight change in the armature current produces considerable alteration in its distribution, destroys the balance required for non-sparking, and leaves the brushes in an incorrect position. Again, some of the devices mentioned on p. 283 for neutralising the magnetic pull on armatures may effect that object fairly well, but are somewhat apt to produce a different distribution of the field in the top and bottom interpolar gaps when the cross magnetising forces of a large armature current are called into play.[1]

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[1] Swinburne, *Journal Inst. Elec. Eng.* vol. xix. part 85, p. 109.

[2] *Vide* Swinburne, 'The Theory of Armature Reactions in Dynamos and Motors,' *Journal Inst. Elec. Eng.* vol. xix. No. 85, pp. 95, 96.

Our previous equations have so far furnished us with a limiting value for the ampère-stream of an armature, beyond which it is inadvisable to go; since, if this maximum be exceeded, the dynamo will at least be sensitive to changes of load, and will probably in everyday working spark badly. The question of the armature ampère-stream does not, however, by any means exhaust the problem, so far as to determine whether a machine will work successfully without sparking at full load. We must further consider the extent to which the armature winding is subdivided into sections by the commutator segments, for upon this largely depends the magnetic effect of each short-circuited coil, owing to the proportion which its ampère-turns bear to the total ampère-turns. For a given number of inductors, and given normal armature current, the reaction of each section of the winding will depend upon the number of turns included within it; the fewer the number of commutator segments, the larger the number of turns per section, and the greater its effect on the field. Thus an armature with 160 inductors divided into 40 sections, each containing four inductors, will not be so free from sparking as the same armature with the winding divided into 80 sections of two inductors each: the reaction of the coil of four turns will be more than twice as great as that of the coil with two turns, and it will therefore require to be reversed in a stronger field, will require more lead, and in every way will be more sensitive to slight changes of load. Further, the proportion which the ampère-turns of the coil with four turns bear to the total number being so much greater, the short-circuiting of two such coils simultaneously if not correctly performed will produce a much greater quivering or vibration of the field, and this effect will combine to render the tendency to spark greater.

The subdivision of the armature winding into a large number of small sections is therefore an essential requirement for the avoidance of sparking, and the extent to which it may advantage-

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[1] *Vide* Esson, *Journal Inst. Elec. Eng.* vol. xix. No. 86, p. 280.

ously be carried is only limited by the question of expense in manufacture and the difficulty of dealing with very thin commutator segments. The armatures of closed-coil machines for high voltages of from 500 to 1,500 volts necessarily have a considerable number of turns per section, since they are wound with a large number of inductors; hence, even though the current of such machines may be comparatively small, special care is required to render them sparkless in working. In the case of ring-machines for moderate voltages of from 50 to 200 volts, the number of turns per commutator segment seldom exceeds four if the current in each inductor be greater than 10 ampères, while if this current be greater than 30 to 40 ampères, it is generally reduced to three or two turns per section, until finally, with over 70 ampères, it is advisable to adopt as many commutator segments as there are turns. In drum armatures, with two loops per section, containing four inductors, the ampères in an inductor may be raised to about 50; for currents above this, each section seldom consists of more than one loop containing two inductors. In designing large bipolar armatures, especially if for comparatively low voltages, we are met by the further fact that there is a minimum limit beyond which it is inadvisable to reduce the total number of commutator segments—a limit which may be set at about 50. This difficulty is especially felt in the case of large drum armatures, where the total number of inductors required may work out to less than 100; the maximum possible number of segments is thus less than 50, even when one section of the winding is limited to a single loop of two inductors. The required armature current may be as much as 1,000 ampères, and therefore each section when short-circuited is carrying 500 ampères. The magnetic reaction of such a loop upon the field is very great, and the short-circuiting of two such loops out of a total numbering less than 50 has proportionately great effect; hence, if the reversing field in which it is short-circuited be not of the precise strength required, the sparking will be violent. Under such circumstances two courses are open to the designer. The one is to subdivide the armature winding into more than two parallel circuits from brush to brush, the segments of the two or more windings being formed into one commutator, and each brush having sufficient width to simultaneously cover as many segments as there are pairs of paths from brush to brush.[1] The advantage thereby secured is that the commutation of the two or more subdivisions is not contemporaneous, but one is always in advance of the other: hence at any moment the magnetic reaction of the pair on the field is less than that of the simple undivided loop, and any quivering of the field due to incorrect commutation, although more continuous, is reduced in amount. Further, the eddy-currents in the armature bars are reduced. The alternative course is to entirely change the design, and adopt a multipolar armature with its winding arranged in as many parallels as there are poles.

The brushes of closed-coil dynamos are now most commonly made of copper or brass gauze of a fine mesh: this is folded over on itself, stitched together with fine copper wire, and pressed flat. The one end is then lightly soldered together, while the other, which is cut diagonally across the mesh to avoid fraying at the tip, is trimmed to an angle so as to bed evenly on the commutator surface over its entire length and thickness. Less frequently brushes are made of fine copper wires or thin sheets of copper soldered together at one end. An average thickness for the insulating strips between the segments of smooth-surface commutators is $\frac{1}{25}$ of an inch, and the amount by which the width of contact of the brush-tip should exceed this depends upon the length of time during which it is advisable to short-circuit each section in order to minimise the sparking. With a large arc of contact, if the neutral zone between the poles be narrow, or if the coil be of considerable width, an excessive current may be generated at the commencement of the short-circuit; while if the brush be too thin, the time during which a section is short-circuited is never long enough to allow the current to be reversed and raised again to its normal strength. As a general rule, it is best not to short-circuit more than one section at a time, unless it be for a mere instant, and therefore (except in the case of double- or treble-wound armatures, alluded to above) the width of brush contact should not exceed the width of one segment and one insulating strip by more than a fraction of a segment. In ordinary cases this is obtained by using a brush of $\frac{1}{4}''$ or $\frac{5}{16}''$ thickness, so set that it makes an angle of about 45° with a line drawn tangentially to the commutator surface.

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[1] *Vide* Mather and Platt's Patent, 1884, No. 768; and Nebel's Patents, 1890, No. 4824; and 1891, No. 20840.

The contact produced by the pressure of the flexible brush on the commutator surface being at best somewhat imperfect, an appreciable resistance is presented to the passage of the current into or out of the brush. Hence with large currents it is important that the total area of brush-contact should be sufficiently large to pass the current without undue heating of the commutator or brushes. Assuming that the brushes are set approximately as described above, a current density of 200 ampères per square inch of brush cross-section may be taken as a normal allowance, insuring sufficient contact-surface on the commutator. In some cases carbon brushes are employed, since by their greater resistance of contact and non-metallic nature they assist in suppressing sparking: the current-density, however, must not be allowed to exceed 50 ampères per square inch, and hence for large currents a great number of brushes and length of commutation surface are required. The width in which brushes are usually made varies from ¾″ to 2″, but the latter dimension is seldom exceeded, since it then becomes troublesome to maintain proper contact along its entire bearing surface. Hence, to carry any considerable current, two or more brushes are mounted in line on each arm of the rocking-bar, forming in effect one wide brush. This arrangement further renders it possible to adjust each brush separately, or even to remove one temporarily, without interrupting the current; and this advantage is so great that every dynamo which is more than a toy should invariably be furnished with at least two brushes on each arm, each brush being of such width that, if one be removed, the other can temporarily carry the whole current.

The pressure of the brush-tips on the commutator may be adjusted by altering the tension of the 'hold-on' spring. 'Jumping' of the brushes, due to vibration of the machine when running, must be carefully avoided, since it will give rise to sparking, and on this account a substantial rocking-bar with strong but light brush-holders, capable of being firmly fastened, is an essential part of a well-designed and well-built dynamo. The brushes should then bear lightly and evenly on the commutator. Any pressure beyond this should be avoided, since it will not only cause increased wear, but may itself create sparking. Occasionally, one or two segments in a commutator wear down below the general cylindrical surface of the rest, and form what is known as a 'flat;' as the brushes pass over the faulty spot, the circuit is momentarily broken, and sparking occurs, which rapidly increases the evil. The development of a flat may be caused by inequality in the wear-resisting properties of the segments, but it may also often be due in the first instance to sparking. Owing to a want of uniformity in the spacing of the winding on the armature surface, a particular section may be short-circuited when in an incorrect position; its passage under the brushes is then accompanied by sparking, and the segment to which it is attached becomes worn. If an armature wire is broken or its connection to the commutator becomes loose, violent sparking may be set up, and the faulty coil may then be located by running the machine until one segment becomes pitted by the sparks.

Passing to the case of open-coil armatures, we are met by many of the same phenomena as occur in closed-coil armatures, save that the brushes must now be placed near to the line of maximum field, and that there is no short-circuiting of a coil as it passes from one field to another. The conditions for non-sparking will be readily apparent from Chap. X. Each coil should cease to be connected in parallel with its more active neighbour at the exact moment when the current flowing in the former has been stopped by the higher E.M.F. of the latter. We are now, however, able to see why, in the diagrams of that chapter, the line *m m* is shown displaced forwards in the direction of rotation, and is not exactly at right angles to a line of symmetry drawn vertically between the two poles. The reaction of the armature current in open-coil dynamos produces effects of exactly the same nature as in closed-coil dynamos; the field is displaced forwards, and consequently, when the armature current is considerable, the line of maximum action, while still approximately at right angles to the neutral line *n n*, no longer coincides with a line passing through the centres of the poles (cp. Chap. XIX. II. *Open-coil Machines*). It will readily be understood that in such machines, owing to the small number of coils the abruptness of the changes made in their mutual connections, and the high voltages induced per coil, a certain amount of sparking is almost unavoidable. Such sparking, however, is not by any means so detrimental as might be thought; the commutator segments being insulated on the surface by air-gaps, the sparks pass from the overhanging edge of the segment to the brush, and do not, therefore, destroy the surface of the commutator. Hence the apparently violent sparking of open-coil armatures is not nearly so injurious to their commutators as very much less sparking would be when acting on the smooth-surface commutators of closed coil armatures.

The brushes of high-tension open coil dynamos are usually made of thin sheet copper of considerable springiness. Each brush is slit up longitudinally for some distance from its bearing end, so as to form three or four teeth, and is then set almost tangentially to the commutator surface.

CHAPTER XVIII

THE HEATING OF DYNAMOS

WITH the exception of 'sparking,' probably no subject is of such importance, alike to the designer, the purchaser, and the attendant, as the question of the heating of dynamos. The continuous generation of heat in the armature and magnet-windings of all dynamos, so long as they are at work, is a necessary consequence of the passage of the current through their coils, and the appearance of this heat implies that a corresponding amount of energy is 'lost,' in so far as no useful work is derived therefrom. All that can be done from the point of view of economy is to minimise the amount of the heat which is thus generated, so as to obtain a reasonably high efficiency, such as is suited to the circumstances of any given case. Apart, however, from the question of the amount of heat produced every second, or its rate of generation in watts, there is the further and equally important question of the temperature to which any part of the dynamo is thereby raised. Whether it be the field-magnet coils or the armature which is the source of heat in question, when the machine is set to work the temperature of their mass gradually and continuously rises above the temperature of the surrounding air, until, finally, the rate at which the heat is generated is balanced by the rate at which it is carried off by radiation, convection, and conduction : after attaining the temperature which satisfies this condition, no further rise takes place. Evidently, therefore, the *rise* of temperature depends essentially upon the amount of cooling surface provided, and its actual effectiveness in dissipating heat, and, this being so, it follows that it may be regulated so as not to exceed a certain maximum, if the amount of cooling surface be duly proportioned to the watts expended. Before, however, we enter upon this point, we have first to consider in what way a high temperature is actually detrimental to a dynamo, and also the closely connected question, in what way a large range of temperature is injurious to its working: it will be found that they are so in three ways.

In the first place, the higher the temperature of any portion of the electrical circuit of a dynamo, the greater is the loss of energy, due to the passage of a given current through it. For each degree Fahrenheit that the temperature of a piece of copper rises, its electrical resistance rises about 0·21 per cent.; if, therefore, its temperature be raised from 60° F. to 120° F., the increase in its resistance will be 12·6 per cent. of its resistance at 60° F. The limits of temperature within which dynamos are worked under average conditions may be taken as 60° F. and 120° F., the former corresponding with an average value for the temperature of surrounding air, and the latter to an ultimate temperature which it is frequent for the coils to attain when the dynamo is worked continuously, or for many hours together, at its normal output. Hence the increase in the resistance of the copper wire on an armature which rises 60° F. is as much as 12·6 per cent., or R_a hot $= 1{\cdot}126\ R_a$ cold; and when a given current is passed through the armature, the loss of volts over its resistance when hot is, of course, increased by the same percentage. Not only must this increase be allowed for in the design, but it further involves an equally increased loss of energy. Again, the heating of the series coils on the field-magnets of series- or compound-wound dynamos increases the loss of power in them for the same output at the terminals; while in the case of separately-excited or shunt machines the E.M.F. at the ends of the exciting coils must be raised if the same number of ampères is to be passed through their turns when hot as when cold, so that, again, more energy is expended after the machine has become heated.

Armatures are seldom wound with more than one or two layers of wire, and therefore the temperature of the outside, as measured by a thermometer placed in contact with the outer insulating covering of the conductors, may be taken as indicating approximately the actual temperature of the conductors themselves.

But in the case of a large number of layers wound closely over one another, as in a field-magnet coil, an appreciable difference of temperature is required to produce the flow of heat to the outer surface, by conduction, partly through the length of the wire itself, and partly from layer to layer across the intervening cotton covering, this latter being a bad thermal conductor. Hence the temperature of the inner layers of the winding is considerably higher than that of those on the surface, and in calculating the resistance of a coil of many layers, the average temperature of the whole length of wire must be taken as higher than that of the outer layer as measured by a thermometer applied externally. The allowance to be made for the increased temperature will vary with the depth of winding and the closeness with which the coil fits the magnet; for the field-magnet coils of ordinary machines, the ratio of the actual increase in resistance to the increase as calculated from the temperature of the surface varies from about 1·25 for a depth of winding of ·75″ to about 1·5 for a depth of 2½″ or 3″. Thus, if the depth of winding in a field-magnet coil be 2½ inches, and the temperature of its surface rises 55° F., the actual increase in its resistance will be about $0·21 \times 55° \times 1·5 = 17·3$ per cent. of its resistance when cold.[1] Such considerable percentages show that the effects of heating must on no account be neglected in designing machines or in estimating their efficiency.

In the second place, the rise in temperature of a dynamo when at work produces a disadvantageous effect upon the regulation of its voltage. As already mentioned, the separately-excited machine requires the E.M.F. applied to its exciting coils to be raised, if the same number of ampère-turns is to be maintained when the field-winding is hot as when it is cold; while if the terminal E.M.F. of the dynamo is to be kept constant, its internal E.M.F. must be increased in order to compensate for the increased loss of volts over the heated armature coils, and this necessitates either a further increase in the exciting E.M.F. or a higher speed of rotation. Similarly, the self-regulation of the compound-wound machine for constant potential is injuriously affected by the differences in the resistances of its shunt, series, and armature coils when hot and when cold: if correctly compounded when cold, the constancy of the potential must necessarily be inferior when it is hot, or *vice versâ*. In fact, in designing compound-wound machines it is especially important that the rise of temperature of the field-magnet winding be not so great as to seriously affect the compounding action of the two sets of coils, and it should preferably be limited to about 50° F. at the most.

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[1] Compare pp. 319, 329, and *Electrician*, October 11, 1895, 'The Heating of Dynamos.'

Thirdly, and of chief importance, if the temperature of any coil becomes very high, the cotton or other fibrous material commonly used for the insulating covering of the copper wires will be burnt or charred; the insulation between neighbouring turns is thus broken down, and the short-circuiting which ensues is only terminated by complete collapse. A 'burnt-out' armature may be the result of an accidental short-circuiting of the machine, the heat from the excessive current almost instantaneously raising the temperature so much as to literally burn the insulation. Quite apart, however, from such accidental heating, the result of continually working a machine at a high temperature is a gradual deterioration in the toughness and mechanical strength of the insulating coverings of the wires. All materials in ordinary use for insulating purposes are alike subject to this gradual decay. Slowly but surely they become charred and rotten, the cotton or calico crumbling away when touched, and the blackened paper and fibre becoming excessively brittle; so that, although the insulation resistances may still remain very high, the liability to a breakdown is enormously increased. When old Gramme armatures are taken to pieces for repairs, the cotton covering of the internal layers of wire which have been continuously subjected to high temperatures may be found to be a mere charred powder, which can be wiped away with the finger.

Finally, among the drawbacks incidental to a great variation in temperature of the armature may be mentioned the expansion and contraction which take place in its metal framework as it heats or cools; for example, if the rise be very great, heat is conducted along the armature shaft, and its expansion, either within the bearings, or lengthwise between the collars that limit the end-play, may set up undue friction, or even cause it to seize. Under ordinary circumstances, however, such troubles are rather to be charged to bad workmanship, and hardly require to be separately considered here.

It will now be sufficiently evident that the ultimate temperature attained by a dynamo when at work is, of all questions, the most important. If its wires are insulated with the usual materials, the frequent attainment of a very high temperature is incompatible with a long life, and this consideration of durability leads us to fix a maximum temperature, which no part of the machine should exceed in continuous working.

Some inventors have endeavoured to make use of insulating materials which may by contrast with cotton and paper be called fireproof; the wires may then be allowed to reach very high temperatures, such as 250° F., provided that considerations of efficiency allow of it. But such attempts have not met with any great success; even if the insulation be satisfactory as regards its mechanical and heat-resisting qualities, it may occupy too much room; further, the heat gradually spreads to the commutator and brushes, while, lastly, the second consideration of p. 396, viz. the great difference in the electrical resistances when hot and when cold, introduces difficulties in the regulation of the voltage. Returning, therefore, to the more usual case of armatures and field-magnet coils insulated with cotton, silk, tape, fibre, or paper, the maximum temperature which they should be allowed to attain in continuous work cannot be set higher than about 190° F. at the most, and even then it must be remembered that, if a coil consist of a large number of layers, the limiting temperature of the surface must be appreciably lower.

Having thus fixed upon a maximum permissible temperature, it is at once evident that the number of degrees by which a dynamo may be allowed to rise in temperature is entirely dependent upon the starting-point from which this rise takes place; in other words, upon the temperature of the surrounding air during the working of the dynamo. Thus, in the case of dynamos working in hot atmospheres, for instance, in the engine-room of a steamer in the tropics, where the normal temperature may be, and frequently is, as much as 115° F., the permissible rise is much smaller than in the case of a dynamo working in a well-ventilated central station on land, where the temperature will seldom exceed 70° F. Now the output of a dynamo is dependent upon the *rise* in temperature which is permitted, and it therefore follows that the output is indirectly limited by the normal temperature in which it is to work, and from which the rise is reckoned. It is seldom, however, that in specifications of dynamos the actual limits of temperature are stated, inasmuch as they must necessarily be somewhat vague, and it is more frequent to find the maximum *rise* of temperature alone specified, this rise being such as will not be likely to endanger the insulation under ordinary conditions. Thus it is generally stipulated that the machine must run satisfactorily during a test of six hours' duration at full load without undue heating of any part, and that at the end of the run the temperature of the armature or field-winding must not exceed the temperature of the surrounding air by more than 70° F. The rise of temperature is usually obtained by comparison of the readings of two thermometers, the one registering the temperature of the room within a few feet of the dynamo, and the other placed in contact with the exterior of the winding, and therefore indicating the temperature of the surface. Hence, assuming a normal temperature of 70° F. for the surrounding air, it will be seen that the maximum temperature which the surface of the coils may attain is 140° F., and this limit is found to give thoroughly satisfactory results in practice.[1]

The great importance of the safe *rise* of temperature lies in the fact that it determines the output of any dynamo, and this it does chiefly by limiting the maximum current that may be passed through a given armature in continuous working. Of course, the permissible armature current may be determined in the case of continuous-current dynamos by the question of sparking as explained in Chap. XVII., or in the case of alternators by the self-induction of the armature causing too great a drop in the terminal volts (p. 501); but in by far the largest number of cases these limiting considerations do not take effect until the output is decisively fixed by the serious heating which would occur in everyday working if the armature current were increased.

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[1] In the specifications of the British Admiralty, from which the rise of 70° F. has largely been copied, the temperature of the room during the test run is further given as 120° F., and hence the permitted final temperature would be as much as 190°. The high temperature thus permitted is, however, largely discounted by another stipulation, viz. that at the end of the six hours' run, and *one minute after stopping*, no accessible part of the dynamo shall have attained a temperature more than 30° F. above that of the room. Although this latter condition cannot be defended on strictly scientific grounds, still the combined effect of these two clauses, as well as of other conditions, has sufficed to secure for the Admiralty dynamos of the very best quality. Indeed, the Admiralty specification for combined sets of engines and dynamos has been largely instrumental in formulating the requirements of engineers in general, and is well worth studying as the result of the accumulated experience of many years. In the United States, dynamos are frequently worked at considerably higher temperatures than would be tolerated in England.

The rise of temperature deemed permissible by different makers varies somewhat according to the different ultimate temperatures and starting-points which they have in view, and consequently the normal output at which one and the same dynamo would be rated for continuous working will likewise be different; but under average circumstances a rise of 70° F. may be taken as a thoroughly practical guide in determining the output of any dynamo.

The time taken for the armature and field-coils of a dynamo to attain their ultimate temperatures is dependent upon their size. At the commencement of the run the rate at which heat is produced is almost as great as when the machine has attained its maximum temperature, but part of this heat is absorbed in raising the temperature, not only of the winding, but also of the core or magnet on which it is wound. The larger this mass which has to be heated, the longer will be the time taken in raising its temperature, until the final state is reached in which the heat has to be dissipated almost entirely by radiation and convection-currents of the surrounding air. A certain difference of temperature between the cooling surfaces and the surrounding air must then have been established sufficient to enable the heated masses to part with their heat as fast as it is generated. If a machine be run with constant load for several hours, and the rise of temperature of armature or field-winding, as taken at intervals of, say, one hour, be plotted as ordinates to a horizontal axis of time, the curve so obtained will be found to gradually bend over, and become more and more flat; finally the readings will fall almost in a straight and horizontal line, showing that a steady temperature has then been attained. Such an experiment enables us to be certain that the final state has been reached, and the time which we find that a dynamo of given size takes to attain its maximum temperature will serve as a clue to the number of hours for which a machine of similar size should be run at full load in order to test it thoroughly. While a ring armature, of which the core dimensions are 9″ diameter × 12″ length, will attain its final temperature after about four hours' run at full load, a drum armature 15″ diameter × 19″ long will barely reach its maximum rise in six hours, and larger machines will require to be run for still longer periods. The thermometer employed to measure the temperature of the surfaces should preferably be of a sensitive chemical type, the graduated glass stem having a very fine bore, and the small cylindrical bulb containing but little mercury; after being laid or held in close contact with the winding, it should be covered with some material (such as a piece of rag) which is a bad conductor of heat, and then allowed to remain undisturbed for several minutes until the mercury entirely ceases to rise. When a rotating armature is stopped at the end of a run or at any time for the purpose of taking thermometer readings, the temperature of its exposed surface continues to rise for some minutes after the rotation has ceased. The generation of heat ceases with the rotation, but the simultaneous cessation of the air-currents set up by the revolving parts virtually amounts to a large reduction in the cooling power of the surfaces, and in consequence the fall of temperature between the inner or hottest parts and the external surface is reduced; in other words, the outside rises by conduction to a higher temperature, more nearly the same as that of the centre of heat. Both the cotton insulation of the wires and the shellac or other varnish with which they are coated are bad thermal conductors, and it therefore takes an appreciable time for the heat to penetrate through them to the outside of the armature. It may further be mentioned that, owing to the low thermal conductivity of insulating materials in general, if a heated armature be cursorily felt with the hand, the bare metal of the binding-wires, or even of the commutator, will appear hotter than the insulated wires, and these latter may seem comparatively cool to the touch: any such conclusion is, however, entirely illusory, and the continued application of the hand to the conductors will usually suffice to correct the error.

We have now to consider the predetermination of the rise of temperature in a dynamo—a subject which is naturally divisible into two portions, dealing respectively with the field-magnet coils and the armature.

In the case of the exciting-coils the question is fairly easy of solution. The heat generated in them is simply that due to the passage of the current through their resistance, and the watts expended over them are therefore easily calculated as the product of the square of the current into the resistance. Further, the coils are usually stationary, and are therefore by no means so much affected by currents in the surrounding air as is the rotating armature. Hence their rise of temperature is solely dependent on the ratio which the watts bear to their cooling surface, and apart from external draughts can be very approximately predicted. We have already, in Chap. XIV. p. 264, given a basis for calculating the cooling surface of a coil, or at all events a surface to which the cooling effect may approximately be regarded as proportional. A certain difference must, however, be drawn, according as the coil fits closely to the magnet all round its inner surface, or is wound on a former of which one or more of the sides stand away from the iron magnet. In the latter case a free circulation of air is possible between the former and the magnet, and if they be vertical a slight current of air will thus ascend and materially assist in carrying off the heat by convection. Against the advantage of this greater cooling power must, however, be set the fact that the length of each turn of wire is slightly increased by the margin of clearance between the bobbin and the magnet core. In the case of close-fitting coils of double-cotton-covered wire varnished with shellac on the outside, the final rise of temperature of the exterior in degrees Fahrenheit will be approximately

$$t^\circ \text{ F.} = \frac{90 \times \text{w}}{\text{s}_c}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

where w=the rate of generation of heat in the coil in watts, and s_c=the cooling surface of the coil in square inches measured over the external surface only of the wire. If an effective air-passage is obtained between at least one side of the former and the magnet, the constant is reduced, and we have

$$t^\circ \text{ F.} = \frac{80 \times w}{s_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45a)$$

It will readily be understood that the values which should be assigned to the numerical coefficient in these formulæ are considerably affected by the shape of the machine and the position of its coils. The above equations may, however, be taken as applicable to stationary field-coils of usual size and under ordinary conditions; that is, when they are well exposed, and to a slight extent cooled by the draughts which an armature revolving in close proximity sets up in the surrounding air. In both cases the cooling surface of the end-flanges has not been reckoned, and as their influence becomes more important the greater the depth of the winding, the temperatures deduced from our equations will be somewhat too high in the case of large machines having coils more than 2½ inches deep. If the permissible rise of temperature be taken as about 50°–60° F., it will be seen that 1·5 sq. inches of cooling surface must be allowed per watt, and this ratio may frequently be found in practice. A greater rise, although perhaps permissible on the score of the final temperature reached, is seldom advisable owing to the difference which it causes in the exciting power according to the time during which the machine has been at work.

When we pass to the question of the final rise of temperature in an armature, the problem before us is far more complex, and this for two reasons. In the first place, it is difficult to calculate with accuracy the watts actually expended in heating the armature. The sources of the heat are threefold, viz. (1) the loss of electrical energy due to the passage of the total current through the armature resistance; (2) eddy or Foucault currents set up in the copper winding, in the armature core, or in its supporting framework, including the shaft; and (3) the magnetic loss due to hysteresis. The first and third of these can be calculated with a close degree of approximation, but the second loss is very largely indeterminate, inasmuch as it is dependent on a number of conditions, such as the width of the armature inductors, the strength of the field, and the speed.

In the second place, the rate at which a given armature can part with the heat generated within it is most materially affected by its speed of rotation, and also by the shape and number of the pole-pieces which surround it, in so far as they allow of or actively promote a more or less free circulation of the air over the surface of the winding. The exact effect of either of these causes does not admit of very accurate generalisation, since it necessarily varies under different conditions. How important a part is played by the peripheral speed of a rotating armature in increasing its cooling power is at once evident from a comparison of the watts which it can dissipate with the watts which a stationary field-coil can dissipate: if their cooling surfaces be reckoned on a similar basis, and the rotating armature have a peripheral speed of 2,000 ft. per minute, roughly speaking, it will be found that the latter can dissipate nearly twice as many watts per square inch of cooling surface for the same rise of temperature in the two cases. But even the increase in the cooling power, due to a given peripheral speed, depends largely upon the nature of the winding, the construction of the armature core, the disposition of the pole-pieces, and the ratio of their area to the total cooling surface.[1] If the surface of the armature winding be broken, so that the air can play upon more than the outer face of the inductors, or even reach freely to the core, the cooling action is increased. Again, the numerous poles of large multipolar machines break up the currents of air, and this effect is very marked in the case of discoidal-ring alternators, revolving between a number of poles on either side of the armature. It is therefore impossible to lay down any general formula, expressing the cooling power of a given surface at different speeds, which will meet the case of entirely different types of machines, and all equations connecting rise of temperature with a certain ratio of watts per square inch must necessarily be only approximate.

Confining our attention for the present to ordinary 2-pole continuous-current dynamos, with ring or drum armatures wound almost uniformly over their entire surface, we shall find that a number of formulæ have been published for the determination of their rise in temperature. Not one, however, can be regarded as truly reliable, or as possessing even the approximate accuracy which may fairly be expected; since when applied under conditions differing apparently but slightly from those contemplated by their inventors, they may lead to results which were certainly far from being contemplated. The reason for this is to be found in the fact that the eddy-current loss has never been assigned a separate and prominent place in calculating the total watts expended over the armature; it has in most cases been entirely left out of sight, owing to the difficulties both of ascertaining its exact value in a machine under test and of estimating its probable amount when designing a new machine. In order that the reader may judge of the importance of the eddy-current loss the following table is given, which embodies the results of tests made on ordinary commercial machines, constructed by different well-known makers: the losses are those which occur when the dynamos are run at their rated outputs and speeds, and in each case the eddy-current loss has been determined by actual experiment.

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[1] See *Electrical Engineer*, August 10, 1894.

No.	—	Dimensions of core in inches		Output in kilowatts	Volts	Amps.	Revs. per min.	Periph. speed in ft. per min.	Watts			Z_a in C.G.S. lines $\times 10^6$	B_a	B_g
		Dia.	length						$C_a^2R_a$	Hyst.	Eddies			
1	Drum	20	36	120	135	900	375	2,100	2,300	694	2,730	30·6	11,400	5,300
2	Drum	10	12	15	130	115	865	2,420	520	187	530	4·65	17,500	4,470
3	Ring	7½	7	3·5	130	27	1,250	2,600	167	75	265	1·9	18,600	4,200

To these may be added[1] the three following, of which full accounts have been published by Dr. Hopkinson (reprinted in 'Original Papers on Dynamo Machinery,' pp. 79–133 and pp. 135–139); but in these cases the eddy-current loss is obtained by subtracting from the measured waste power in the armature the calculated hysteresis loss.

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[1] In the case of a discoidal-ring continuous-current armature, described by Mordey, *Journal Inst. Elec. Eng.* xviii. 1889, p. 620, the watts expended in hysteresis and eddy-currents at 900 revs. per min. were respectively 291 and 1,240. The designed output was 18 kilowatts, but it is not mentioned at what speed the dynamo was intended to be run.

No	—	Dimensions of core in inches		Output in kilowatts	Volts	Amps.	Revs. per min.	Periph. speed in ft. per min.	Watts			Z_a in C.G.S. lines × 10^6	B_a	B_g
		Dia.	length						$C_a{}^2R_a$	Hyst.	Eddies			
4	Drum	$9\frac{5}{8}$	20	35	110	320	780	2,150	1,160	215	585	10·95	13,500	6,800
5	Ring	$9\frac{3}{8}$	$9\frac{1}{8}$	13	100	130	1,050	2,750	900	142	148	3·85	17,400	4,600
6	Drum	$7\frac{1}{2}$	15	11·2	140	80	880	1,830	330	65	360	4·7	9,700	5,200

Thus the relative proportion of the three armature losses varies considerably in machines of different construction and size. As compared with the watts expended over the resistance of the armature, the hysteresis loss is relatively most important in the case of small machines driven at a high speed, and therefore having a high periodicity; in the case of large drum armatures coupled direct to the crank-shaft of the driving engine, not only is the periodicity low, but the induction is seldom pressed so high as in the case of smaller ring machines, and therefore the loss per cubic centimetre per cycle is less.[1]

On the other hand, the figures above cited show that the eddy-current loss is usually of considerable importance; the precautions taken to minimise eddies have already been described in Chap. XIII., but their complete elimination is altogether exceptional; and it is clear that no formula can be of much value which does not specifically take them into account when calculating the rate at which heat is developed in the armature. If only the watts lost over the armature resistance and the hysteresis loss are calculated, a formula expressing the rise of temperature in terms of these watts can at best only apply to armatures of very similar construction in which the eddy-current loss may be treated as approximately proportional to the other two losses. This, however, is very far from being the case if the width of the inductor, or the value of the induction in the air-gap or armature, or the degree of lamination of the core, or the peripheral speed, be largely altered. Many of the published formulæ are either implicitly or explicitly applicable only to some more or less standard peripheral speed, such as 2,000 ft. per minute ; or, again, to armatures wound with round wire ; and are therefore entirely unsuited to drum armatures wound with large bars of rectangular section. In fact, an alteration which might at first sight seem unimportant in any one of the conditions above mentioned may radically affect the amount of the eddy-currents, and may therefore entirely vitiate any results calculated from an incomplete formula.

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[1] Alternators are not here under consideration, and in these the hysteresis loss may become important owing to the high periodicity. As a general rule, however, the armature is not worked at a high induction, and the effect of hysteresis is thereby lessened.

Considering, in detail, the three sources of heating in an armature, the first, or the electrical loss over the copper resistance of the winding, is entirely independent of the speed, and is simply equal to the product of the square of the armature current and the resistance of the armature from brush to brush, i.e. $C_a^2R_a$ watts. In the process of designing a dynamo the resistance of the armature can easily be calculated, and from it the loss of watts due to the passage of the normal armature current. It is only necessary to remark that if the resistance of an armature from brush to brush be actually measured, it will usually be found to be appreciably higher than the resistance as calculated from the length and area of copper used. This discrepancy is due to the inferior conductivity of the soldered joints, even when carefully made. In a drum armature of which the winding is composed, for example, of 180 bars united by separate end-connectors, there are as many as 360 joints, each of which has a slightly increased resistance owing to the want of homogeneity in the conducting material, and the combined effect of these may cause the actual resistance of the armature to differ from the calculated by as much as 25 per cent. Owing to the low resistance of a large armature with few inductors, the most convenient method of measuring its actual value is by passing through it a known current of considerable strength and measuring the difference of potential between opposite commutator segments, where the current leaves and enters. The quotient obtained by dividing the measured difference of potential by the known current is, of course, the resistance of the armature. From our knowledge of this resistance, after making due allowance for its increased value when hot, the $C_a^2R_a$ loss can be accurately determined.

It is in the second or eddy-current loss, that most difficulty lies. With a given core, a given winding, and a given field, this loss will be directly proportional to the square of the speed; since if the speed be increased x times, the E.M.F. of an eddy will be equally increased, and this will increase the current of the eddy x times, so that the product of eddy E.M.F. × eddy current will be increased x^2 times. It may therefore be expressed by a coefficient multiplied by the square of the speed, the value of the coefficient being different in different cases. Recently Mr. Kapp and Mr. Housman have independently devised a simple method by which the exact eddy-current loss in a given dynamo, when run at different speeds, can be experimentally determined.[1] A brief description of this method will be found in Chap. XX. Two points alone require to be further mentioned. In the first place, the value of the eddy-current loss, as thus determined, includes any eddies set up in the pole-pieces of the magnets by the rotation of the armature. Such loss of power in the pole-pieces will be reserved for consideration in a later part of the present chapter, and does not here concern us. In continuous-current machines, having conductors wound uniformly over a smooth core (as is usual in the majority of machines), the loss in the pole-pieces is quite inappreciable, and it may therefore be assumed that in such cases all the eddy-currents which the above method measures are generated in the armature itself. In the second place, during the experiments, the distribution of the field is approximately the same as that at no load, and it has already been shown, in Chap. XVII., that this distribution may be, and usually is, considerably modified at full load. Hence the eddy-current loss at full load may be greater than the experiment would show, and when the results of such tests have been checked by other methods, it has been found in practice that this difference in the distribution of the field does sensibly alter the amount of the eddy-currents. We are, however, justified in assuming that the method above alluded to furnishes us with a ready means for determining, at least approximately, the eddy-current loss, and that we can thence calculate under given conditions the value of the coefficient, which, when multiplied by the square of the number of revolutions per minute, gives the loss in watts. Let F be the value of this coefficient for a given armature with a particular excitation; then the rate of loss by eddy-currents in the armature when run at N revolutions per minute is equal to FN^2 watts. There still remains the difficulty that, in calculating the watts expended in a armature, we must, during the process of designing, estimate the value of the coefficient F from the coefficients of machines previously tested. In so doing, not only must any alteration in the length and volume of the core be taken into account, but, as previously stated, it is equally important to estimate the probable effects of alteration in the value of the induction in the air-gap or armature, the degree of lamination of the core, or the width of the inductors; in especial, the question of whether the inductors are stranded or twisted in order to eliminate Foucault currents will materially modify the value that should be assigned to the eddy-current coefficient.

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[1] Vide *Electrician*, vol. xxvi. 1891, pp. 699, 700, and 740, and vol. xxvii. p. 162.

The third source of heat is the 'magnetic' loss by hysteresis, or the work that must be done in changing the direction of magnetisation of the core, as explained in Chapter XII. For a given maximum induction in the core it is simply proportional to the number of complete cycles per second, and may therefore be expressed by a coefficient, h, dependent on the volume and induction of the core, multiplied by the number of revolutions per minute. Hence, for a 2-pole dynamo, the loss by hysteresis is equal to h N watts, where $h = \frac{kc}{60}$, the symbols having the same meaning as on p. 226.

Combining the three losses together, we obtain, as the total watts expended in heating the armature,

$$W = C_a{}^2 R_a + FN^2 + hN\,; \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (46)$$

and it is the ratio of this value to the cooling surface of the rotating armature that determines its rise of temperature. When the total watts are thus taken into account, the authors have found the following formula to approximately hold good for ordinary 2-pole continuous-current dynamos with ring or drum armature having a smooth exterior surface. The maximum rise of temperature of the outside of the armature in degrees Fahrenheit is

$$t^\circ\ \text{F.} = \frac{115 \times \text{w}}{\text{s}_c\left(1 + \frac{\sqrt{v}}{30}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where w = the total watts of the preceding equation, s_c = the cooling surface of the armature in sq. inches, and v=the peripheral velocity of the exterior of the armature in feet per minute. The cooling surface, s_c, is reckoned for rings and drums alike as the external surface ($=\pi dl$) taken from the outer edge of the commutator lug to the extreme opposite end of the armature, plus the area of the two ends $\left(=2 \times \frac{\pi d^2}{4}\right)$. The value of the coefficient may be determined by measuring the rise of temperature for a given number of watts when the armature is stationary;[1] it certainly will vary somewhat with different methods of construction or different proportioning of the main dimensions; still, the authors believe that the general form of the equation will prove serviceable in practice, even if the exact value of the coefficient should be found to require modification to suit armatures of different make. A very considerable influence has been assigned to the peripheral speed in increasing the effect of a given cooling surface—more, indeed, than has usually been attributed to it. The explanation to this may lie in the fact that the watts dissipated at high speeds are apt to be under-estimated. Since the eddy-current loss increases as the square of the speed, the relative importance of our second term, FN^2, increases greatly as the speed is increased; if, therefore, experiments be made on the difference in the cooling power of an armature at different speeds, and the eddy-current loss be not accurately determined, too low a value may be assigned to w at high speeds, and in consequence, to deduce the observed rise of temperature, we are led to under-estimate the increase in the cooling power due to the higher speeds.

For a peripheral speed of 2,000 ft. per min. a rise of 65° F. is approximately obtained if $\text{s}_c = \cdot 71\text{w}$, *i.e.* if nearly three fourths or

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[1] See *Electrician*, October 11, 1895 (Wilson), 'The Heating of Dynamos.'

·75 sq. inch of cooling surface be allowed per watt expended over the armature. In the above formula the cooling surface which has been taken into account has been confined almost entirely to the outside of the armature; yet in many dynamos the inside of the armature permits of the free passage of air into and through it. Especially is this the case with ring armatures, in which the radial depth of the core is comparatively shallow, in order to allow of the requisite connecting wires passing through the interior; even in the case of drum armatures the discs are frequently supported on the arms of a hub, and a clear air-space is thus secured between the internal circumference of the discs and the cylindrical hub. Further, open passages are frequently arranged between the connectors and the shaft, and even in some cases between the inner sleeve of the commutator and the shaft; it is thus possible for the air to pass through the inside of the core from the one end of the armature to the other. This tendency has in some cases been assisted by giving the supporting arms of the spider or hub a curved shape, so designed that they act as the blades of a ventilating fan, drawing air in at the one end and expelling it at the other. Again, in many cases, at certain intervals along the length of the armature, distance-pieces of vulcanised fibre are arranged between neighbouring core-discs, so as to keep them entirely apart; free passages are thus obtained from the inner air-chamber to the outside through openings on the surface of the core between groups of inductors, so that the air may be drawn out by centrifugal force. Owing, however, to the confined nature of such internal apertures and passages, and the low peripheral velocity of ordinary armatures running at moderate speeds, the draught of air which is thus set up through the interior openings is but small, and it is doubtful whether much advantage is obtained by the supposed internal ventilation. If the inside of an armature be accessible to a thermometer, and its temperature at the end of a long run be measured, it will usually be found to be from 5–10° F. higher than that of the exterior. In fact, the temperature of the core rises above that of the inductors by just so much as to cause the heat which is generated within it to be conducted to both the internal and external surfaces, and to be there dissipated. In drums the internal temperature of the core can seldom be so certainly ascertained; no great difference of temperature can, however, be required to cause the heat to pass through the insulation to the external surfaces, even when the core-discs are keyed directly on to the shaft, while, further, a somewhat higher temperature of the inside does not endanger either the working or the durability of the drum machine. Still, any internal cooling surface must remain of some value, and hence the formula above stated can only be held to apply to cases in which its effect is small as compared with the cooling power of the exterior. On this account multipolar armatures of large diameter, whether ring- or drum-wound, stand on a different footing: in these the radial depth of the core bears a much smaller proportion to the diameter than in bipolar machines, and not only is the peripheral speed of the outside comparatively high, but the linear speed of the interior is also proportionately great; hence the cooling power of the internal surface approximates much more closely to that of the outside. Further, in the case of drum-wound armatures for more than four poles, the end-connections become comparatively shallow, and may be supported at some distance from the shaft; a free circulation of the air is thus obtained through the inside, which renders its cooling effect comparable with that of the outside. Hence, the output of such armatures is more often limited by the reaction of their ampère-turns on the field than by the heating of the winding.

Finally, it is worth remarking that a cool machine is by no means necessarily efficient; although in most cases these two desirable qualities are attained by the same means, still it should be remembered that, while the efficiency is dependent on the ratio of the lost watts to the useful output, the rise of temperature is determined by the ratio which the lost watts bear to the cooling power of the surfaces.

It remains to consider the question of eddy-currents in the pole-pieces: their existence is due to sudden changes in the distribution of the field over the bored face of the pole-piece, caused by rotation of the armature. If the core of the armature have a smooth surface, and be uniformly wound with inductors, the length of the interferric gap is constant, and the magnetising effect of the armature current is very uniform; the only fluctuations are those which are produced when a section of the winding is short-circuited by the brushes (p. 378). Hence in such cases the eddy-currents set up in the pole-pieces are so slight as to be negligible. But if the armature core have Pacinotti teeth or iron projections, and these be few in number and of large size, as in the Brush machine (p. 442), the eddy-currents in the pole-pieces may be very considerable; in fact, the temperature of the latter may be raised so much in continuous working that they become unbearably hot to the hand. The manner in which these prejudicial currents are created will be seen from fig. 156, (i) and (ii), which shows part of a discoidal-ring armature in plan as viewed from above. While there is a dense flow of lines between the pole-pieces and the tops of the teeth, very few pass between the pole-pieces and the recesses, owing to the reluctance of the air-gap being considerably greater in the latter case than in the former. Fig. (i) shows the armature in one position, fig. (ii) being a second position after it has moved forward a little. Now as the armature moves forward, the dense portion of the field moves with it; and it results that those portions of the pole-pieces which at one moment have but few lines passing through them immediately afterwards are traversed by a large number, or *vice versâ*. The number of lines passing through any small portion of the polar surface is in this manner continually changing as the armature rotates, and such change must necessarily induce E.M.F.'s, and consequently currents in the iron mass of the

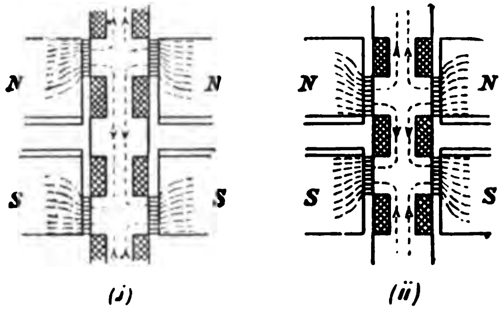

FIG. 156.—Discoidal-ring armature with Pacinotti teeth.

pole-pieces. Such currents do not spread to any great depth into the pole-piece, but whirl round in the metal near the surface facing the armature: they run upwards or downwards opposite each projecting tooth, and, dividing, curve round to complete their circuit opposite the recesses between the teeth. This distribution continually moves forwards as the teeth themselves advance. If the machine be running fully loaded, we have seen that the reaction of the armature current crowds the lines into the trailing corners of the poles; hence it is reasonable to conclude that the eddy-currents and the heating will be more intense in the trailing corners than in the leading corners, and in practice this effect is distinctly marked.

The remedy for such eddy-currents lies in making the distance between the tops of the teeth and the polar face bear a considerable proportion to the width of the recesses between the teeth. Under these circumstances, as the armature rotates, the lines, instead of following the advance of the teeth, snap across from one tooth to the next, and this slight divergence of the magnetic tufts produces but little alteration in the distribution of the lines over the face of the pole-pieces. Since, however, it is required to keep the teeth close to the poles, it follows that it is necessary to reduce the width of the recesses until they become narrow grooves; hence in continuous-current ring or drum armatures with toothed core-discs (p. 251) the number of the teeth and grooves is usually large, and their width correspondingly small. In practice, it is found that if the width of the grooves be not more than about twice the clearance between the pole-piece and the teeth, sufficient steadiness of the field is obtained to avoid any serious eddy-currents in the poles.[1]

Owing to the inferior electrical conductivity of cast iron as compared with wrought iron, it is advantageous to use cast-iron pole-pieces in connection with toothed armatures, since, for the same E.M.F., the magnitude of any eddy-current will be much less in cast iron than in wrought iron.

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[1] *Vide* Esson in *Electrical Review*, November 14, 1885, and December 18, 1891, p. 693, and (Sayers) *Journal Inst. Elec. Eng.* vol. xxii. No. 107, p. 393.

CHAPTER XIX

DESCRIPTIONS OF TYPICAL DYNAMOS

So many and so varied are the dynamos now manufactured in this and other countries that space does not permit us to describe all of even the leading types of machines. Hence no attempt will here be made to exhaustively classify and describe the diverse makes of dynamos; and it is only proposed to examine a few typical designs, sufficient to illustrate the principles of foregoing chapters.

I. *Closed-coil continuous-current machines.* (A) *Bipolar.*

(1) The ring dynamo manufactured by Messrs. Johnson and Phillips affords a good illustration of the 2-pole overtype machine with ring-wound armature such as has been already shown in fig. 2. It is constructed for outputs ranging from 3 to 12 kilowatts for belt-driving at speeds of from 1,300 to 1,000 revolutions per minute. Fig. 157 shows a perspective view of a 5-kilowatt machine giving 130 volts and 38 ampères when run at 1,200 revolutions per minute. The armature core is built up of a number of annealed charcoal-iron discs ·025″ thick; on the inner periphery of these are stamped three keyways by which they are threaded over the three arms of a gun-metal hub, after the manner shown in fig. 113. The hub is secured to the shaft of mild steel by a long sunk feather. In order to check eddy-currents in the core, an insulating ring of thin paper is inserted between each pair of discs, the net volume of iron being thus about 87 per cent. of the gross volume of the core. The discs are tightened by a gun-metal three-fingered star driven up by a nut screwing on the shaft; a set screw passing through the nut into the shaft finally locks the whole, and prevents any slacking back through vibration

or jarring. The external and internal diameters of the discs are $8\frac{1}{4}''$ and $5\frac{1}{4}''$ respectively—giving a radial depth of iron of $1\frac{1}{2}''$ or $\cdot 182$D—while the length of the core is $10\frac{1}{2}''$. The surface of the core is smooth, and is insulated with varnished paper and tape, additional protection being afforded to the internal connecting wires of the Gramme winding by covering the three arms, and lining the inside of the core, with thin sheets of vulcanised fibre. The armature is wound with 216 turns of double-cotton-covered round wire, arranged in 72 coils of three turns each : the bare wire is $\cdot 105''$ dia., and the winding forms a single layer on the outside

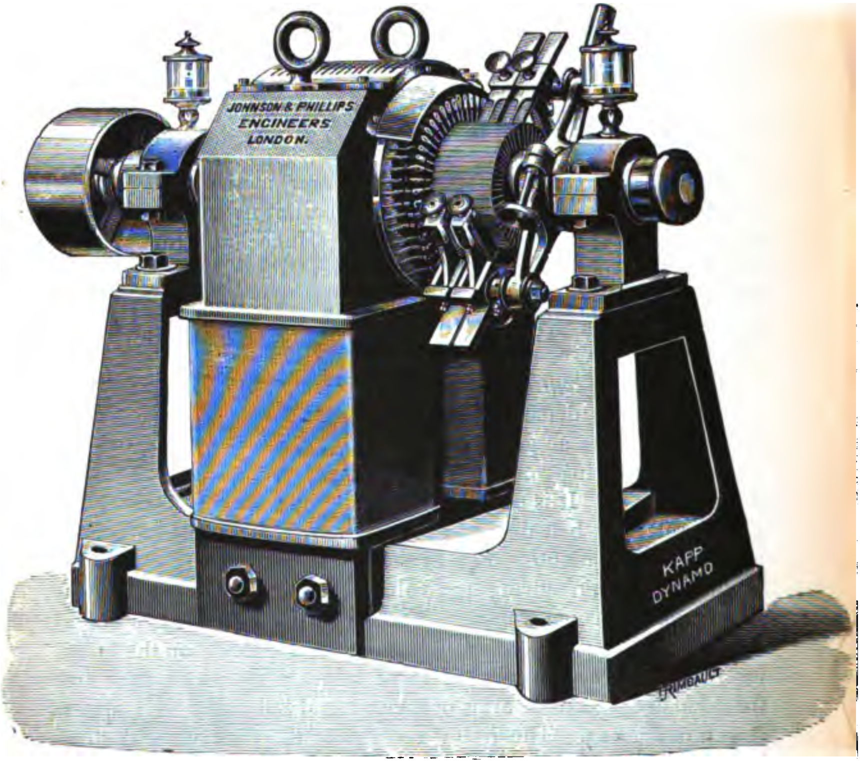

FIG. 157.—Ring dynamo of Messrs. Johnson and Phillips.

and two layers on the inside, an air-space remaining between the internal wires and the gun-metal hub for ventilation. The beginning and the end of each pair of contiguous sections are connected to the lugs of a 72-part commutator : these lugs are made of thin copper strip, one end being soldered into a saw-cut in the commutator segment, while the other is bent round and soldered to the two armature wires. The commutator segments are of hard-drawn copper, insulated with mica, and held together, as shown in fig. 120, by a gun-metal sleeve and collars, with insulating rings of ebonite. The length of the commutator is sufficient to take two brushes, each 1″ wide, side by side, and to allow of a slight lateral movement of the pair of brush-holders along the gun-metal brush spindle in order to distribute the wear evenly over the surface of the commutator. There are thus four brushes in all, two on each arm of the rocking-bar, as shown in fig. 149. Each brush is of copper gauze ¼″ thick, and its holder is provided with a pressure-spring, a hold-off catch, and a pointer ; the latter indicates the correct length to which the brush must be pushed through the brush-holder, as wear takes place, so as to maintain the proper width of contact at the brush-tip.

The magnet of this machine is composed of two slabs of annealed wrought iron, each 10″ wide × 4¾″ thick, held to the cast-iron yoke in the bedplate by two wrought-iron bolts passing through from one side to the other (see fig. 2). The magnet limbs are let into recesses in the bedplate in order to obtain sufficient area of joint through which the lines of induction may pass into the cast iron, and both limbs and recesses are machined to ensure an accurate fit. In order to increase the polar area without unduly lessening the section across the neck of the pole-piece, strips of cast iron are screwed to the tips of the wrought-iron poles, so as to form polar extensions both above and below the armature (cp. fig. 135). These projecting 'horns' approach each other more closely at the top than at the bottom, so as to equalise the fields above and below a horizontal diameter, and prevent a downward magnetic pull on the armature (p. 283).

The cast-iron yoke has an area about 75 per cent. in excess of the area of the wrought-iron limb, and together with the bedplate and bearing standards forms one casting. The bearings are bushed with gun-metal and are oiled by sight-feed lubricators, the feed of which can be adjusted by a thumb-screw to the required number of drops per minute. The armature is thoroughly protected externally from injury by a gun-metal grid screwed on the magnet top and by guards, formed of portions of copper spinnings, which encircle the ends of the armature.

The machine is shunt-wound, and the field-winding is of ·042″ round wire, double-cotton-covered to ·057″, wound on two sheet-iron 'formers,' which are slipped on over the magnet-limbs. On each former there are eleven layers, and the two bobbins are connected in series. The resistance of the shunt is 50 ohms, whence $c_s = \frac{130}{50} = 2{\cdot}6$; the resistance of the armature is ·11 ohm, or, say, ·12 when warm after a run of some hours. The loss of volts over the armature at full load is therefore $c_a r_a = 40{\cdot}6 \times {\cdot}12 = 4{\cdot}9$, whence the internal E.M.F., $E_a = 130 + 4{\cdot}9$. The total number of lines in the armature at full load is therefore by equation (11)

$$Z_a = \frac{E_a \times 60 \times 10^8}{\tau \times N} = \frac{134{\cdot}9 \times 60 \times 10^8}{216 \times 1{,}200} = 3{,}120{,}000.$$

The floor-space required by this dynamo is 3′–4″ × 1′–7″, and the weight of the machine complete is 11·3 cwt.

(2) The 'Manchester' dynamo manufactured by Messrs. Mather and Platt is illustrated in fig. 158, and is the original from which is derived the type of double-horseshoe 2-pole magnet that has often been previously mentioned as the 'Manchester' field (fig. 125). The cylindrical wrought-iron magnet cores are let into circular recesses in the massive cast-iron pole-pieces, and a large contact surface is thus obtained between the two metals, while the area of the cast iron itself is very considerably greater than that of the wrought iron. Two set screws serve to further bind the vertical cores into the lower pole-piece. In some cases the pole-pieces are bored out to a radius slightly larger than will be required to give the necessary clearance round the completed armature, and are subsequently brought nearer to each other; the length of air-gap thus becomes slightly less on a vertical diameter than at the polar tips, and the result is to concentrate the field more densely at the centre of the pole-pieces. The armature core

is constructed of thin iron discs passed over arms which project longitudinally from a brass frame or spider, keyed to the shaft at the one end ; at the other a brass end-plate is driven up so as to compress the discs, and is held in position by a nut and lock-nut.

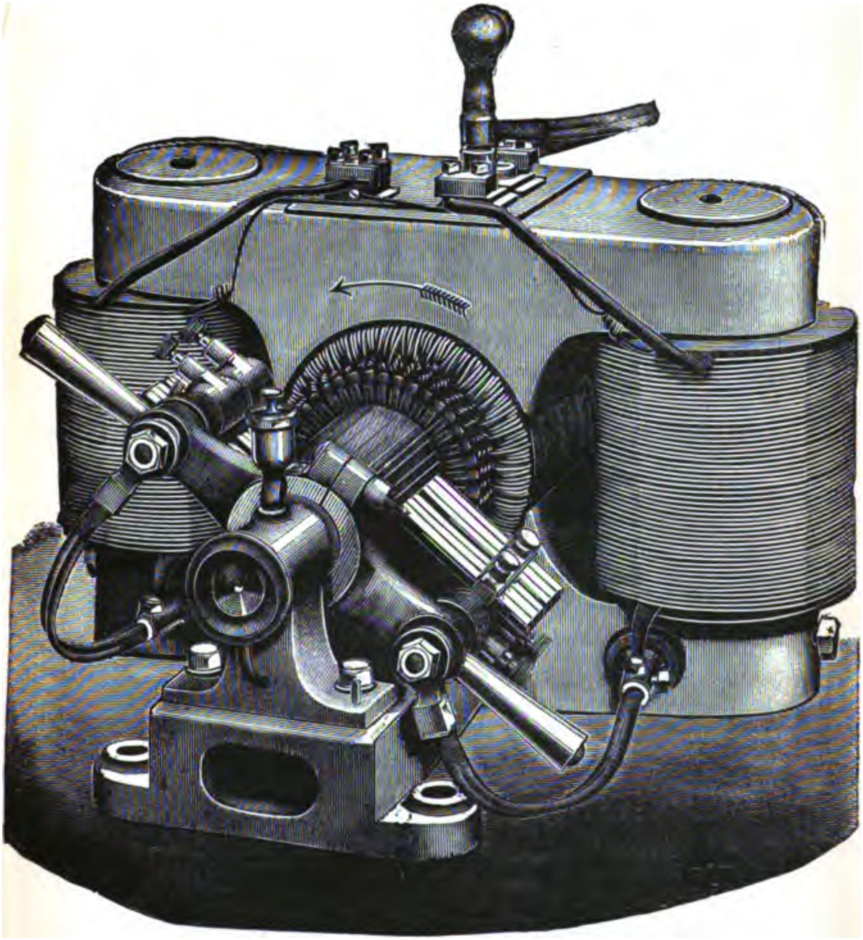

Fig. 158.—Manchester dynamo of Messrs. Mather and Platt.

There is thus a free internal space between the shaft and the discs, through which pass the inner wires of the Gramme or ring winding. The commutator segments are of hard-drawn copper insulated with mica. The armature shaft is of Bessemer steel, and the bearings of gun-metal, supported on extensions from the lower pole-piece, which forms the base-plate. Thus the axis of rotation is kept low, giving great steadiness of running at high speeds, while the whole is compact and occupies but little floor-space. On the top of the upper pole-piece is mounted a terminal board of polished wood. The machine illustrated has an output of 100 volts and 130 ampères at a speed of 1,050 revolutions per minute, and is compound-wound for incandescent lighting, with the series coils outside the shunt. It is also series-wound for lighting arc lamps in series, in which case the output is 600 volts, and 15 to 20 ampères, at a speed of 1,100 revolutions. The standard pulley for belt-driving is 12″ in diameter with 8″ width of face, and the over-all dimensions of the complete machine are 3′–8″ length from outer edge of pulley to end of commutator bearing, 2′–8″ width over the outside of the field-coils, and 1′–6″ height to top of pole-piece. The weight of the machine complete is 12½ cwt., and of the armature alone 2½ cwt. Full particulars of the dimensions and winding of this dynamo for incandescent lighting have been published by the Drs. Hopkinson in 'Phil. Trans.' Part I. 1886,[1] from which paper the following details are taken :—

External diameter of core	9½″
Diameter of internal opening through core .	5½″
Length of core over end-plates . . .	9½″

Thus the radial depth of the core discs is 2″, or ·21 D. The gross sectional area of the two sides of the ring is 38 sq. in., and the net area of iron, after deduction for the paper insulation between the discs, $38 \times {\cdot}9 = 34{\cdot}2$ sq. in., or 220·5 sq. cm. The winding consists of 160 turns of wire, ·134″ diameter (No. 10 B.W.G.), forming a single layer on the outside, and divided into forty sections of four turns each. The resistance of the armature is ·047 ohm.

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[1] Reprinted in *Original Papers on Dynamo Machinery*, pp. 79–133.

Length of magnet-cores between pole-pieces	10¼"
Bore of pole-piece	10⅛"
Width of gap between the poles	3⅜" = $10·8\, l_g$
Angle subtended by the pole-pieces	139°
Diameter of magnet-core	6"

whence the area of the two cores is 365 sq. cm., while the smallest area of the cast-iron pole-pieces is 955 sq. cm., or more than twice as great.

On each magnet-core there are 2,196 turns of copper wire ·058" diameter (No. 17 B.W.G.), and these two shunt coils are connected in series, their total resistance being 26·85 ohms. There are thus approximately 8,200 ampère-turns furnished by the shunt on each of the two magnetic circuits. The resistance of the series coils is ·0245 ohm, giving, when warm, a loss of about 3·5 volts at full load. The loss of volts over the armature, when warm, being 6·9, the total number of lines required at full load is

$$z_a = \frac{110·4 \times 60 \times 10^8}{160 \times 1,050} = 3,950,000.$$

Hence $B_a = 17,900$, and the average value of the induction over the area of the air-gap, after allowing for a fringe beyond the bored polar face of a width $= ·8 l_g$, is $B_g = 5,000$. Assuming that in the magnet-core $z_m = 1·49 z_a$[1] $= 5,890,000$, $B_m = 16,100$. The total ampère-turns required at full load are approximately 13,000 on each magnet-core.

The following table gives the results of efficiency tests made with this machine, approximately at full and at one-third load :—

Revs.	Output		Power supplied in watts	Percentage lost in				Efficiency	Lead of brushes
	Volts.	Amps.		Armature	Shunt	Series coils	Other losses		
1,098	100·1	139	16,684	5·8	2·2	3·0	1·9	87·1	17·5°
1,094	103·8	43·48	5,367	2·0	7·5	1·0	5·5	84	5°

In larger sizes an efficiency of 93 per cent. is obtained.

(3) The 'Manchester' type of field is again illustrated by fig. 159, which shows a machine manufactured by Messrs. Brown,

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[1] See p. 311.

Boveri & Co., of Baden, Switzerland. The circular magnet-cores are here not let into the pole-pieces, but are fastened by screws to flat seatings. Across the centre of each pole-piece runs a deep V-shaped groove, the object of which is not only to lighten the magnet (cp. p. 277), but also to increase the reluctance of the

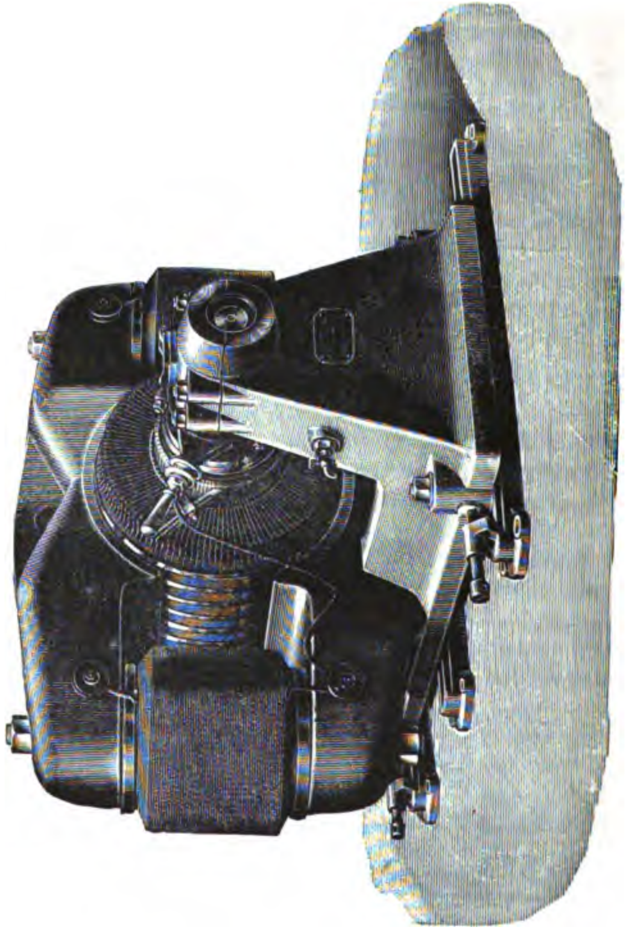

FIG. 159.—Dynamo designed by C. E. L. Brown.

cross-induction path, and so to lessen the displacement of the field by the cross ampère-turns of the armature (cp. p. 374). A web uniting the two halves of the upper pole-piece supplies a means of lifting the entire machine. Three slide rails are seen in fig. 159, on which the dynamo can be slid backwards or forwards by the two tightening screws on the outside rails, so as to keep the driving belt at its due tension.

(4) The Edison-Hopkinson dynamo is also manufactured by Messrs. Mather and Platt, and with its drum-wound armature and undertype single horseshoe field is one of the best-known machines. At once scientifically designed and simply but well constructed, many of its features have been extensively copied, especially in England, where the majority of large continuous-current dynamos have been modelled after its pattern. Taking as their starting-point the original Edison machine, Drs. J. and E. Hopkinson so improved its design (especially by perfecting its magnetic circuit), and so simplified its construction, that its new form became at once greatly more efficient, less bulky, and less costly to manufacture. The general design of the undertype 2-pole field (cp. fig. 123) has been already discussed (p. 266), and the special form adopted in the Edison-Hopkinson machine is illustrated by fig. 160. The whole of the magnetic circuit is composed of wrought-iron forgings of annealed hammered scrap or in modern machines of cast steel. Pole-piece and magnet-core are in one piece, and the two limbs thus formed are joined together by a massive rectangular yoke screwed on at the top. A footstep of zinc raises the pole-pieces of the magnet to a considerable height above the cast-iron bedplate, on which are bolted the two low cast-iron plummer-blocks carrying the armature shaft.

The armature core is built up of a number of soft wrought-iron discs, insulated one from another by thin sheets of paper. A plain circular hole through their centre enables them to be passed over the shaft, which is of Bessemer steel, and they are then compressed between two stouter end-plates, one of which is secured by a collar shrunk on to the shaft, while the other is driven up tight by a nut and lock-nut screwed on the shaft (cp. fig. 115). Since the core-discs are threaded directly on the shaft, there is no internal ventilation of the core. In the first instance the discs were insulated from the shaft, the central hole being larger in diameter than the turned shaft; but experience shows that if the discs of an armature of this type are well insulated from each other in planes at right angles to the shaft, this precaution may be dispensed with. The commutator is built up of copper bars, after the fashion shown in fig. 119, with mica insulation.

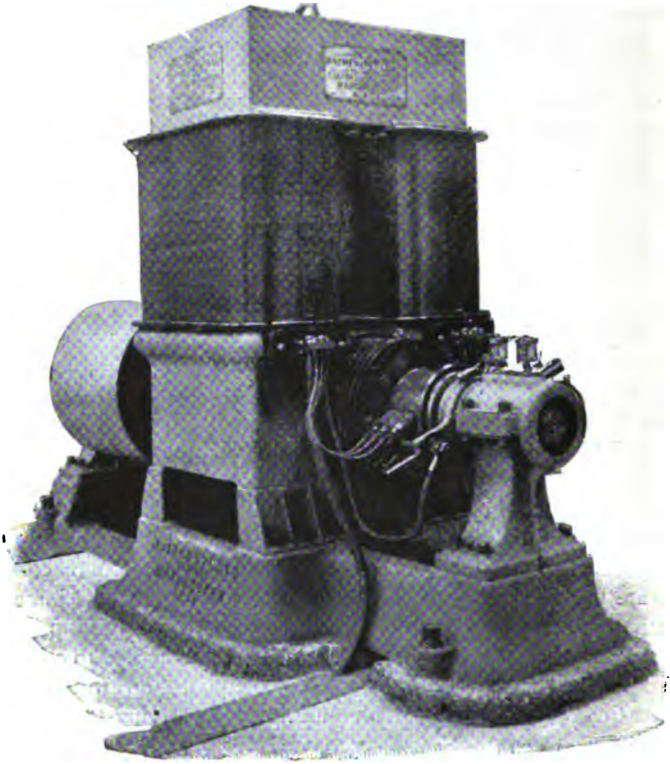

FIG. 160.—The Edison-Hopkinson dynamo.

For small outputs the armature is drum-wound with loops of round wire, the convolutions at the ends of the core being wrapped round the shaft. But for large outputs the armature inductors are bars of rectangular or trapezoidal section, distinct from the end-connectors. To form the end-connections Dr. Hopkinson devised a system of strips of copper forming two series of spirals in two planes at right angles to the shaft (Patent No. 4,884, 1886). One method of forming these is shown in fig. 161. A number of wedge-shaped copper pieces, each insulated from its neighbours, are assembled in a ring round a wooden hub; into a saw-cut in each of these are soldered two copper strips, which are bent in opposite directions, so as to span the diameter of the core, and their ends are soldered to a long and a short armature bar respectively. Each complete connector is entirely insulated, and the current, therefore, flows from a short bar into one curved strip, along the copper segment and out through the other curved strip into a long bar. At the one end of the armature the copper wedge-shaped pieces are continued outwards to join the commutator segments; thus the ends of a loop situated in a vertical plane are connected to segments lying in a horizontal plane. In consequence the plane of commutation in a vertical 2-pole dynamo becomes approximately horizontal, and the position of the brushes is rendered convenient for observation and attention.

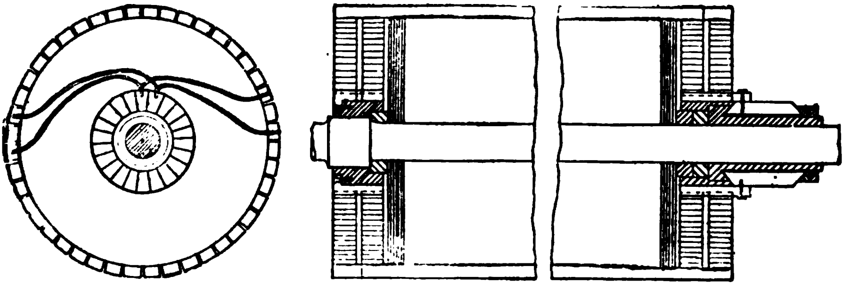

FIG. 161.—Hopkinson end-connector for drum armature.

A complete account of an Edison-Hopkinson machine with wire-wound armature for an output of 105 volts 320 ampères when running at 750 revolutions per minute is contained in the previously cited paper of the Drs. Hopkinson. The leading dimensions are as follows :—

Diameter of core	$9\frac{5}{8}''$
Diameter of internal hole	$3''$
Length of core over end-plates	$20''$
Diameter of shaft	$2\frac{3}{4}''$

The armature is wound with forty loops ($\tau=80$) of a stranded conductor containing sixteen wires, ·069″ diam.: the loops are wound in two layers and are connected to a 40-part commutator.

Width of magnet-core parallel to shaft	$17\frac{1}{2}''$
Thickness of magnet-core	$8\frac{3}{4}''$
Length of magnet-core	$18''$
Width of pole-piece parallel to shaft	$19''$
Bore of field	$10\frac{13}{16}''$, whence $l_g=\frac{19}{32}''$
Height of zinc base	$5''=8\frac{1}{2}\,l$
Width of gap between pole-pieces	$5''=8\frac{1}{2}\,l_g$
Polar arc	129°

The field is shunt-wound; each coil consists of eleven layers of wire, ·095″ diam., wound directly on the limbs, the two together giving 3,260 turns with a resistance of 16·93 ohms at a temperature of 57° F. Thus the normal magnetising current when the shunt is warm is about 6 ampères, giving 19,500 ampère-turns.

The resistance of the armature also at 57° F. is ·009947, or, say, ·0115 ohm when hot. Thus the loss of volts over the armature is $\cdot 0115\times 326=3\cdot 75$:—

whence $$E_a=105+3\cdot 75=108\cdot 75,$$

and $$z_a \text{ (at full load)}=\frac{108\cdot 75\times 60\times 10^8}{80\times 750}=10{,}875{,}000.$$

Allowing 6·5 per cent. of the length of the armature core for the thickness of the insulation between the discs, and, on the other hand, making a slight allowance for induction through the shaft, the effective area of iron in the armature may be taken as 810 sq. cm.; whence $B_a=13{,}500$. The effective area of the air-gap is 1,600 sq. cm.; whence $B_g=6{,}800$, a figure showing the great intensity of the field. Assuming $z_m=1\cdot 32\ z_a$ (p. 312), the induction in the magnet-cores is $B_m=14{,}700$.

The weight of the dynamo complete is $51\frac{1}{2}$ cwt., and of armature alone $5\frac{1}{2}$ cwt. With standard pulley, 20″ diameter × 12″ wide, the over-all dimensions are 6′-1″ length, 2′-11½″ width over bedplate, 4′-1″ total height. With bar armature the same machine is constructed to give 105 volts 350 ampères at the lower speed of 650 revolutions per minute.

The commercial efficiency which is attained in these machines is remarkable, and is guaranteed to be from 90 to 94 per cent.; in tests of a machine similar to that described above, and intended for a normal output of 110 volts 320 ampères at 780 revolutions, the following results were obtained:—

Revs.	Output		Power supplied in watts	Percentage lost in			Efficiency
	Volts.	Amps.		Armature	Shunt	Other losses	
764	110·12	363·36	42,917	3·17	1·66	1·94	93·23
798	110·6	356·24	42,326	3·11	1·7	2·09	93·1
811	125·8	156·56	21,900	1·235	4·26	4·535	89·97
801	127·2	79·15	11,884	·63	8·04	6·73	84·6
810	129·1	28·85	5,425	·24	18·1	12·73	68·93

An Edison-Hopkinson machine with bar-armature for an output of 110 volts 475 ampères at 430 revolutions per minute, directly coupled to a high-speed Willans engine, gave the following results:—

Resistance of shunt, 15·6 ohms. Loss over shunt, 756 watts = 1·4 % of output.
„ armature, ·0058 ohm. „ armature, 1,386 watts = 2·6 „

$$\text{Over-all efficiency of combination} = \frac{\text{E.H.P. at terminals}}{\text{I.H.P of engine}} = 83{\cdot}3 \text{ per cent.}$$

The three large machines built by Messrs. Mather and Platt for the generating station of the City and South London Electric Railway are similar to fig. 160: each is belt-driven and has an output of 450 ampères at 500 volts, or 225 kilowatts, when running at 500 revolutions per minute. In these the pole-pieces are of cast iron, and the weight of the complete magnet is 11 tons. The armature weighs nearly 3 tons, and is carried by three bearings, a separate bearing being provided outside the pulley to take part of the strain of the belt and give additional security. The whole machine with its bedplate weighs 17 tons.

The Edison-Hopkinson dynamos at the electric-light station of the Manchester Corporation are of the same type as the above, and give 410 volts 590 ampères or 240 kilowatts at 400 revolutions per minute: each machine weighs 24 tons, and yields 600 watts per revolution per minute. When tested by the method described in Chap. XX. they gave an efficiency of 95·2 per cent.,[1] any increased friction due to the pull of the belt being neglected.

(5) In fig. 162 is shown an overtype drum dynamo for belt-driving, constructed by Messrs. W. H. Allen, Son & Co. While the field-magnet is of the construction shown in fig. 2, the armature is in this case drum-wound, and is built up in the following manner. On the shaft, which is of Siemens-Martin steel, is turned a collar near the pulley end, and a strong cast-iron end-plate, bored to fit the shaft, is passed along until it abuts against this collar. Along the entire length of the core four keyways are accurately machined to fit corresponding projections stamped on the inner periphery of the discs. These are of annealed charcoal-iron, ·025″ thick, and in order to insulate their flat surfaces from one another they are varnished or separated by thin paper. After the discs are threaded over the shaft, a second cast-iron end-plate is put on, and the whole is tightened up by a nut screwing on the shaft, and locked by a set screw. The length of the completed armature core is slightly greater than the length of the pole-pieces parallel to the shaft, and the solid end-plates, being of smaller diameter than the core-discs, are kept well out of the magnetic field. The bearings are self-lubricating, oil-rings threaded over the journals and dipping in a reservoir serving to distribute the lubricant when the machine is at work.

The following are the leading dimensions of a belt-driven machine for an output of 65 volts and 170 ampères when running at 700 revolutions per minute. Armature core 10½″ diameter × 10″ long, with a radial depth to the discs of 2⅛″ : wound with 144 inductors, each composed of three strips ·200″ high × ·054″ thick. The centre strip is lightly insulated with silk wound spirally round it, the side strips being shellacked : the laminated bar thus formed is twisted at its centre so as to eliminate eddy-currents (p. 247). The connectors have a width of 1¼″ and are ·040″ thick ; it will thus be seen that the sectional area of the inactive connector is over 1½ times the area of the active inductor. The commutator has 72 segments built up as in fig. 120, and the resistance of the armature is ·0202 ohm. The magnet limbs have

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[1] *Electrician*, October 6, 1893.

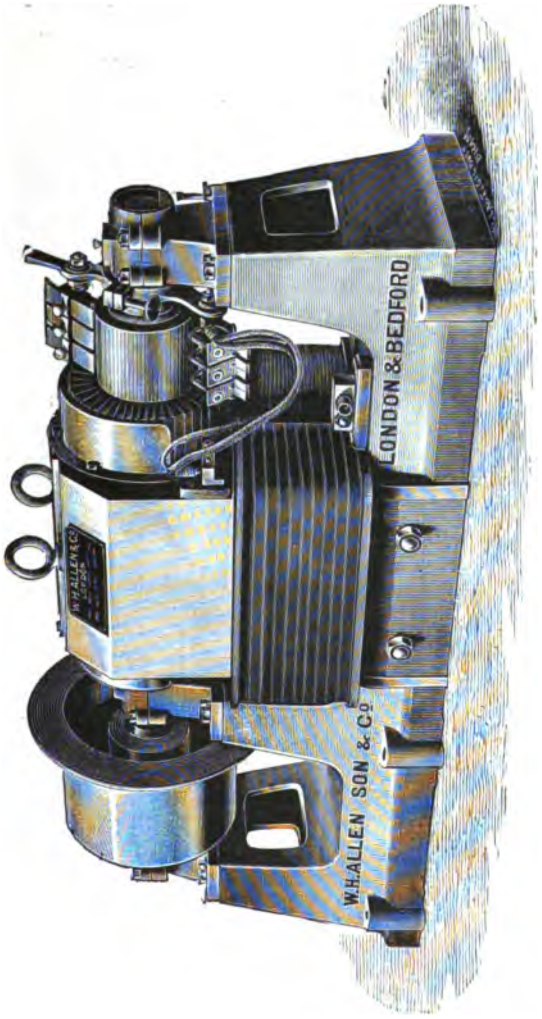


FIG. 162.—Belt-driven drum dynamo of Messrs. W H. Allen, Son & Co.

a cross-sectional area 9¼″ × 5½″, and the length of winding on each former is 9¼″. The field is compound-wound, fifteen layers of ·069″ shunt wire and one layer of ·310″ × ·200″ rectangular 'main' wire being wound on each former. The two shunt-coils are connected in series, while the two 'main' coils are in parallel. The rise of temperature in continuous working does not exceed 55° F. in any part of the machine. When

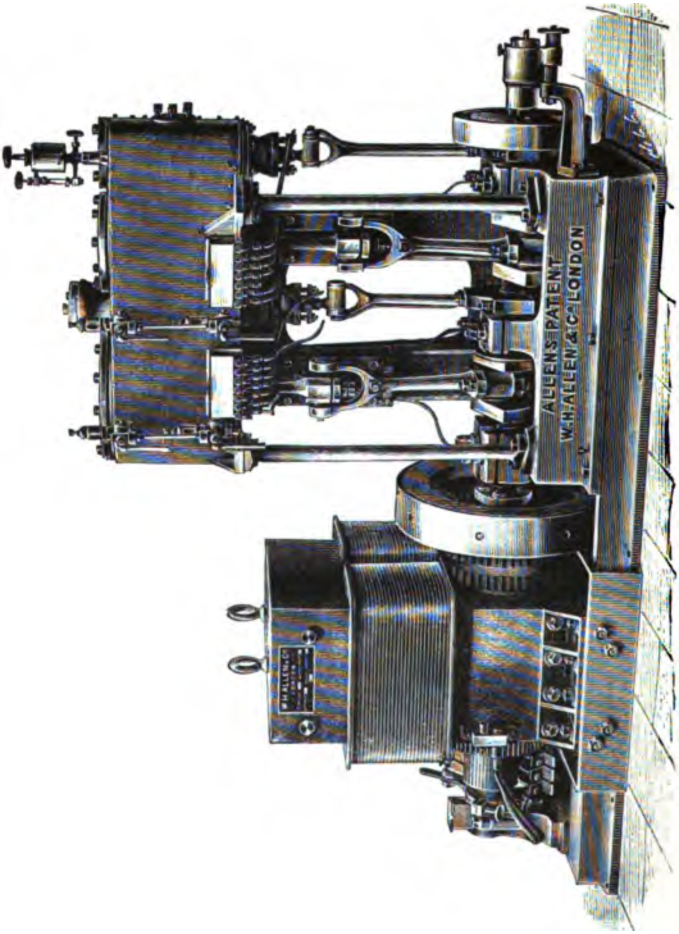


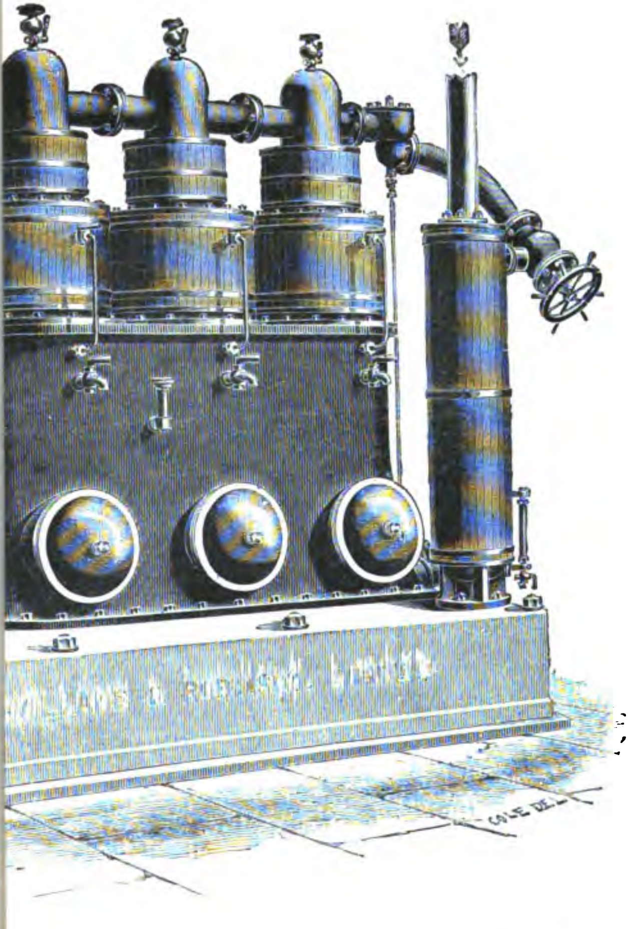
Fig. 163.—Engine-driven dynamo of Messrs. W. H. Allen, Son & Co.

provided with a fast and a loose pulley, a third bearing, carried on an extension of the bedplate, is placed outside the pulleys; two brackets fitted to the base-plate carry a screwed spindle, on which are mounted two travelling arms forming a belt fork. On rotating the spindle by a hand-wheel, the belt is shifted by the arms from the fast to the loose pulley, or *vice versâ*. The weight of the machine complete, with fast and loose pulley, third standard, and belt-shifting gear, is 19 cwt.

Fig. 163 shows an engine-driven dynamo manufactured by the same firm. The shaft of the armature has a solid coupling forged on it, and is coupled directly through the fly-wheel to the crank-shaft of an Allen double-acting engine. The undertype field-magnet is built up of wrought-iron forgings after the manner of fig. 123, and each limb is supported on a pair of gun-metal angle brackets, the one screwed to the pole-piece by manganese-bronze screws, and the other bolted to the cast-iron base-plate. The arrangement keeps the axis of rotation low, and at the same time enables each magnet-limb to be slid horizontally away, should it be desired at any time to examine or remove the armature. Further, the minimum distance between the iron of the pole-piece and the iron of the base-plate being considerable, the loss of lines by magnetic leakage is extremely small. In a machine having an output of 80 volts and 500 ampères, at a speed of 200 revolutions per minute, the armature core is 18″ in diameter, and is built up of discs ·025″ thick, insulated with paper: the winding of the armature consists of 104 laminated bars, each composed of six strips ·072″ × ·510″ united to a 52-part commutator. The sectional area of the connectors is ·3 square inch, and the resistance of the armature when warm is ·005 ohm. The current is collected by five 1¾″ gauze brushes. The polar angle is 136°: at the extremities of the pole-pieces furthest from the yoke, the wrought-iron polar extensions are forged solid with the limbs, and approach one another more closely than do the cast-iron 'horns' screwed on at the upper edges of the poles (cp. fig. 135). By this means the field is distributed evenly round the bore, and any excessive upward pull of the magnet on the armature is prevented. The field is shunt-wound with 34,000 ampère-turns, the wire having a rectangular section in order to economise the space occupied on the formers.

(6) To Messrs. Siemens Bros. & Co. belongs the credit of having built what are probably the largest 2-pole direct-coupled dynamos in existence, and with an account of these we conclude our description of bipolar continuous-current dynamos. Their output is 250 kilowatts, viz. 250 volts at 1,000 ampères, and they are coupled directly to double-acting engines running at 350 revolutions per minute: thus the ratio of their watts of output to revolutions per minute (cp. Chap. XX.) is as much as 715. The over-all diameter of the armature is 30 inches, the width of the core or of the magnet parallel to the shaft being 40 inches. The dynamo illustrated in fig. 164 is similar but of smaller size: it is one of three combined sets which were shown at the Royal Naval Exhibition held in London in 1891, and which have since been in constant use as part of the generating plant of the central power station for driving Messrs. Siemens Bros. & Co.'s works at Woolwich. It is capable of maintaining continuously an output of 1,500 ampères at 120 volts or 180 kilowatts, and is coupled directly to a single-acting engine running at 350 revolutions per minute. The engine, which is of the standard I.I.I. size, manufactured by Messrs. Willans & Robinson, is capable of indicating 300 horse-power with a steam pressure of 130 lb. per sq. in., and when tested at the Exhibition, shortly before its close, the over-all efficiency of the combination at full load, or the ratio of the electrical power at the terminals to the power indicated in the cylinders of the engine, was as high as 84 per cent. Assuming the brake H.P. of the engine to be about 90 per cent. of the I.H.P., the commercial efficiency of the dynamo is thus 93·5 per cent.

The dynamo is of the undertype single-horseshoe form shown in fig. 123 with a drum-wound armature. The latter is 24″ in diameter with a length of 36″, and is built up of soft-iron discs mounted directly on a 6″ steel spindle, which measures 7 feet 2½ inches from the commutator end up to the face of the forged coupling by which it is bolted to the engine crank-shaft. Owing to the small number of commutator segments that are necessary to obtain the voltage, and the large size of inductor that is required to carry the current, the armature is treble-wound in order to avoid sparking and heating by eddy-currents (p. 390). Each winding consists of 48 bars, making 144 bars in all: these are



lans engine. *To face p.* 432.

connected to a joint 72-part commutator, in which the segments of any one winding occur alternately with those of the other two windings. The bars are composed of a number of strands twisted together (p. 248) and compressed into a rectangular shape. The commutator is 12½″ in diameter, and has a wearing surface 9″ long, the sections being of hard-drawn copper, insulated with mica. There are three brushes on each of the two brush-spindles, and each brush is formed of four copper gauze brushes, each 2″ wide ×¼″ thick, laid one above the other, the great width of contact which such a composite brush gives being necessitated by the fact that with the treble-winding each brush must cover at least two commutator segments and the adjacent insulating strips simultaneously. For greater convenience in moving the heavy cast-iron rocking-bar and brush gear, the whole is arranged to be rotated round the commutator by a worm and worm-pinion.

The magnet is composed of three massive wrought-iron forgings, having a sectional area, 34½″ × 16½″, bolted together after the same manner as in the last-described dynamo. It is supported by gun-metal brackets screwed to the limbs and bolted to the cast-iron combination base-plate, the minimum distance between iron and iron being such that the magnetic leakage is kept within reasonable limits: two of these brackets on the one side of the dynamo are seen in the illustration. The whole of the weight of the fly-wheel and armature is taken by the inner main bearing of the engine and the outer bearing of the dynamo, the latter being bushed with a gun-metal sleeve 13½″ long, and fitted with two sight-feed lubricators. The field is shunt-wound, and the two formers are provided with shackles for the purpose of lifting them off or replacing them on the limbs. At the ends of the bobbins are substantial hard-wood flanges, and upon an extension of the upper flanges above the commutator are mounted the gun-metal terminal blocks: each of these is connected to the brush-holders by two flexible cables having a total cross-sectional area of 1·6 sq. inch.

The total weight of the dynamo is 13½ tons, and of the armature nearly 2½ tons.

I. (B.) *Multipolar closed-coil continuous-current dynamos.*

(7) The 4-pole dynamo of fig. 165, which represents a machine

manufactured by the Maschinenfabrik Oerlikon in Switzerland, exemplifies a type of field which has found great favour for large outputs, especially on the Continent and in the United States. A diagram of the field-magnet has already been given in fig. 129. The machine illustrated has a ring armature wound with 400 loops of a stranded cable, which forms a single layer on the outside and a double layer on the inside: the output is 270 ampères and 625 volts, or 170 kilowatts at 500 revolutions per minute. The diameter of the armature core is 37¾″, and its length 19½″, the radial depth of the core-discs being nearly 6″. Two loops of the armature winding are connected to each of the 200 segments of the commutator, the current in each quarter of the winding being 67½ ampères (cp. p. 389), and opposite pairs of brushes are connected in parallel. The radial poles are cast in one with the two halves of the yoke-ring, the exciting coils being passed over them from the inside.[1] Later machines of the same type have circular yoke-rings and drum armatures with curved evolute end-connectors.[2]

(8) The drum dynamo (fig. 166), manufactured by Messrs. Crompton & Co. for large outputs, affords a good illustration of the peculiar 4-pole field with two distinct magnets, such as has been typically shown in fig. 130. Each magnet is composed entirely of rectangular wrought-iron forgings, the yoke being bolted between the two limbs, on which are wound a pair of magnetising coils. The horseshoes being placed horizontally on either side of the armature, four equal interpolar gaps are obtained, two on a vertical and two on a horizontal diameter: in the illustration these are hidden by substantial gun-metal distance-pieces, screwed to the corners of the poles, and serving to stay the whole magnetic system. Each horseshoe is supported from the bedplate by high brackets of gun-metal, so as to minimise the magnetic leakage from the two lower limbs and pole-pieces into the bedplate. Sheet-

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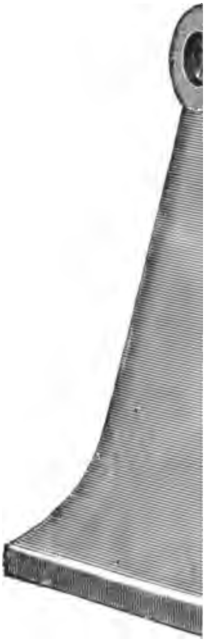
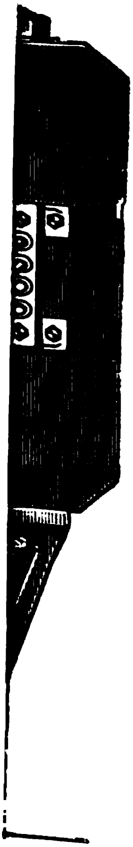
[1] For a machine of the same type but with 6 poles, constructed by Messrs. Brown, Boveri & Co., to give 1,025 ampères at 400 volts for the Heilmann locomotive, see the *Engineer*, February 9, 1894.

[2] Similar designs have been adopted by the General Electric Company of America for electric traction work, and are fully described and illustrated in *Engineering*, August 16, 1895.





ce p. 435.

metal formers with wooden flanges, insulated with mica and varnished tape, are used to carry the field-coils : these, as well as the armature, are carefully dried in an oven, both before and after winding.

The armature core is composed of charcoal-iron discs, insulated from one another by varnish : they are mounted on several aluminium bronze radial arms, which are fitted into projections running along the length of a cylindrical cast-iron hub, the latter being itself keyed to the steel shaft. The driving force is transmitted from the core to the armature-winding by means of hard fibre driving-teeth securely fitted into the substance of the core and projecting through the armature bars. These are of Crompton's patent stranded cable, compressed into a rectangular shape, sufficient insulation being maintained between the separate strands after they are compressed to avoid eddy-currents. The end-connectors of the drum-winding span approximately a quadrant of the armature, and are of copper strip bent to the required form ; at the engine or pulley end of the armature the copper strips are soldered into a segmental hub, built up and insulated very much after the fashion of a commutator, while at the other end they are soldered directly into the commutator segments, the necessity for a separate commutator lug being thereby avoided (cp. fig. 161). The armature-winding is connected into either two or four parallels, according as a high or low voltage is required.

The machine illustrated has an output of 50 volts, 1,000 ampères at 450 revolutions per minute, and is arranged for rope-driving with a bearing on either side of the grooved pulley. There are only two sets of brushes, 90° apart. The commercial efficiency is 90 per cent., and after a full-load trial for ten hours the rise of temperature did not exceed a limit of 70° F.[1]

Similar machines have been supplied for central-station work, coupled directly to Willans high-speed engines, and giving 200 or more volts for use in connection with a 3-wire system of mains.

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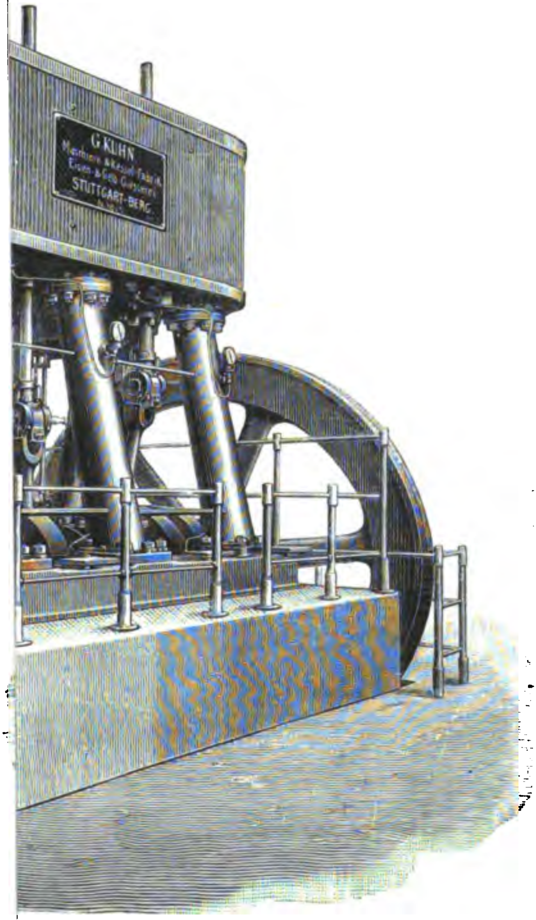
[1] In a similar machine, described in *Industries*, April 25, 1890, for 2,500 ampères and 50 volts at a speed of 400 revolutions per minute, the armature conductors, being few and massive, had faced joints and were screwed together with steel bolts and nuts: four sets of six brushes, each 2" wide, were employed to collect the current.

One such, coupled to a Willans I.I. engine, and giving 225 volts, 600 ampères at 350 revolutions per minute, was exhibited at the Crystal Palace Electrical Exhibition of 1892: the armature core was 28″ diam. × 16″ long, and was wound with 214 bars, while the magnet-slabs had a sectional area of 14″ × 12″. As compared with 2-pole dynamos of equal output, Messrs. Crompton & Co. claim a decided superiority for the above type on the grounds of its lesser liability to troubles from armature reaction and sparking, and its greater cheapness to manufacture.

(9) The 10-pole dynamo of fig. 167, manufactured by Messrs. Siemens & Halske, of Berlin, brings before us a remarkable type of machine, almost unknown in England, but much favoured on the Continent. In this type, which is peculiarly suitable for large central-station dynamos, the poles are internal and stationary, while the external ring-armature revolves. The machine illustrated was exhibited at the Frankfort Electrical Exhibition of 1891, driven directly by a triple-expansion condensing engine of the marine vertical type by G. Kuhn, of Stuttgart-Berg: the output of the dynamo was 150 volts and 2,200 ampères, or 330 kilowatts, at 75 revolutions per minute, the engine being of over 500 H.P.; at 100 revolutions per minute the output may be raised to 600 kilowatts, or 200 volts, 3,000 ampères, without unduly loading the machine. The internal magnetic system consists of ten poles, radiating outwards from a massive central yoke-ring, very similarly to the internal field of fig. 73; the whole is cast in one solid piece and bolted to the end of the engine bedplate. Each pole is wound with a magnetising coil 14″ long, the entire field-winding forming a shunt to the external circuit.

The armature core is constructed of thin iron plates, supported from a large cast-iron star-frame. The latter has thirty arms radiating outwards from it, and through a boss at the end of each arm projects a steel bolt; holes through the middle of the segmental core-plates enable them to be threaded over the steel bolts, and they are then compressed by nuts at the ends of the bolts. Each of the thirty bolts is carefully insulated from the core, in order to prevent the formation of eddy-currents along its length.

The engine crank-shaft is continued through the centre of the system of poles, and to its outer end is keyed the star-frame. The



ke.

To face p. 436.

completed core is thus held out, concentric with and overhanging the internal poles. The external diameter of the armature is approximately 10 feet, while the diameter of the circle drawn round the periphery of the curved pole-pieces is but little less than 9 feet: these dimensions sufficiently illustrate how small, as compared with the diameter, is the radial depth of iron in such a multipolar armature. The Gramme winding consists of 810 turns of solid copper, each turn being composed of a bow-shaped piece, which embraces the interior and sides of the ring, and an external connecting piece riveted and soldered to the end of one inductor and the beginning of the next at the outer edges of the armature periphery. A striking feature is the absence of any separate commutator, the collecting brushes being arranged to press directly on the bare exterior connectors. The possibility of this in a ring armature has not been hitherto alluded to in our pages, but a reference to fig. 98 will show that in the ring-winding the potential rises progressively as we pass from loop to loop, from one neutral point of collection to the other, after the same law as in the commutator sections, and that if the armature wires were bare at any part of their length, the brushes might be shifted so as to touch immediately on the actual winding. In the present machine the external bars are bare, each being merely insulated from its neighbour by an intervening strip of a kind of compressed paper or cardboard: in shape they are broader and deeper than the internal inductors, being nearly $\frac{7}{16}''$ wide, and the surface of the whole set is turned up in place on the armature. The ten sets of brushes are supported by as many horizontal arms so as to bear on the outer periphery of the armature, as seen in the figure; in each set there are three brushes, $1\frac{3}{4}''$ wide, formed of rectangular copper wires laid side by side and soldered together at their outer ends. The five positive and five negative sets of brushes are respectively connected in parallel to two common terminals. The stellate brush-carrier, with its ten radial arms, is bolted to the outer bearing; a hand-wheel and pinion gearing into a spur-wheel rim on the brush-carrier enables the whole to be rotated, while by moving a lever all the brushes can be simultaneously raised from or lowered on to the armature. The outer bearing, 26″ long, is bolted to a cast-iron base, which is itself bolted to a pair of rails below the level of the lowest brush-spindle; thus, on loosening the bolts, the base and all the brush-gear can be slid back, leaving the armature entirely free. The diameter of the shaft where the armature hub is keyed on is $13\frac{3}{4}''$, and within the bearings on each side, $11\frac{3}{4}''$,

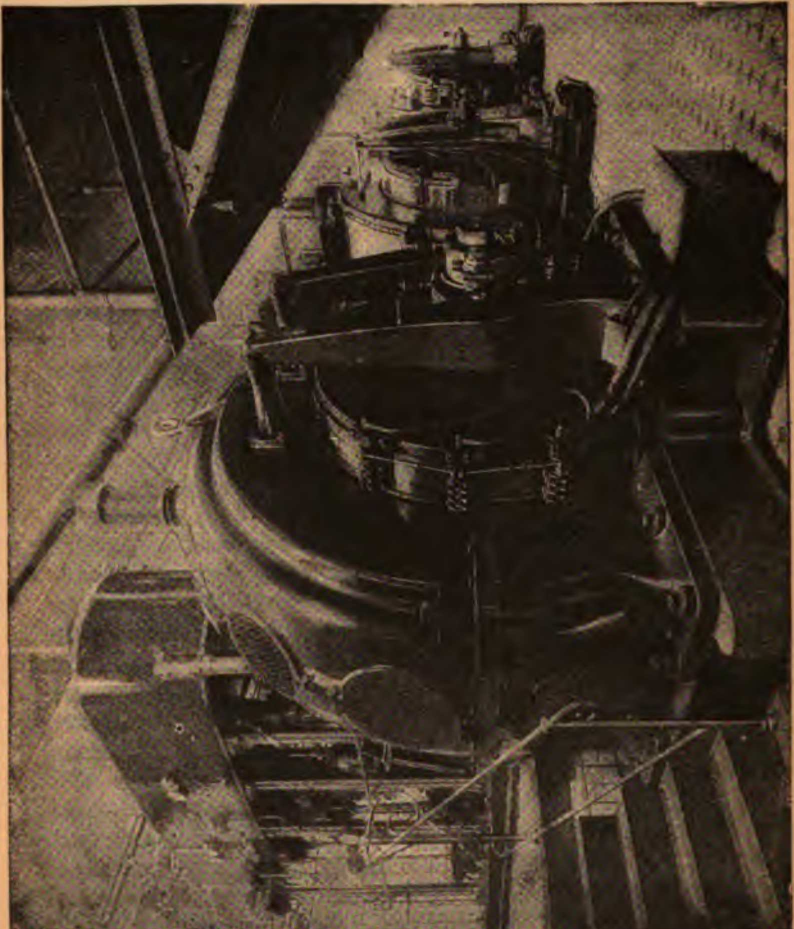


FIG. 168.—Schuckert dynamos in Hanover electric light station.

while the distance from centre to centre of the bearings is nearly 8 feet.[1]

At the Berlin central stations there are several dynamos of the same or very similar construction, with outputs of 400–500 H.P., and so far as can be judged by our present knowledge, the mechanical construction of the overhanging armature with its insulated bolts has proved entirely satisfactory, no difficulties being experienced arising from any de-centering of the ring. The internal position of the magnets reduces the weight of iron very considerably, since the mean diameter of the yoke-ring is so much smaller than would be the case if the poles were external to the armature, and the yoke-ring external to the poles. It is worth remarking that the centrifugal force on the armature is opposed by the pull of the field-magnets on the iron core. In large multipolar machines for central station work, having a separate commutator perhaps 3 or 4 feet in diameter, and with several hundreds of sections, the cost of this commutator and its connections is a very serious item, both when first manufactured and when it requires renewal. In the present machine this cost is largely reduced by the adaptation of the exterior winding to serve the purpose of a commutator: the only disadvantages are the high peripheral speed of the brush surface (2,000–3,000 feet per minute) and the somewhat inconvenient position of the lower sets of brushes.

(10) The Schuckert flat-ring dynamo of figs. 168, 169 is another form of large multipolar machine which may be found in many Continental central stations: it is manufactured by the Elektrizitäts-Aktiengesellschaft (formerly Messrs. Schuckert & Co.) of Nürnberg. This dynamo has a discoidal-ring armature revolving between two crowns of numerous poles, the divided magnetic circuit being of the type represented diagrammatically in fig. 132. The magnetising coils are so wound that opposing poles are of the same sign, while the poles on one and the same side of the armature are alternately N and S. The cast-iron magnet-cores are oblong in section, and are cast in one with the two halves of the yoke-ring on either side; these are surrounded by overarching

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[1] Esson (*Electrical Review*, xxix. 1891, p. 342) to whose excellent description we are indebted for many of the above details. *Electrician*, August 28, 1891.

flanges, which, when the two sides are bolted together, form an enveloping casing pierced with ventilating holes. The lower halves of the yoke-rings are secured to cast-iron girder bed-plates; the girder at the commutator end also carries the outside bearing, and from the latter rise two massive arms, forming a fork, to which the upper half of the magnet casting is braced.

The armature core is formed by coiling soft iron ribbon upon a brass foundation-ring, a continuous strip of thin paper being wound on at the same time in order to insulate adjacent convolutions and prevent the flow of eddy-currents. At intervals radial screws pass through the layers of iron ribbon into the ring, and serve to bind the whole together (fig. 169). As these screws are embedded in the centre of the core in a vertical plane, they do not cut the lines of the field, and therefore do not require to be insulated. The brass ring is mounted on a cast-iron six-armed hub, keyed to the shaft. In order to prevent end-play of the armature, the outer journal has turned on it two thrust-collars, which run in corresponding grooves in the bearing, and there are also two screws for setting the armature centrally between the poles, so that there may be no unbalanced magnetic pull (cp. p. 275).

A striking feature of these central-station machines is the size of the commutator. The dynamo illustrated is one of several which have been supplied to the Hanover and Altona electric light stations. While the outer diameter of the armature is 10 feet, the diameter of the commutator is as much as 6 feet 5 inches; it contains 700 segments, or fifty per pole, the total number of turns on the armature is 1,400, the current in any inductor being 70 ampères, and to this large number of sections is due the absence of sparking at the brushes. The segments are cross-connected (p. 254), so that only four sets of brushes are used. These are arranged, two on either side, in a convenient position for examination and attention, the angular distance between each pair corresponding to the pitch of a pair of poles. In each set there are three brushes each 2″ wide. By means of a system of links one lever lifts or lowers all the brushes simultaneously. A facing on the magnet-frame enables a tool to be attached thereto for the purpose of turning the commutator in place. The output

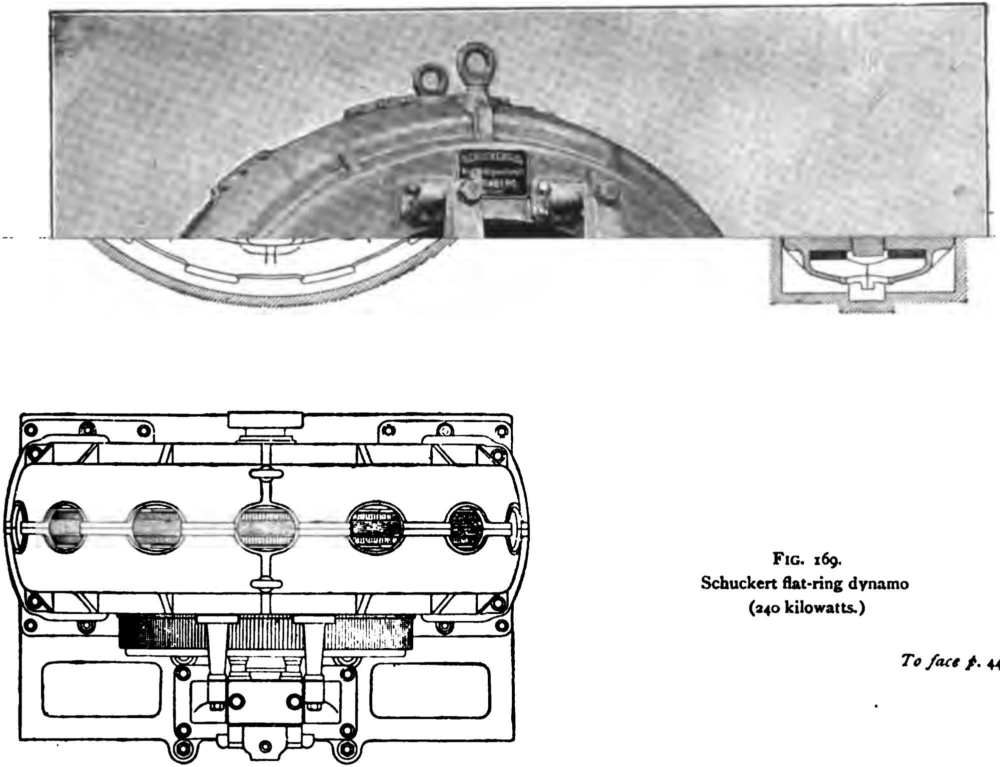

FIG. 169.
Schuckert flat-ring dynamo
(240 kilowatts.)

To face p. 440.

is 220 volts and 1,000 ampères at 110 revolutions per minute, or, when charging accumulators, from 270 to 335 volts, while the speed rises to 135 revolutions and the current decreases to 800 ampères; thus the average output is approximately 240 kilowatts at 120 revolutions, or 2,000 watts per revolution per minute. It is coupled directly to a 350–450 H.P. engine by Schichau, which stands on an entirely separate foundation: the engine has three cranks, and from the nature of the armature no separate fly-wheel is required. The voltage of the dynamo adapts it for connection across the two outer mains of a three-wire system, the third wire being coupled to the middle of the battery of accumulators which is in parallel with the dynamo, and is charged by it at the higher limit of voltage. The combined efficiency of the engine and dynamo is 82 per cent., that of the dynamo alone being 93 per cent. The field is shunt-wound, and approximately 33 ampères are employed for excitation, or 3½ per cent. of the output. In itself the double set of magnets is uneconomical in field-wire, but against this must be set the fact that the magnet-cores have an economical section. Finally, the discoidal-ring armature possesses the two recommendations that it is excellently ventilated and can be driven at a high peripheral speed, the nature of the winding being well adapted to withstand centrifugal force.

II. *Open-coil continuous-current machines.*—In this class two machines, and two only, have attained to complete practical success: they are the Brush and the Thomson-Houston dynamos, the former of which is manufactured by the Brush companies in England and America alike, while the latter is manufactured by the Thomson-Houston Electric Company. Although entirely dissimilar in construction, both machines are adapted to give a high potential, amounting in the larger sizes to as much as 3,000 volts, and both are pre-eminently arc-lighters. Their current, although pulsating slightly in each revolution, is uni-directed, and has a steady mean value, while, further, by special devices it is maintained very approximately at the same mean value, even when the resistance of the external circuit is considerably altered. They are thus admirably suited to the lighting of a number of arc-lamps arranged in series at considerable distances apart, and requiring a constant current with varying potential; in fact, provided the mean value of the current be maintained constant, its pulsatory character would appear to be of positive advantage in steadying the light, probably because the pulsations keep the mechanism of the lamp in a constant state of tremor, and so render its various parts less liable to stick or become sluggish in their action.

(1) *The Brush dynamo* may be shortly classified as a discoidal-ring armature with large Pacinotti teeth placed in a horizontal double-horseshoe field. Fig. 171 shows in outline a side and end view of a machine manufactured by the Brush Electrical Engineering Co., and known as their 8 L size; when running at 800 revolutions per minute it is capable of lighting 55 arc lamps in series, each taking 50 volts × 10 ampères, its output being therefore 10 ampères, and nearly 3,000 volts at its terminals.

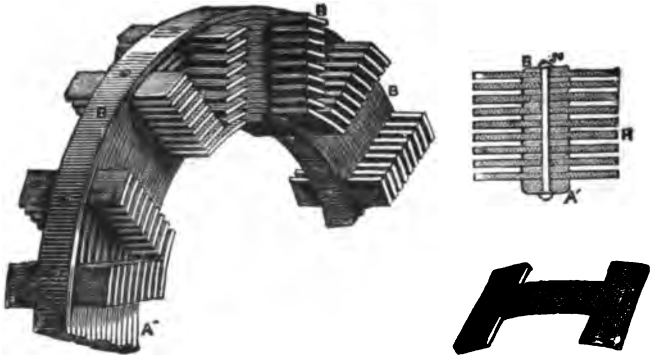

FIG. 170.—Construction of armature of Brush dynamo.

The construction of the laminated armature core is indicated in fig. 170. On a foundation-ring (A) of wrought iron is wound a continuous soft-iron ribbon (B) ·059″ thick: between each successive convolution, and at due intervals round the circumference, are inserted H-shaped iron stampings of the same thickness as the ribbon. Bolts passing radially through ribbon and stampings into the ring A, and riveted over at either end, serve to secure the whole. The H-pieces being all of the same size and placed one above the other, their projecting teeth form deep grooves with parallel sides, and in each such groove a coil of many turns is wound. The adjacent projections from two sets of H-pieces converge towards the inner periphery of the ring, and thus form large Pacinotti teeth, which reduce the length of air-gap, and drive the inductors through the magnetic field. Before winding the armature coils, the recesses are insulated with a wrapping of strong canvas, saturated with shellac varnish, and with sheets of tough paper, also treated with shellac. The coils are then wound on, all in the same direction, wrappings of calico being inserted occasionally between the layers. The inner ends of each pair of coils, situated at opposite ends of a diameter, are soldered together and carefully insulated : the outer ends, also insulated, are led through a hole bored lengthways in the shaft, through the bearing, and so to two opposite segments of a commutator. The machine illustrated has four pairs of coils, and therefore two double commutators, the arrangement of which has been already described in Chap. X. and illustrated in fig. 87. The segments are of thin metal, screwed to insulating rings, and separated by air gaps: this construction allows of the easy renewal of the segments, which are comparatively quickly worn out by the sparking at the brushes (cp. p. 392). The brushes are of thin sheet-copper slit at the one end into a number of teeth (p. 393); each pair is adjustable by rotating a lever round the shaft, and connection is made to the terminals mounted on a slate slab in the bedplate by flexible copper strips of zigzag shape, as shown in fig. 171.

The double-horseshoe field is of the type shown in fig. 127, but with the centres of the poles on a horizontal diameter. The cores on which the field-coils are wound are of cast iron, and are bolted to the cast-iron vertical end-frames which form the yokes; the curved pole-pieces are extended considerably on both sides of the magnet-cores, and thus cover a large portion of the sides of the armature. The similar poles which face each other on either side of the armature ring are not united by any pole-shoe, and therefore the lines enter the core only from the sides, and chiefly by the Pacinotti teeth. Before being insulated, each magnet-core is surrounded by a thin sheet of copper soldered together at its edges : the field-winding is in series with the armature and external circuit, and hence the pulsating nature of the main current, which is also the magnetising current, tends to produce fluctuations in the magnetism of the field, and consequently eddy-currents in the cores: the object, therefore, of the copper envelope is to damp down any fluctuations in the field, the iron core being as it were shielded by the induced currents which are set up in the copper. Over the copper sheaths is wound insulation of varnished vulcanised fibre and stout paper, and at intervals the layers of wire are further insulated by strong cotton sheets, well varnished with shellac. Indeed, careful insulation is throughout of great importance, owing to the high difference of potential, 3,000 volts at full load, which may exist between the winding and the ironwork of the machine.

The reaction of the armature on the field in this type of dynamo is very considerable; as the result, the field is powerfully distorted, and the lines are densely concentrated at the trailing corners of the pole-pieces. The line of maximum field thus passes through the trailing corners, these being in the figure the upper corner of the right-hand, and the lower corner of

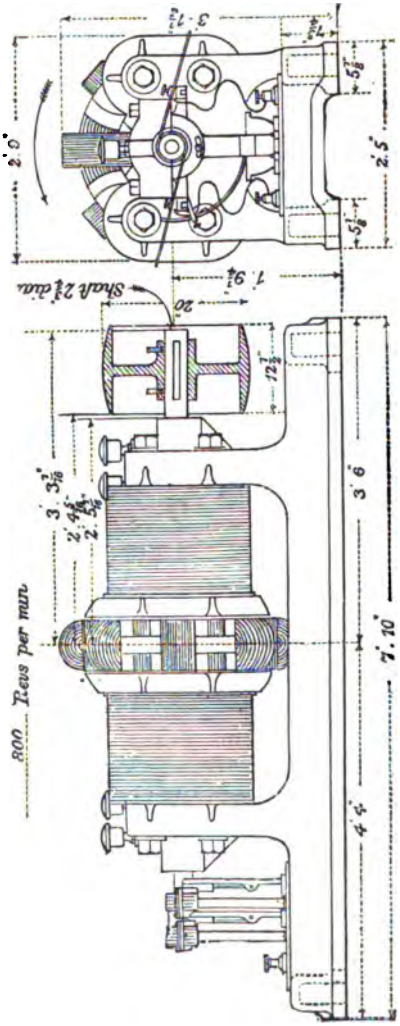

FIG. 171.—8 L Brush arc-lighting dynamo.

the left-hand pole-piece. As, however, the dense fringe does not extend much beyond the trailing pole-edges, the diameter of commutation is somewhat behind the line of maximum field, in order that the time of contact of a coil may approximately coincide with the time during which a strong E.M.F. is generated in it. Thus, in the diagrams of Chap. X., since the subject of armature reaction had not then been discussed, the line marked *m m* is not so far in advance of the horizontal diameter as the line of maximum density of field would in reality be; it corresponds rather to the diameter of commutation and indicates the lead of the brushes ahead of the line of symmetry. The influence of the Pacinotti teeth in heating the poles has been alluded to in Chap. XVIII., p. 413; hence at the pole-corners the normal rise of temperature of the field, viz. 70° F., is exceeded, without, however, any prejudicial effect on the working or life of the machine.

The commercial efficiency of the open-coil dynamo is not so high as that of the closed-coil machine of equal output, owing to the pulsating character of the current, the sudden changes in the connection of the few sets of coils, and the inevitable sparking at the brushes, together with a greater liability to eddy-currents in both armature and pole-pieces. An average value is probably not more than 75 per cent., although large sizes are built under a guarantee of 80 per cent. commercial efficiency.

When employed for arc lighting, the current of the machine is kept constant, even when the number of arcs is varied, by means of an automatic regulator. This consists of a variable carbon resistance placed as a shunt to the field-coils, and so arranged that when the current rises above the normal the carbon resistance is decreased: a greater proportion of the main current is thus diverted from the magnetising coils, and passes through the shunt, the result being that the strength of the field and the induced E.M.F. are reduced, and the main current is brought back again to its normal value. Similarly, when the main current falls below the normal, the resistance of the carbon shunt, and consequently the induced E.M.F., are increased until the normal strength of current is re-established. The resistance consists of a number of thin blocks of retort carbon resting one above the other: the closer these are pressed together, the less is their resistance. The whole series is acted on by a lever connected to the armature of an electro-magnet, and this latter is excited by a solenoid, round which the main current passes. When the main current rises or falls in value, the pressure exerted by the lever upon the carbon blocks is increased or diminished; thus the resistance of the shunt is automatically varied, and the magnetising current round the field coils is altered so as to maintain a constant external current. The self-regulating quality of the machine is the special feature which renders it peculiarly adapted for high-tension series arc lighting. It may be short-circuited with impunity, and is therefore less likely to be injured by overloading or by sudden and large fluctuations of load than machines having approximately flat characteristic curves (cp. p. 342).

In the United States large Brush machines have been employed, coupled directly to high-speed engines; these give as much as 6,250 volts at 525 revolutions per minute, and are thus capable of feeding as many as 125 arcs in series. The field has four poles with magnet cores of soft steel, and the twenty-four coils of the armature, each containing 528 turns, are cross-connected into six sets. Each set is composed of the four coils which have the same position relatively to the four poles, or are situated 90° apart, and these four coils are connected in series. The six sets of coils are joined to three commutators; instead, however, of each commutator having only four segments, it is divided into eight segments, those which are diametrically opposite being cross-connected. Hence, although there are four poles, only one pair of brushes is necessary for each of the three commutators, and these brushes are joined in series as described on p. 174. The characteristic curve of the machine rises gradually to a maximum of 9·6 ampères and then falls rapidly. Thus its output is 60 kilowatts, its weight being five tons, and its over-all dimensions approximately 4 feet in each direction.[1]

(2) *The Thomson-Houston dynamo* is the joint invention of Professors Elihu Thomson and E. J. Houston, and is unique alike in its action and construction. Fig. 172 gives a perspective view of the machine, while fig. 173 is a longitudinal section of the

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[1] See *Electrician*, October 13, 1893, and *Electrical Engineer*, April 5, 1895.

field-magnet, exposing to view the armature. The main features are a spheroidal armature wound ring-fashion, with only three sets of coils, a 3-part commutator combined with a remarkable device for maintaining a constant current, and a peculiar 2-pole magnet with hemispherical pole-pieces, between and within which the armature revolves. The more theoretical consideration of the

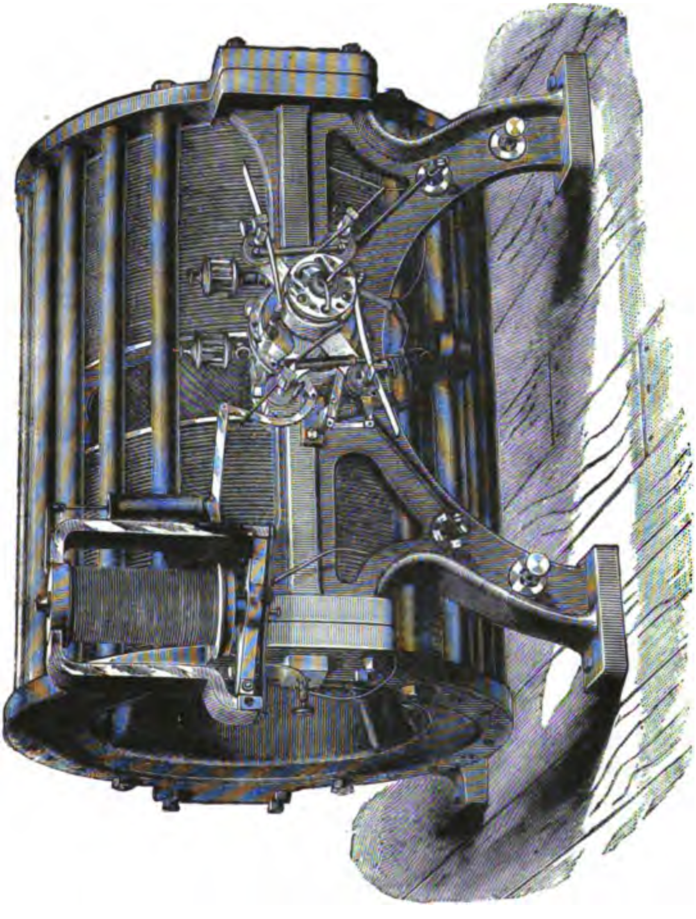

Fig. 172.—Thomson-Houston dynamo.

armature and commutator has already been dealt with in Chap. X. pp. 176–81, and it only remains to describe the constructive details.

Formerly the armature was made of iron wire and was wound as a drum, but by a more recent construction it is formed of iron discs threaded on the arms of a gun-metal spider. The circular discs are reduced in diameter from the middle of the core to the ends, so that when completed it has a spheroidal shape (fig. 173 *a*); it is then wound with six sets of ring coils, radiating outwards from the centre, and spaced so that any section can be easily rewound or separately repaired. Each set of coils is connected in series with the diametrically opposite set (numbered with corresponding figures in the diagram). The three symmetrically placed ends of the three pairs of coils are united to a common copper ring insulated from the shaft. The remaining three ends are then insulated and led through the hollow shaft to the segments of the 3-part commutator outside the bearing. Each segment is nearly

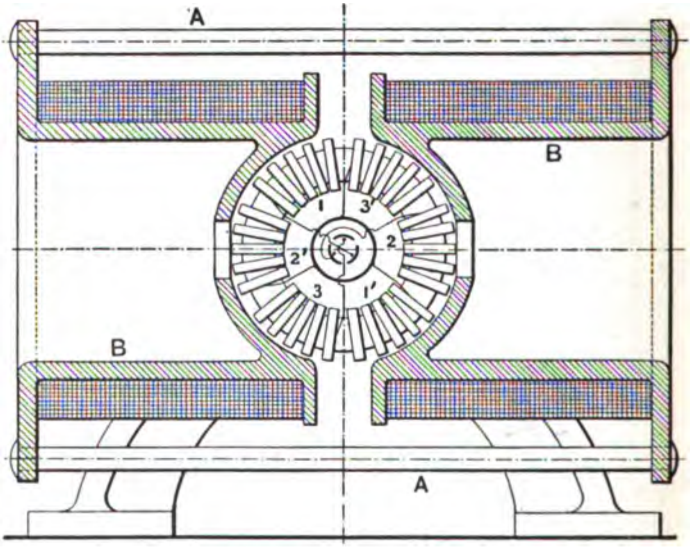

FIG. 173.—Section of Thomson-Houston dynamo.

120° wide, and insulated from its neighbour by small air-gaps. On this commutator rest four brushes, coupled two-and-two together, as shown in fig. 91, each pair being equivalent to one brush with a wide arc of contact.

The shape and design of the field magnet will be seen from figs. 172 and 173. The hollow cylindrical magnet-cores, B B, are cast in one with the cup-shaped pole-pieces, and terminate in circular flanges, between which the field-coils are wound. To the outer end-flanges, which are of larger diameter, are bolted a number of wrought-iron bars, A A, stretching lengthwise across the machine outside the field-coils, and forming the yoke. Strong standards, bolted to ears projecting from the end-flanges, carry the field-magnet and the bearings of the armature. The cupped pole-pieces have each a central hole to assist the ventilation of the armature, and are accurately turned on the inside to a spherical surface. In this, as in the Brush machine, the line of maximum field passes approximately through the trailing edges of the pole-pieces, and, therefore, a diameter passing centrally through the brushes falls near these edges. Not only does each composite brush embrace an extraordinarily wide arc of contact, but the arrangement is unique in that this width is variable and dependent upon the current of the machine. As explained in Chap. X. the normal width of each pair of brushes at full load is 60°, that being the angle which separates the following brush from the leading brush. But if, for any reason, such as the switching-off of some

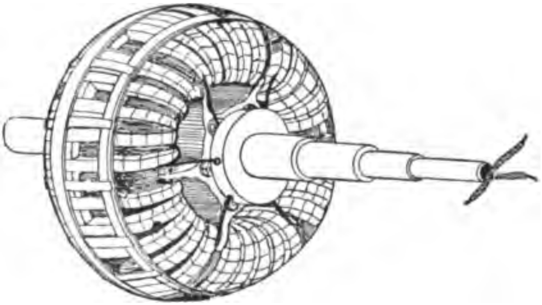

FIG. 173*a*.—Ring armature of Thomson-Houston dynamo.

of the arc-lamps, the current rises above its normal value, it acts upon an electro-magnet (seen to the left in fig. 172), and this, by means of a system of levers, separates the two portions of each brush to a still greater distance apart. The following brush is shifted backwards and the leading brush forwards; the latter is, however, only moved through one-third of the angle through which the following brush is moved, such an amount of forward movement being necessary in order to lessen the sparking that would otherwise occur, owing to the field being displaced further forwards by the increased reaction of the armature current. Since, when so moved, each brush covers more than 60°, it follows[1] that for certain periods in each revolution the + and – brushes both touch the same commutator segment; the armature is then short-circuited on itself, and the effective E.M.F. of the machine is reduced, until the current is once more brought to its normal value. When the current falls below the normal, each pair of brushes is closed up, the result being to cut out an inactive coil, and lessen the time during which it is in parallel with a coil of greater activity. A constant current is thus automatically obtained, although in a very different manner from that lately described in connection with the Brush machine. As might be expected, there is considerable sparking at the brushes, and in order to reduce this evil the inventors have adopted the novel and bold expedient of blowing the sparks out as they are formed, by means of small air-blasts delivered on the tips of the two leading brushes, and occurring just at the moment when each segment breaks contact with the brush. The blower is placed behind the commutator in a small circular case, from which two nozzles project, one in front of each leading brush. Three times per revolution each nozzle automatically delivers a blast, which impinges on the rear edge of the segment as it leaves the brush.

The field-winding is in series with the armature, the current passing through one magnet coil, then through the external circuit of lamps, and returning through the second magnet-coil to the armature. The current is very considerably steadied by the self-induction of the magnet, and it is not surprising to find that if the field-magnet be separately excited, and the external circuit be joined directly to the armature, the proper working of the machine becomes impossible.[1]

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[1] See Chap. X. pp. 179–181.

The normal currents required by the two standard sizes of Thomson-Houston arc lamps are 9·6 and 6·8 ampères respectively, and for these currents the standard machines are usually wound. One of the larger sizes, capable of supplying 9·6 ampères to 35 arc lamps in series when running at a speed of 820 revolutions per minute, has an armature 2 ft. in diameter, the resistance of the armature and of the field-winding being 10·5 ohms each. The floor-space required is 5½ ft. × 4 ft., and the standard pulley is 16″ diameter × 8″ wide. The machine is surprisingly light for its output; a result due to the fact that the amount of iron in both the armature core and magnet is extremely small, while the proportion of the weight of copper to the weight of iron is correspondingly high.

III. *Alternators.*—(1) *The Lowrie-Parker alternator*, of which a general view is given in fig. 174, owes its title to its joint designers, Messrs. W. Lowrie and T. Parker (Patents Nos. 3,710, 1882, and 12,907, 1888), and is manufactured by the Electric Construction Company, Limited, formerly Messrs. Elwell-Parker, of Wolverhampton. It belongs to the type represented in fig. 73, with a drum-wound stationary armature, within which revolves the field-magnet system: the latter is formed by a number of magnet-cores mounted radially on a common yoke-ring, and presenting a series of pole-faces, alternately N. and S., to the internal surface of the cylindrical armature core. The machine of fig. 174 gives 2,000 volts and 75 ampères at 386 revolutions per minute, and is driven directly by a Willans engine. It has 26 poles of alternate sign, whence its periodicity is $\frac{26 \times 386}{120} = 83{\cdot}3 \sim$ per second when running at its stated speed. The shaft carrying the magnet system is 7″ diameter in the centre, and is supported by bearings 15 in. long × 4½ in. diameter, and coupled directly to its outer end is the exciter. This supplies energy at the rate of 1,400 watts for

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[1] Milton Thompson, 'A Study of an Open-coil Arc Dynamo,' reprinted in the *Electrical Review*, June 19, 1891.

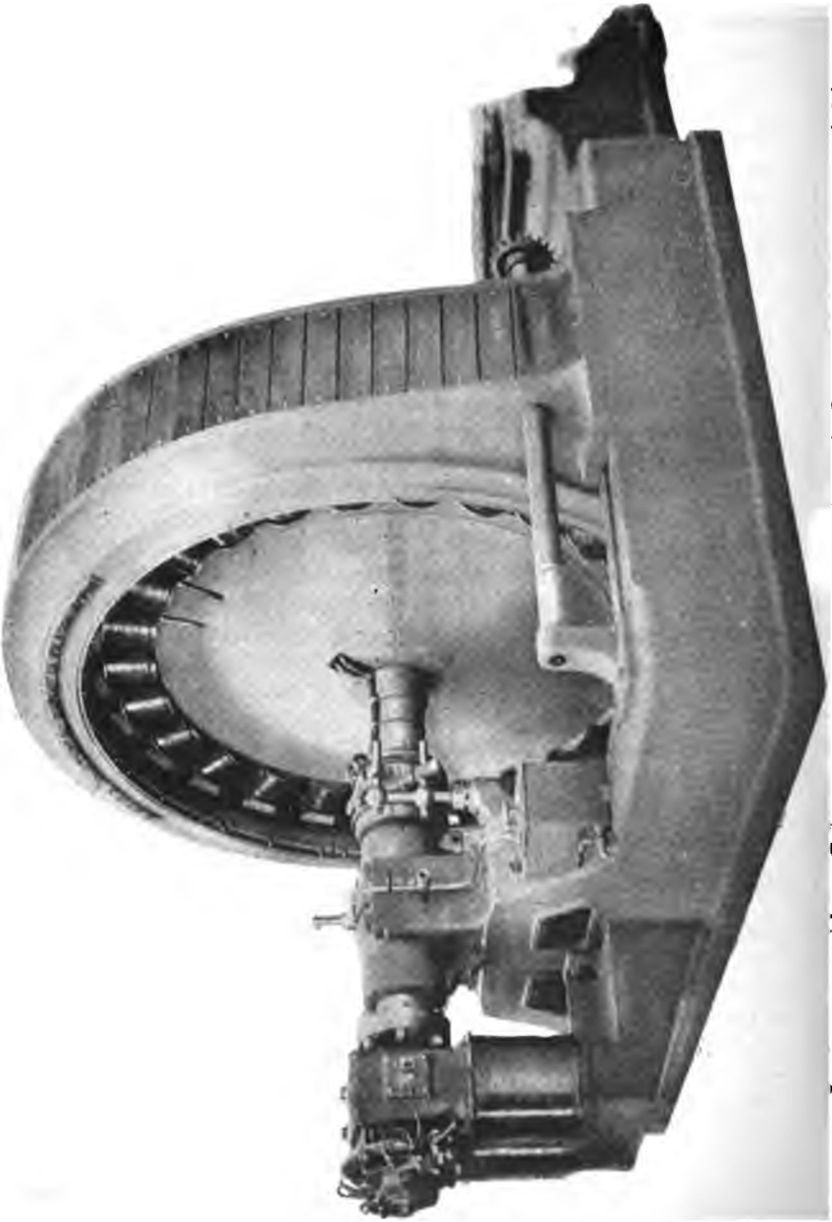
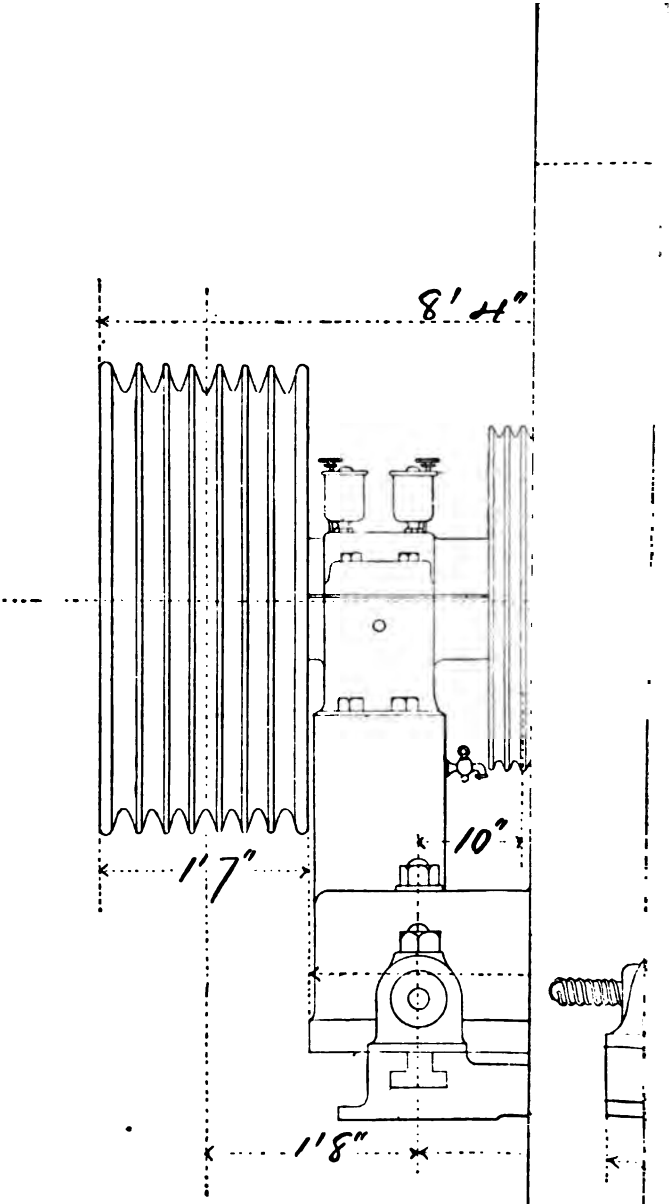


Fig. [illegible] of Electric Construction Company.



lternator.

full load, the commercial efficiency of the alternator and exciter combined being 92 per cent. at full load and their weight 9 tons. The armature ring is provided with a screw traversing gear to allow of its being shifted for inspection of the magnet-poles or armature coils. The construction is more clearly seen in fig. 175, which shows in outline a side and end elevation of a 100-kilowatt alternator, giving 2,000 volts and 50 ampères at 400 revolutions per minute. On a shaft of mild steel 6″ in diameter is keyed a cast-iron wheel, the spokes of which are united by a solid rim of mild steel shrunk on. To this are bolted a number of soft wrought-iron magnet-cores, on each of which is slipped an exciting coil. Each magnet-core is surmounted by a pole-piece, the edges of which project over the exciting coil; two long bolts pass from the pole-piece through the core and rim of the wheel, and are fastened by nuts on the under side of the latter. Both magnet and coil are thereby secured in place, and the stress due to centrifugal force, which is very considerable, is taken by the rim of the wheel in a thoroughly mechanical manner. In order to obviate the loss of power which would be caused by the arms of the wheel churning the air, the ends are closed by thin plates of sheet-iron screwed on to either side of the rim. The poles being twenty-eight in number, the periodicity or 'frequency' of the machine is $\frac{28 \times 400}{120} = 93 \sim$ per second.

The armature core is composed of segmental plates of thin charcoal iron, insulated by paper, and bolted between two cast-iron end-rings having a channel section; to the inner surface of this core are fastened the twenty-eight armature coils. These are formed of thin copper tape, and are wound separately from the armature, each on a wooden former of the same thickness as the width of the tape and rounded at its ends; a flat coil of rectangular shape with rounded ends is thus obtained, similar to those shown in fig. 73. The finished coil is, however, not truly flat, but is curved to the radius of the inner periphery of the armature core, and hence when placed against the inside face of the latter it fits closely to its cylindrical surface. The coils are laid close to each other, and are fastened in place by strips of wood; the internal surfaces of the cast-iron end-rings are lined with curved wood strips, and to these are screwed other segmental strips outside the line of path of the magnet so as to clamp down the ends of the coils. The axes of the coils being radial, the edge of the copper tape is exposed on their flat surface facing the poles: they are therefore free from any liability to droop or sag inwards, and no binding-wire being required, the clearance between the revolving poles and the stationary inductors can be reduced to a minimum. The width of the internal insulating piece in the centre of each coil is equal to the width of the pole-piece, the outside width of each coil being of course equal to the pitch (cp. Chap. VIII. p. 142). Not only are the coils well insulated from the core, but the latter is itself insulated from the cast-iron frame of the machine on which it is carried, a double insulation being thus obtained between the armature circuit and earth. As the armature coils are simply secured to the inner face of the core, each can be easily removed and replaced. They are joined together in series, the connections being made in the channels of the end-frames, and the free ends of the winding are brought to terminals mounted on porcelain insulators: these being inclosed within the wood lagging which encases the whole armature, it is impossible to accidentally touch any portion of the high-tension circuit. Small terminals are also provided to enable a voltmeter or regulator to be placed across one or two of the armature coils. The magnet-coils are joined in series, the ends of the winding being brought to two gun-metal rings insulated by ebonite washers from each other and from the shaft on which they are mounted. The brushes by which the exciting current is passed into the rotating field-magnets are supported with their holders on two arms projecting from the base-plate: against each ring two brushes bear, either of which is fully able to carry the maximum exciting current, when the other is lifted off for the purpose of adjusting or trimming it. The base-plate is of a strong box type, well ribbed on the under surface: on it are two planed facings, to which the armature frame is bolted. The bearings are each 20 inches long, with brasses of phosphor bronze or lined with white metal: each end of the bearing is provided with an oil-thrower and an oil-chamber, from which the waste oil is conducted into tanks in the base-plate of the machine. The exciter is driven off the alternator shaft by a pulley grooved

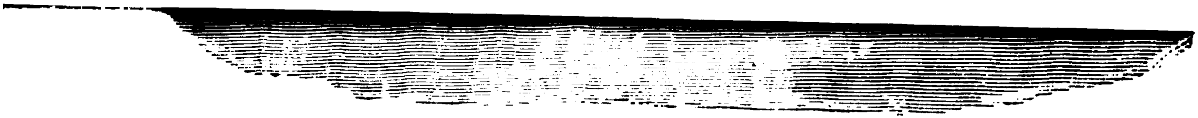


FIG. 176.—Hopkinson alternator.

To face p. [illegible]

for four ropes, ¾ inch diameter: it runs at a speed of 800 revolutions per minute, the energy necessary for excitation at full load being 3,250 watts. The alternator is itself driven by a second and larger pulley grooved for seven ropes, 1½ inch diameter. The total weight of the alternator is 9 tons 6 cwt., and that of the exciter 15 cwt., the over-all efficiency of the combination being about 87 per cent. at full load. A noteworthy feature of these machines is that they work well in parallel with a low synchronising current, and are so worked in several central stations.

(2) *The Hopkinson alternator* (fig. 176), manufactured by Messrs. Mather and Platt, may be described as the inverse of the previous machine, since in the present case the field-magnet system is stationary and external to a revolving drum-wound armature. Our illustration shows a 30-kilowatt machine, giving an output of 30 ampères at 1,000 volts when running at 800 revolutions per minute. The radial magnet-poles, alternately N. and S., are twelve in number, whence it follows that the frequency is 80 ~ per second: they are of wrought iron, screwed to a massive cast-iron yoke-ring which is bolted to the bedplate. Any magnet-core can be removed or replaced without disturbing the remainder or taking out the armature. The width of each pole-face is as much as three-fourths of the pitch, or even more, this large width being permissible owing to the fact that the armature coils are not laid flat upon the surface of a cylindrical core, but are wound upon short teeth of laminated iron: these project radially from the armature core and take the place of the wooden centres mentioned in connection with the coils of the last-described machine. As the coils are nearly as deep radially as the iron projections, the width of the winding for a given number of turns is much less than if the coil were wound in a single flat layer. The armature coils and plates are held by bolts passing through two thick plates keyed on to the shaft, and these end-plates are furnished with numerous openings to allow of free ventilation of the interior of the core. Any section of the armature can be removed without interfering with the rest. The armature resistance is ·55 ohm, and that of the magnet-coils 2 ohms. The field is excited in the machine illustrated by a 'Manchester' dynamo carried on a bracket bolted to the bedplate, and with its armature shaft coupled

directly to the alternator shaft. At full load the necessary exciting current is 21 ampères, requiring an expenditure of about 900 watts, or 3 per cent. of the output. The Hopkinson alternator is successfully worked in parallel, and owing to its high self-induction it may be short-circuited with impunity, the current not rising to so much as twice its normal value.[1]

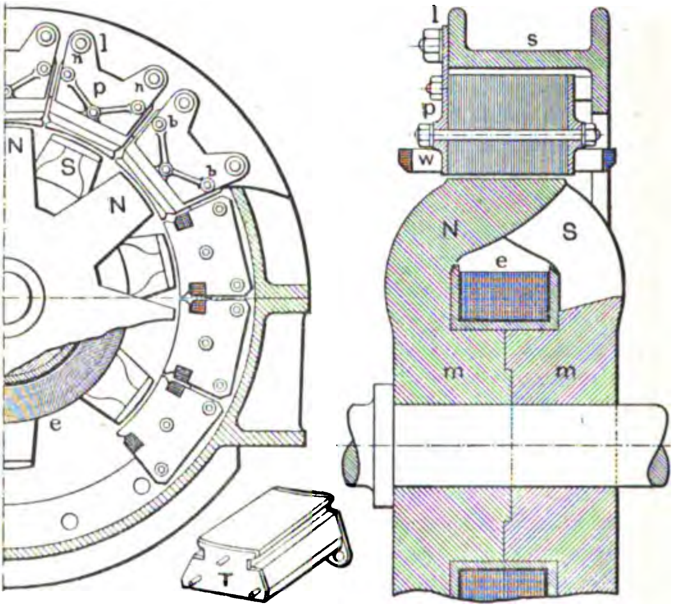


FIG 177. – End-view and section of Kapp alternator.

(3) *The Kapp alternator.*—The more recent type of alternator designed by Mr. Gisbert Kapp and manufactured by Messrs. Johnson and Phillip (fig. 177), differs entirely from the discoidal-ring machine described in the first edition of this book. The armature instead of rotating is now stationary, and is drum-wound as in fig. 73; the internal revolving field-magnet is, however, of different construction, and merits attention as an example of a multipolar machine with a single exciting coil (cp. p. 159).

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[1] See *Electrical Engineer*, November 9, 1894.

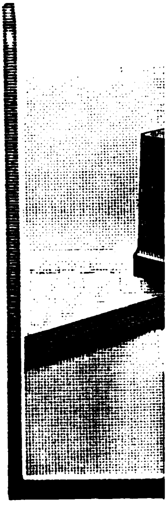

Fig. 177*a* illustrates in greater detail the construction of the armature and field-magnet. The latter consists of two steel castings (*m m*) provided with a number of polar claws curving over and presenting pole-faces on their periphery concentric with the shaft. These are bolted together in such a way that the claws on the one side project into the gaps between the claws of the other side, the single circular exciting coil (*e*) being embraced between them. Or the coil may be wound on a cast-steel core to which are bolted the two star-shaped castings. Thus a series of poles of alternate sign turned to a true circle on their circumference face the armature yoke-ring which surrounds them concentrically; the magnetic lines flowing up out of *e.g.* each N. pole bifurcate and pass through the armature core into the nearest halves of the neighbouring S. poles. The leakage directly across from pole to pole is large, but this disadvantage is more than counterbalanced by the economy of wire and exciting energy due to the use of a single coil. The exciting current is conveyed to the coil by a double set of brushes, resting on two contact rings of gunmetal which are mounted on and insulated from a cast-iron bush keyed to the shaft.

The armature is of novel and ingenious design. The annular core is divided circumferentially into as many truncated wedge-shaped sections as there are poles. One of these sections (T) is shown separately in fig. 177*a*: it is composed of thin annealed sheet-iron stampings, insulated from each other and strung on insulated steel bolts, which compress them between gun-metal end-plates (*p*) stiffened by ribs. The heads of two of the bolts (*b b*) in each sector are sufficiently long to engage in the flange of the cast-iron supporting shell *s*. The gun-metal endplate on the other side is provided with lugs *l l*, by which the sector is bolted to the outer flange of the shell. Thus to remove any armature section it is only necessary after disconnection of the wire of its coil to unscrew the nuts (*n n*) and draw it out in a direction parallel to the shaft. Each coil (w) is wound separately and completely insulated with lappings of mica cloth: it is then embedded in the longitudinal grooves which run along the sides of the sector near to its inner surface, and these grooves themselves may be lined with troughs of fibre. Thus thorough insulation between coil and core can be obtained by increasing the thickness of the enveloping material to any required extent without addition to the air-gap or increase in the exciting power, and further any risk of damage to the insulation from threading the inductors through holes in the iron is avoided. The insulation between coil and core is tested by subjecting it to an alternating difference of potential equal to at least twice the normal voltage of the machine.

When the sectors are in place the core plates of contiguous sections almost touch each other; the clearance left being merely enough to allow of the withdrawal of any section. Hence, the armature, although built up of sections distinct from each other, is magnetically almost equivalent to one with a continuous core, and its internal cylindrical surface being practically smooth and unbroken, there is very little change in the magnetic flux for different relative positions of armature and field.[1] The supporting shell of the armature is made in halves, and the lower half is bolted by suitable feet to the bedplate of the machine.

(4) *The Ferranti alternator*, illustrated in figs. 178 and 178*a*, has attained a widespread fame, not only from the fact that the largest alternators in the world, viz. the two 1,250 horse-power 10,000-volt machines at the Deptford Central Electric Light station, were designed by Mr. S. Z. de Ferranti, but also by reason of the great inventive ingenuity which is displayed in every detail of the machine's construction. It is of the disc type, represented in fig. 75, with an armature of large diameter and narrow width revolving between two circles of magnets: these latter on any one side of the armature are alternately N. and S., but in sharp distinction to a discoidal-ring machine such as that of fig. 169, the poles directly facing each other on either side of the armature are of opposite sign, so that the lines of induction flow directly across and through the armature from pole to pole, thus forming a series of fields of alternate direction.

The field-magnet system consists of a number of wrought-iron magnet-cores set in a supporting framework formed by two massive cast-iron yoke-rings. Each of these is divided on a vertical diameter into two halves, which are cast separately. The union of the cores to the yoke is effected in the simplest manner by casting the yoke on to them; half the number of magnet-

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[1] Patent No. 17,341, 1893.

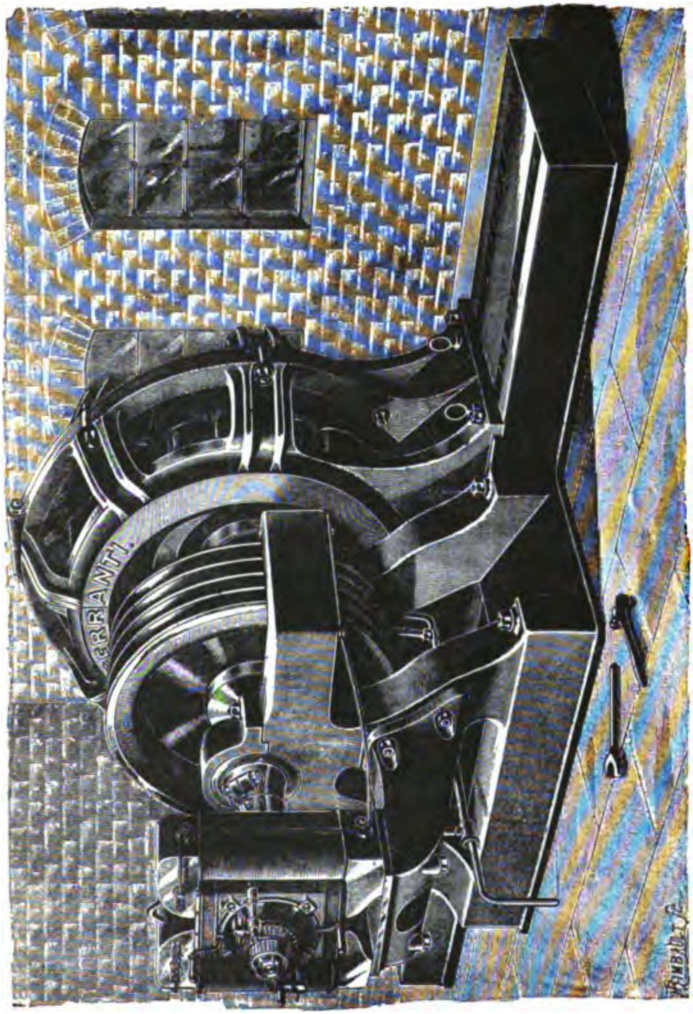

Fig. 178.—Ferranti alternator.

cores that go to form one complete crown are set up in the mould, and the cast iron is run in round them to form a half yoke-ring. On the magnet-cores are wedged the formers carrying the exciting coils. Curved arms springing out from the half-yoke ring form arches external to the magnets, and are united at their ends into a semicircular ring. When two such halves are bolted together side by side, they form one-half of the complete magnet frame; the distance between the opposing pole-faces of the magnets then only slightly exceeds the width of the armature, so as to allow of a small mechanical clearance on either side of the latter when it rotates, and this distance is preserved by the meeting midway of the central rings through which the connecting bolts pass. The planed base of the half-frame which is thus obtained rests on a planed facing of the cast-iron bedplate, along which it is free to move endways, but not sideways. The second half-frame is similarly formed, and the two are slid together until they unite on a vertical diameter, and inclose the armature between them, the complete frame being finally secured by bolts and fixed in position by steadying pins which drop into the bedplate. The armature is exposed for inspection by the reverse operation of separating the two halves of the field-magnet frame. In the smaller machines a rack is formed at the back of each field-magnet quarter in an extension projecting from the cast-iron bedplate, and levers pivoted on pins which fit into holes in the magnet-frames enable the opening out to be conveniently done by manual power. In fig. 178*a* the magnet frame of fig. 178 has been racked back, and the illustration clearly shows the hand levers and rack-work, together with the exposed armature. In alternators of larger size, this operation is performed by screw- or chain-gear worked by a small subsidiary steam-engine or electric motor.

The steel armature shaft is supported by two spherical bearings lined with white metal, one on either side of the driving rope-pulley: overhanging the one end of the shaft, and outside the bearing, is the armature itself, while to the other end of the shaft is coupled the armature of the exciter. The bearings are carried by massive standards cast in one with the bedplate, and a bracket projecting from the outer standard carries the double horseshoe field-magnet of the exciter (fig. 178). The system of lubrication has been most carefully worked out: the oil is forced from a reservoir placed at some height above the level of the shaft to the

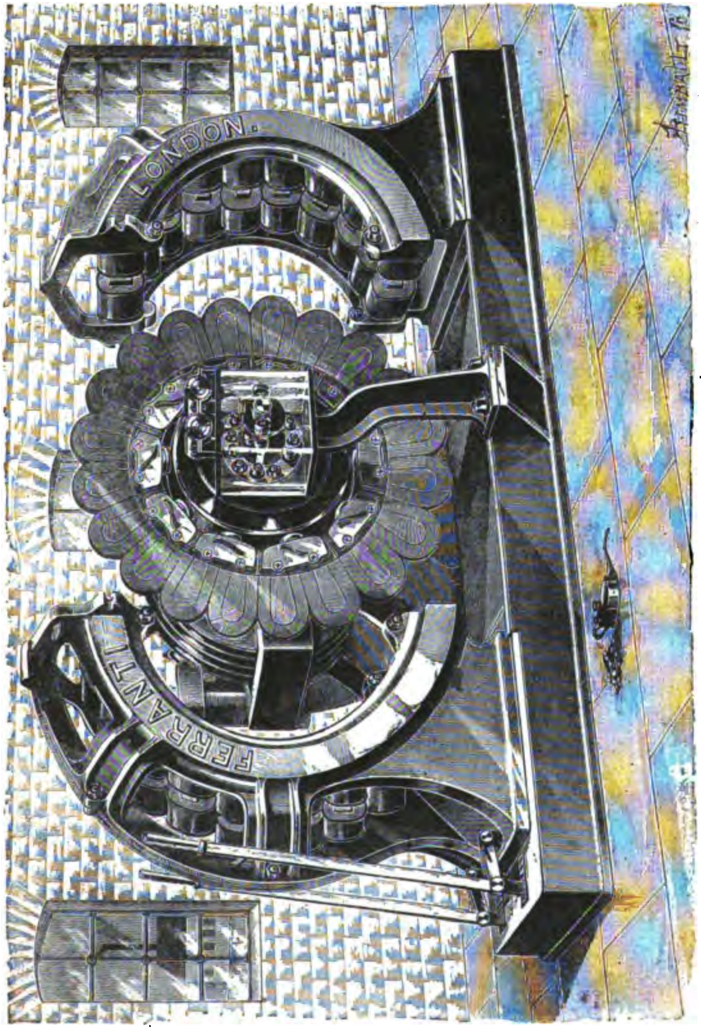

FIG. 176a.—Ferranti alternator opened out.

underside of all the bearings, and when spent is passed by oil-throwers into a tank, whence it is again pumped up into the reservoir by a small pump driven from the main shaft.

The construction of the armature is in many ways unique. The coils, of which there are as many as there are pairs of opposing poles, are wound on laminated brass cores, the direction of lamination being parallel, and not, as is more usual, at right angles, to the inductors. To form the core, a number of pieces of brass strip, each with a central corrugation running up it, are taken and separated from one another by slips of asbestos made tapering: the brass and asbestos thus arranged are put into an hydraulic press, and when under great pressure a brass eye-piece is burnt on at one end, the molten metal being run on to and fusing with the corrugated strip. Thus the core is split into a large number of comb-like teeth, and is egg-shaped in outline, owing to the asbestos strips gradually becoming thicker towards the outer ends of the teeth. When placed in position on the armature, with its small end towards the shaft, the radial teeth of the core have E.M.F.'s induced in them up or down their length; but owing to their being insulated from one another save at their inner ends, no circuit is formed by which a current may pass from one tooth into the next and back again; eddy-currents cannot, therefore, flow save within the limits of the several teeth, and these being narrow, the loss of power and the heating from eddies become unimportant.

On the laminated core is wound a narrow bare copper strip, a central corrugation being impressed upon it in the process of winding, to prevent one layer from slipping over another; in fact, the whole bobbin is locked sideways by the corrugation, so that it cannot become distorted out of its flat shape. Each layer of copper is separated from the next by a thin band of vulcanised fibre, which is wound on at the same time. The inside end of the copper strip is brazed on to the solid end of the brass core. The completed bobbins are mounted in pairs in a forked gun-metal bobbin-holder, and are secured in place by means of screws which pass through the eye-holes cast in the solid ends of each core: thus the holder serves to connect the two inside ends of a pair of coils. The holder is itself lined with fibre, and between the two coils in the same holder, where the maximum difference of potential occurs, is interposed an insulating strip of ebonite. Where the two coils in adjacent holders touch each other, the two outer ends of their windings are brazed together (cp. fig. 75 and p. 142). The bobbin-holders, furnished with their bobbins, are mounted on a gun-metal disc keyed on to the shaft. The rim of this disc is cast with holes through it; the shank of the bobbin-holder, protected by a porcelain ferrule, is passed through one of the holes, and is secured by a nut and pin. Between the nut and the sides of the cavity in the driving ring there is a clearance space all round of at least half an inch, and into this is run a sulphur cement which completely insulates the bobbin-holder and its nut from the ring, while at the same time holding it mechanically firm. The removal of a bobbin for examination or repair is simple, all that is required being to undo the screws connecting it, and the adjacent one, to their respective bobbin-holders; the two can then be lifted out, and, when replaced, connection is at once made by the operation of screwing them to their holders. For purposes of ventilation a small curved blade or wing is attached to each side of a bobbin-holder, as seen in fig. 178.

The armature coils are usually connected in two parallels, as in fig. 75; at opposite ends of a diameter, insulated copper rods pass downwards into the main shaft, and thence along the inside of the hollow shaft to collecting rings. Each collector is formed of two half-rings of brass, pressed together by springs: between the brass and the ring is interposed black lead, which is at once a lubricant and an electrical conductor. Both terminals are entirely inclosed in a glass box, supported on a cast-iron bracket, as seen in fig. 178*a*.

Our illustrations show a 150 H.P., or 112 kw., machine, giving 2,400 volts and 47 ampères at a speed of 500 revolutions per minute: there are twenty coils and twenty poles on each side of the armature, whence the frequency is 83 ~ per second. The armature conductor is ·012″ thick and half an inch wide. The height of the machine is 6 ft. and its length over-all, with its exciter, 8 ft.: when closed its width is 5 ft. 4 in., while its total width, when the magnet frames are slid out to the end of the cast-iron extension plates, is 11 ft. 8 in. The exciter supplies 30 ampères at 100 volts, or nearly $2\frac{3}{4}$ per cent. of the output, and the total weight of the machine with exciter is $7\frac{3}{4}$ tons.

Messrs. Ferranti Ltd. have recently constructed alternators with either the magnet poles or the armature coils mounted on the rim of a massive wheel which is also the flywheel of the engine. The largest of these has been supplied to the Deptford electric-light station, and is for an output of over 1,000 kilowatts, or 100 ampères and 10,700 volts, at 156 revolutions per minute. In this case the armature coils, 64 in number, are of similar design to those previously described; each pair is held between two brass castings which are tightly clamped in an hydraulic press and riveted together to form a carrier, the coils being insulated therefrom by mica sheeting. The carrier is then mounted between a pair of steel supports which are ebonised all over, and the heads of these are set with sulphur within chambers cast in the rim of the flywheel. The coils can be shifted laterally between the pillar supports, so as to bring them exactly into a central plane between the opposing poles. There are 64 poles on each side of the armature, giving a periodicity of 83; they are of circular section, 5″ in diameter, ending in a pole-face of rectangular shape with rounded ends, and they are forced by hydraulic pressure into holes in the four quarters of the yoke ring, where they are further secured by set screws. In order to prevent sparking across from the armature to the pole-faces, enamelled micanite caps $\frac{5}{32}$″ thick are fitted over the pole-faces. The width of the armature strip is $\frac{3}{4}$″, and the clearance between armature and micanite $\frac{11}{32}$″ each side, making a total gap from one iron pole to the opposite iron pole $1\frac{3}{4}$ in. The external diameter of the armature is 22 ft., which gives the high peripheral velocity of 10,800 ft. per minute; the diameter of the flywheel is 16 ft., and the weight of the whole armature 35 tons. The loss of watts over the armature resistance is not more than 0·85 per cent. of the output, and the exciting power is 300 ampères at 50 volts, or less than 2 per cent. The field-winding consists of bare copper strip wound on edge with insulation between the turns. The shaft of the flywheel armature is coupled directly to a three-crank engine of 1,800 horse-power, and is supported by two spherical bearings 12 in. diameter × 36 in. long, lined with white metal.[1]

(5) *The Mordey Victoria alternator*, with which we shall close our descriptions of modern dynamos, is manufactured by the Brush Electrical Engineering Co. to the designs of its inventor, Mr. W. M. Mordey. Belonging as it does to the class of machines which we have called 'unipolar' (Chap. IX.), and of which we have not hitherto described any example, it possesses for us many novel features, chief among which are its single exciting coil and its peculiar revolving field-magnet. The machine has already

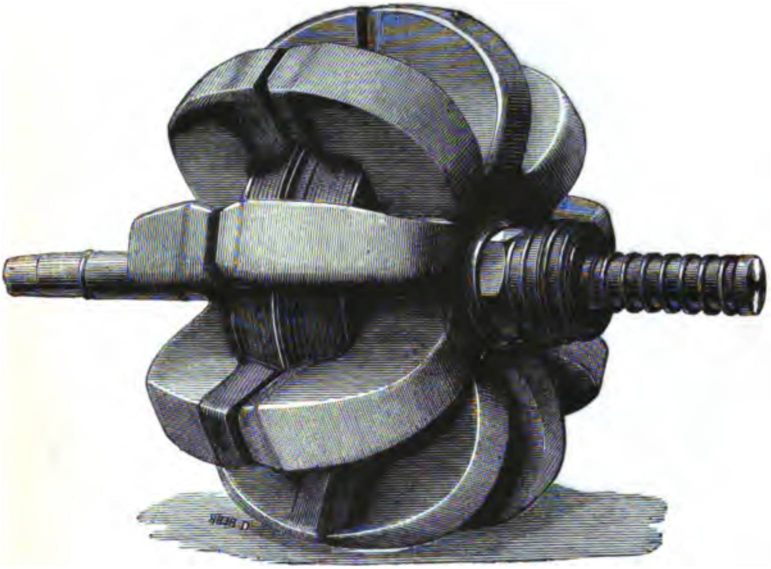

FIG. 179.—Magnet of Mordey alternator.

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[1] See *Electrical Engineer*, Nov. 29, 1895, and *Electrician* of the same date. For a description of the 1,250 H.P. alternators at the Deptford station of the London Electric Supply Corporation, see Fleming's *Alternate-current Transformer*, vol. ii. pp. 328–331. They are worked regularly in parallel with 625 H.P. alternators, supplying current at 2,400 volts transformed up to 10,000 volts.

been diagrammatically represented in fig. 80, and is of the disc type: as such, the bobbins of its armature are somewhat similar in appearance to those of the last-described machine; yet their action is entirely different, owing to the complete dissimilarity of the field-magnet. The construction of this, with its cylindrical core and its two claw-shaped castings of soft steel, one at either end, has been already alluded to on pp. 164, 266. Fig. 179 shows the complete magnet as it was first made, but in later machines

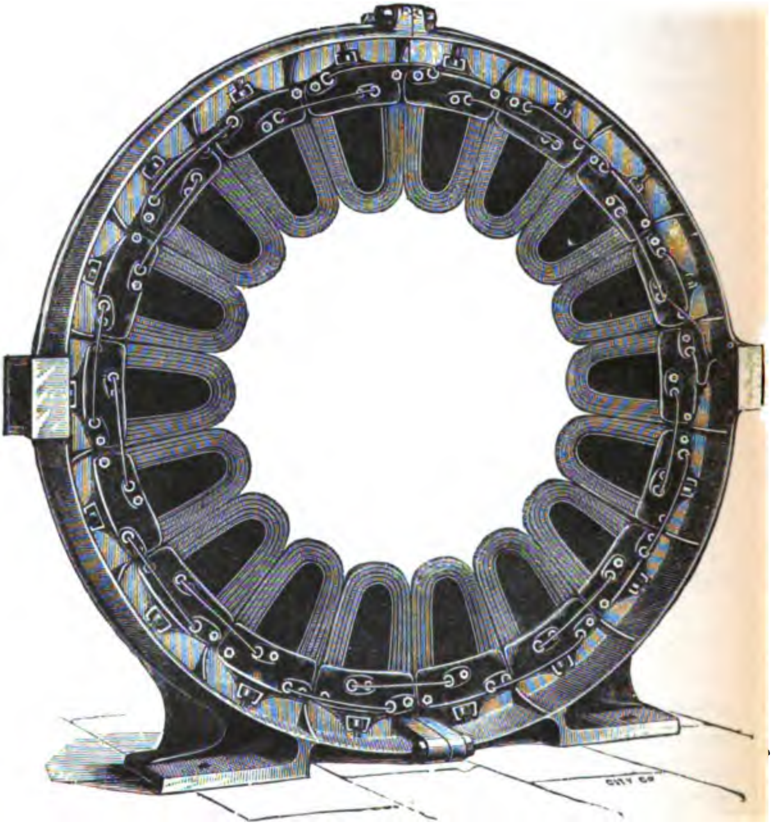


Fig. 180.—Armature of Mordey alternator (front view).

the claws which form the poles on either side are united by a thin external web, so as to present a smooth surface and prevent churning of the air during rotation, and this form is indicated in fig. 80. The heavy mass of the field-magnet revolving at a high speed possesses great moment of inertia, and thereby exerts a very considerable fly-wheel action : this not only has the direct effect of rendering the alternating E.M.F. more regular in its continuous rise and fall in cases where the speed of the prime mover fluctuates in each revolution, but also has the incidental advantage that it helps to keep the machine in step when it is used as a motor or in parallel with other machines. Whatever the number of poles or alternations, there is but one exciting coil wound on the central steel core, which is flanged at either end and keyed to the shaft; as seen in fig. 179, the coil is divided on its surface by a central groove, within which the inner ends of the armature coils extend, the required length of the magnetic circuit being thereby shortened. Since the coil revolves with the magnet, it is necessary to employ rubbing-contacts to pass the low-tension exciting current into and out of the field-winding: accordingly, two gun-metal collecting rings (seen to the right of the magnet in figs. 179 and 181) are arranged side by side on the shaft, from which, as well as from one another, they are insulated by ebonite, and against these ordinary brushes are seen to be pressed in fig. 181, each collecting ring being provided with a pair for greater security. In some machines, however, connection is made with the exciter by means of two flexible bands of folded copper gauze, which hang over the two rings and are pressed against them by weights at their ends. At the right-hand end of the shaft, in fig. 179, there will be noticed several thrust-collars which prevent the magnet from moving endways—a matter of great importance, having regard to the small clearance between the armature and the revolving poles on either side. The shaft is supported by two cast gun-metal plummer-blocks, with bearing surfaces lined with white metal; the use of gun-metal for the entire plummer-block is needed in order to prevent magnetic leakage, since, if cast-iron were employed, there would be a comparatively large flow of lines along the shaft, down through one bearing into the bedplate, and returning through the second bearing into the shaft.

The magnet of fig. 179 forms part of a $37\frac{1}{2}$ kw. machine, giving 2,000 volts and $18\frac{3}{4}$ ampères at 660 revolutions per minute, and figs. 180 and 180*a* show the two sides of the armature for this same machine; since there are nine interpolar fields, and the winding is divided into 18 coils, the 'frequency' of the machine is thus 100~ per second (p. 164). Being stationary, the armature has to resist the tangential pull on its inductors due to the magnetic field, but is not subjected to any stresses from centrifugal force. The armature coils are formed by winding a thin copper tape, $\frac{7}{16}''$

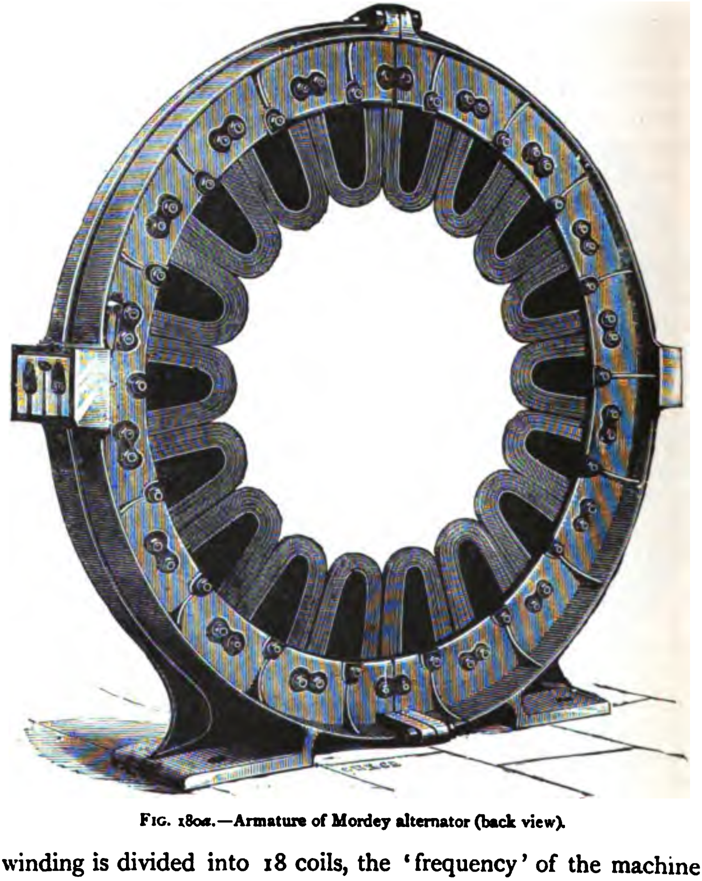


FIG. 180*a*.—Armature of Mordey alternator (back view).

wide, on an ovate core, the turns being separated by a very thin band of fibre wound on at the same time. The cores are of slate, a material which not only possesses the requisite mechanical strength, and is a good insulator, but, further, is entirely free from the hysteresis and eddy-currents which are inseparable from an iron core. In the latest machines of large size a number of small slots are drilled through the slate core down its centre-line (fig. 182, i.), and through these while the coil is tightly compressed are laced bands of whipcord. The surface of the winding is covered with micanite, to prevent sparking from the coil to the poles. Each coil is mounted at its broad end between two German-silver plates, ebonised on their surface, and these are bolted to the inner face of a circular gun-metal ring which forms the external support of the entire armature. All the metal work is outside the path swept by the dense interpolar fields; eddy-currents are, however, liable to be set up in the bobbin-holders by straying lines, and hence German silver is employed in their construction, the high electrical resistance of this metal sufficing to reduce the loss from eddies to a very small amount. The non-magnetic gun-metal ring obviates the greater magnetic leakage, and the correspondingly greater eddy-current loss, which would be occasioned were iron employed.

The tapering shape of the coils is such that when the entire number are assembled they fit tightly together like the wedge-shaped stones of an arch. The bolt-holes in the gun-metal ring are oblong in shape, with their longer axis radial; hence when the winding becomes compressed by the working stress, screws attached to each coil enable them to be individually thrust inwards towards the centre until all are again wedged tightly together (fig. 182, i. and ii.). The connections of the coils to one another are made by short lengths of insulated wire passing through the German-silver bobbin-holders, and protected by porcelain insulators, as seen in fig. 180; the ends of the winding are led to two screw terminals, placed on the outside of the armature ring, and carefully insulated by the hard wood blocks in which they are embedded. The armature frame is cast in two pieces and bolted together at the top and bottom, while a pair of curved arms springing out from either side are bolted to the bearings, and stay the armature on a horizontal diameter. At the base the gun-metal ring is rigidly bolted to the cast-iron bedplate. A foot with guides

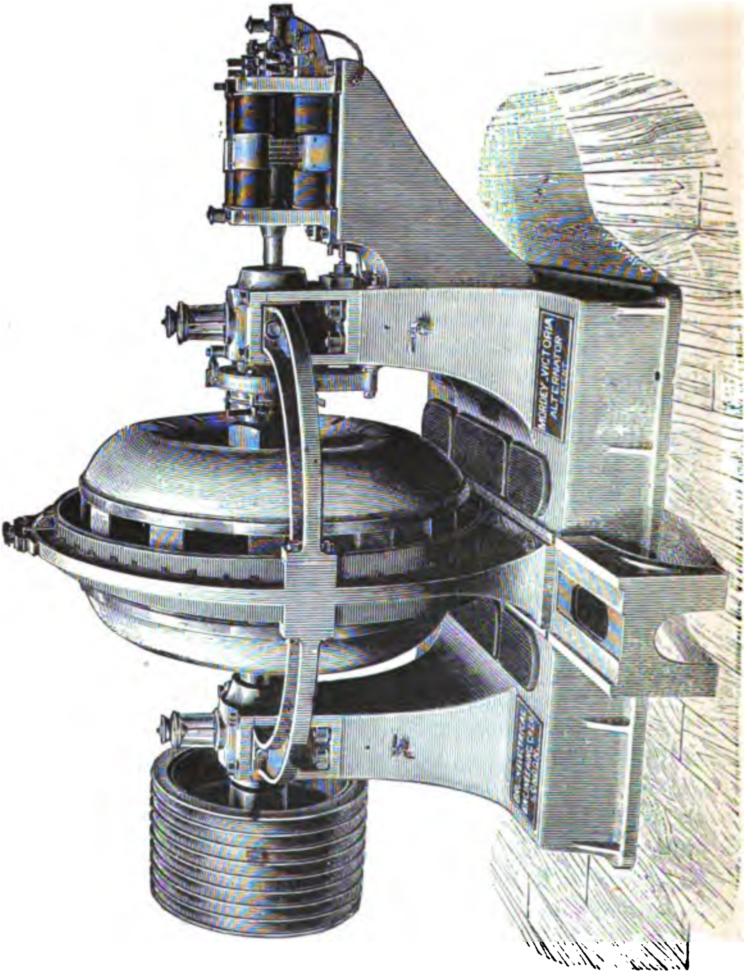
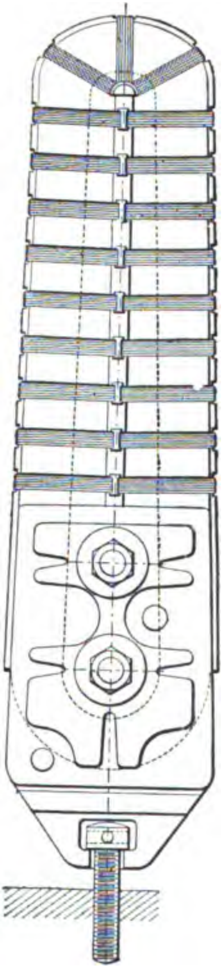
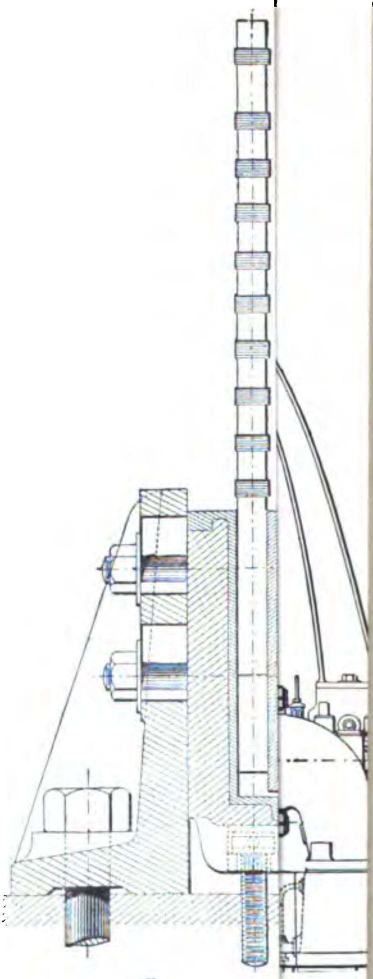


FIG. 181.—75-kw. Mordey-Victoria alternator.

is also provided as an extension to the bedplate in a line with the armature and on to this the half of the armature can



i.



ii.

att Mordey

be slid outwards for inspection. Each coil with its holder being complete in itself, any one can be easily removed and replaced.

Fig. 181 shows a perspective view of a 75-kw. alternator complete with grooved pulley for rope-driving: it has 12 fields and 24 armature coils, and at a speed of 500 revs. per min. gives 2,000 volts and 37·5 ampères with a frequency of 100 ~ per sec. The exciter is in this case a small continuous-current Victoria machine, carried on a bracket projecting from the bedplate and driven directly from the alternator shaft: it supplies an excitation of 17½ ampères at 65 volts, or 1,140 watts.

Fig. 182 shows in outline one of the 200-kilowatt alternators at the Leicester electric-light station.[1] These are driven at 96 revolutions per minute by horizontal engines, to which they are connected by Raworth flexible couplings. The voltage is 2,000 and the periodicity 96, there being 60 poles on each side and 120 coils in the armature. Fig. 182 iv. and v. show the upper half of the machine as supported by girders on the foundation, the lower half being below the floor level. Owing to the large size of the machine, the cast-iron frame, to which the gun-metal armature ring is bolted, is divided into four quadrants hinged on a horizontal diameter: each of these can be unbolted and swung back from the cast-iron end girders for inspection or repair, and one quadrant is so exposed in fig. 182, v. The exciting coil is entirely divided into two halves (as shown in fig. 182, iii.), and the central cast-steel core is pierced with a number of conical radial openings; thus air can pass freely through the core and field-coil to the armature. The side cheeks whence the poles project are in two pieces, bolted together, and they are fastened by bolts and circular feathers to the flanges of the inner core. End-play of the magnet is prevented by shoulders on the shaft abutting against the inner ends of the two bearings. These are of swivel type, and are automatically lubricated by a small oil pump seen in the illustration: they are entirely separate from the engine, and are not in any way affected by the wear of the engine bearings, owing to the interposition of the flexible coupling.

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[1] See the *Engineer*, July 19, 1895, to which we are indebted for the illustrations.

A striking feature of this alternator, as compared with multipolar machines, is the economy with which the field is produced, especially in small sizes: thus the $37\frac{1}{2}$-kw. machine requires 500 watts, or 1·33 per cent. of the output, for excitation of the magnet at full load, and 400 watts at no load. These figures are taken from a paper read by Mr. Mordey before the Institution of Electrical Engineers,[1] and it is there stated that 3 H.P. is required to drive the machine at full speed with full E.M.F., but no external load, and that of this, 1·5 H.P. is absorbed in eddies set up in the armature inductors and their supporting metal-work, the remaining 1·5 H.P. being absorbed by ordinary mechanical friction in the bearings, &c.

The remarkable smallness of these losses, which is to be ascribed to the absence of iron in the armature and the perfectly steady flow of the field-lines, renders the machine extremely efficient at light loads—an important matter in central-station work, where one machine at least is usually running lightly loaded for several hours during the daytime. Assuming that the eddy-loss is the same at full load as at no load (in reality it is probably somewhat greater), we can thence deduce the commercial efficiency as follows:—

Mechanical friction . . .	1,120
Eddy-currents	1,120
Armature resistance . .	875
Excitation	500
	3,615

$$\text{Commercial efficiency} = \frac{37{,}500}{37{,}500 + 3{,}615} = 91 \text{ per cent.}^{2}$$

In a 75-kw. alternator directly coupled to its steam-engine, the exciting energy is 3 per cent. of its output, and in a similar 100-kw. alternator 2·7 per cent., the over-all efficiencies of the plants being respectively 80·5 and 81 per cent.[3]

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[1] *Journal Inst. Elec. Eng.* vol. xviii. part 81, 1889, a paper which contains much useful information on the subject of alternate-current working, among its chief points being a description of tests made with two $37\frac{1}{2}$-kw. machines, and demonstrating their power of working in parallel or as motors.

[2] This calculation does not, of course, include the losses in the exciter itself, which might be an additional 500 watts: taking this into account the over-all efficiency of alternator and exciter works out to about 90 per cent.

[3] Report on the Dover Station, *Electrical Engineer*, January 17, 1896.

In recent alternators of 250-kw. output at 300 revs. per min., a commercial efficiency of 93 per cent. is guaranteed.

Owing to the low resistance of its armature, together with the fact that the reaction of the armature current on the field is small, the drop of volts between no load and full load under constant excitation is very small; or, conversely, to maintain a constant terminal voltage the exciting energy only requires to be varied between small limits, a feature of considerable value in central-station working.

CHAPTER XX

DYNAMO DESIGNING

THE various parts of the dynamo having now been considered in detail, our results may be summed up in the complete design of a machine. The practical art of designing resolves itself into a question of striking a balance between a variety of conflicting considerations, all of which are of importance in different degrees, and each of which will vitally affect the entire design of the machine. Thus a dynamo must be efficient, yet at the same time it must not be too costly to manufacture; it must be compact, yet well ventilated; thoroughly strong, yet not too heavy. Any one feature, however desirable in itself, will, if carried to excess, have some disadvantageous consequence in another direction, and he is the best designer who can effect a series of compromises, such that, while each consideration is given its proper weight, none are forced into undue prominence, and a design well-balanced as a whole results from his practised judgment.

Since the dynamo is primarily a generator of electric pressure, the first consideration must be the production of the requisite volts. This, being a question of the rate of cutting lines, will depend on the speed, the number of inductors, and the number of lines passing through the armature. In the majority of cases, for a given output, the speed may be taken as fixed: either it is directly specified, or it is to a great extent settled by recognised practice or questions of mechanical strength and durability. Thus, to take the case of an ordinary continuous-current dynamo, either it will be belt-driven, or it will be engine-driven. In the former case it will usually fall into one or other of two series, running at speeds which may by comparison be regarded as respectively high and low. In the latter case its speed may again be classified as high or low, according as the engine is of the single-acting class, of which Messrs. Willans and Robinson's central-valve engine is the most prominent example, or of the double-acting class, which is so largely used for ship-lighting. Hence, although makers vary considerably in the speeds which they select for different outputs, yet there is sufficient agreement to enable us to draw up a table giving average values for the speeds of the different classes.

Output in kilowatts	Speed in revs. per min.				
	Belt-driven		Engine-driven		
	High	Low	Single-acting	Double-acting	
1·5	1,500	850	700	350	Single-cylinder engine
5	1,200	750	600	300	
10	1,000	700	550	250	
15	900	650	520	250	
20	800	600	500	320	Compound engine
30	700	500	475	320	
40	600	450	450	300	
50	550	400	400	250	
100	400		375	200	

Suppose that it is desired to design a machine to give 130 volts and 62 ampères when running at a speed of 800 revs. per min., and charging accumulators; to be further capable of lighting lamps directly, and in this case to give 100 volts at 840 revs. when but few lamps are on, and to compound up to 105 volts at 810 revs., when the current rises to 60 ampères, and all the lamps are burning: a machine designed to comply with these several conditions will introduce to us several of the nicer problems of the art, since it will have to be arranged with a shunt field-winding capable of giving the full voltage of 130 volts at 800 revs., with a resistance in the shunt to lower the voltage at the no-load speed, and with a series-winding to make the machine compound between the limits of 0 ampères, 840 revs., and 60 ampères, 810 revs.

For such an output a 2-pole ring- or drum-wound machine would generally be used; the exact type will depend on the class of machine constructed by the maker, but we will here assume that a 2-pole ring armature with over-type horseshoe field has been decided upon (fig. 2).

The determination of the best diameter and length of armature core, without reference to other designs or machines previously built, can only be effected by a method of trial and error. The speed being fixed, the peripheral velocity which is usually regarded as permissible in the type of machine gives a rough idea of a reasonable diameter of core; although, as a matter of fact, the speed has in the first instance become more or less settled by the peripheral velocity which results from the diameter of core that is usually employed in ordinary machines. Assuming, then, a certain diameter of core, D, we can estimate the radial depth which the core-discs would normally have, *e.g.* in a ring armature about ·19 D (p. 258), and adopting an average length of core, say 1 or $1\frac{1}{3}$ times D, we can calculate the net cross-sectional area of iron in the two sides of the ring, i.e. $2ab$ (cp. p. 296). Taking a normal value of B_a, we thus arrive at $Z_a = B_a \times 2ab$, and thence the requisite value of τ for the given speed follows of necessity. Dividing the available circumference of the armature by τ, we find the size of wire that can be employed, and can roughly calculate the resistance of the armature from brush to brush; the question of one or more layers on the outside, as allowed by the air-gap, and of room on the inside, must also be considered. At this point the full-load current is a decisive factor, the output of any continuous-current dynamo being limited by the two considerations of heating and sparking, either or both. If, therefore, we find either the total watts lost in the armature or the ampère-stream too great, the only course open to us is to increase either the length or diameter of core, or both, the value of Z_a being increased, and the value of τ being proportionately reduced. By a series of trials we arrive at such a mutual relation between τ and Z_a as will give a suitable size of armature and a winding of practicable nature which will not overheat or give rise to sparking. In thus determining the size of armature core from first principles, considerable latitude may be permitted to the judgment and discretion of the designer; for within certain limits the relation between D and L may be varied without greatly affecting the cost or efficiency of the machine, although, as a general rule, as small a diameter as is practicable is to be preferred. In the drawing-office of a dynamo manufacturer, however, the result is in most cases quickly arrived at by reference to a list of standard armatures, or at least of standard diameters of core-discs, from which armatures of different lengths may be built up. After constructing a number of machines, a maker is able to draw out a table showing the maximum value which the quantity $\frac{\text{watts of output}}{\text{revs. per min.}}$ has for each of the standard armatures, and this quantity may be taken as the maximum number of watts per rev. per min. which can safely be obtained in continuous working from each armature under average conditions of voltage and speed. If either or both of these be exceptionally high, or if a particularly small rise of temperature is alone permissible, the value which this quantity may have will be somewhat lower than would be the case with moderate voltages and speeds; yet within wide limits, each size of armature core may be regarded as corresponding to a certain number of watts per rev. per min., and by the value of this, its cost to manufacture is almost immediately determined with very fair accuracy. Thus in the present case the designer would look out in his table the standard size of core corresponding most closely to $\frac{8{,}000}{800} = 10$ watts per rev. per min., and we will assume that this leads him to a ring-wound core of discs 9″ external diameter, $5\frac{1}{2}''$ internal diameter, and 12″ long. The net sectional area of iron in such a core after allowing for insulation between the discs is calculated as explained on p. 296, and is found to be 235 sq. cm. A suitable induction in the armature core will be 18,600 lines per sq. cm., whence $Z_a = 18{,}600 \times 235 = 4{,}370{,}000$.

The loss of volts over the armature will be roughly about 6, so that $E_a = 136$ volts; and therefore from equation (11)

$$\tau = \frac{E_a \times 60 \times 10^8}{Z_a \times N} = \frac{136 \times 60 \times 10^8}{4{,}370{,}000 \times 800} = 233.$$

Now for an armature current of about 66 ampères it will not be advisable to have more than three turns per commutator section, and we must therefore choose between $\frac{231}{3} = 77$, involving a slightly increased induction, or $\frac{234}{3} = 78$, with a slightly reduced induction. In drawing-office practice the number of sections may have to be arranged to suit a standard list of commutators, but, apart from such a list, the question only remains as to the number of sections which is most suitable, and will not exceed the maximum that is commercially practicable. With ring armatures, an uneven number of commutator parts is not to be recommended for reasons deducible from p. 387. When the brushes are set exactly opposite to each other on a commutator having an uneven number of segments, the coils at the opposite brushes are short-circuited alternately, first at the positive, and then at the negative brush; then again at the positive brush, and so on. Now, if it happens that at the instant of short-circuiting a large current is set up in the coil, the part of this current which is not balanced by the normal armature current flowing in the opposite sections will tend to send lines round through the core; the result of this tendency is to increase the density of the lines in the portion of the core opposite to the coil, and to decrease the density in the portion that it covers. When the opposite section becomes in turn short-circuited, a similar action, but in the reverse direction, takes place, the total result being the same as though the effects of an alternating current with a periodicity of $s \, . \, n$ (where s is the number of commutator sections, and n the number of revolutions per second) were superimposed on the magnetised core. Thus the difficulty of sparkless commutation may be appreciably increased, as well as the hysteresis and eddy-current losses in the core. In a drum armature no such action will take place, and if anything an uneven is better than an even number of commutator segments.

Returning to the 9″ × 12″ ring machine, we thus decide upon 78 sections and 234 inductors. The size of inductor has next to be determined, and in order to arrive at this, the first step is to divide the available space on the armature by τ. The external circumference of the discs is 28·2″, but at least 1″ should be deducted to allow the winder a little play, so that the available circumference is reduced to 27·2″, and the maximum width of each insulated inductor, if disposed in a single layer on the outside, becomes $\frac{27{\cdot}2}{234} = {\cdot}116''$. Round wires are generally insulated with a double-cotton-covering about 15 mils. (= ·015″) thick, i.e. $7\frac{1}{2}$ mils. a side, and rectangular wires of small section, with about 20 to 25 mils., although for large sizes a braided covering is often employed, ranging from 30 to 40 mils., i.e. 15 to 20 mils. a side. We will here commence with a round wire, and allow 15 mils. for insulation, whence the maximum diameter works out to ·116″ − ·015″ = ·101″. This does not happen to be an exact size in the S.W.G. or B.W.G., and although any size can be specially drawn, it is as well, wherever the exact gauge is not of great importance (as it is in shunt-windings), to adopt the next smallest stock or standard size, since it will be more easily obtainable—in the present case, say No. 13 B.W.G., or ·095″ diameter d.c.c. to ·110.″ From the dimensions given in fig. 135, the mean length of one complete loop round the armature can be estimated to be $28\frac{3}{4}$″ = 2·4 ft. The total length of armature wire will be therefore $2{\cdot}4 \times 234 = 562$ ft: the resistance of 1,000 ft. of ·095 wire at 60° F. is 1·144 ohms; and therefore, by p. 256, the resistance of the armature at 60° F. is

$$R_a = \frac{562 \times 1{\cdot}144}{4 \times 1{,}000} = {\cdot}161 \text{ ohm}.$$

Assuming that the maximum temperature reached in continuous working will be about 120° F., 13 per cent. must be added to obtain the resistance when hot, or $\cdot161 \times 1{\cdot}13 = \cdot182$ ohm; the volts lost over the armature will be $C_a R_a = \cdot182 \times 66 = 12$, and the watts $= 12 \times 66 = 792$. Not only is this loss too great a percentage from the point of view of efficiency, but the armature is too small to dissipate the heat quickly enough, so that it will become seriously overheated. It is therefore necessary to alter the winding, and a simple alteration will be to wind two wires, each ·095″ diameter, in parallel with each other, thereby halving the resistance and the loss over the armature. A length of air-gap of $\frac{7}{16}$″ will not be too great in a machine of our present size, and will permit of the two layers being wound on the outside; but it remains to be seen whether there will be sufficient room on the inside of the ring between the arms or spokes of the gun-metal hub. For 9″ discs three driving arms will be sufficient (as shown in fig. 2), each $\frac{1}{2}$″ wide immediately below the discs. A layer of fibre $\frac{1}{16}$″ thick will be required on the inner surface of the discs, and on either side of the arms a somewhat thicker insulation of fibre, say $\frac{3}{32}''$ thick. Thus the width of each insulated arm is $\frac{11}{16}''$, and their total width, say 2·1″, must be deducted to obtain the available circumference for each inside layer. Further, in each of the three divisions a certain play must be allowed to the winder, say ·2″, to enable him to accommodate the numerous wires without difficulty. On working out the available circumference for each layer, it will be found that five internal layers will be required, and not only will it be troublesome to build up six round wires forming three turns per commutator segment into five layers, but the winding becomes cramped owing to the inner layers approaching so closely to the cylindrical body of the hub ($3\frac{1}{4}''$ diameter). A rectangular wire will, however, pack more closely on the inside, even though it requires a slightly thicker insulation, and will be more easy to build up in two layers on the outside; if its dimensions be ·190″ × ·075″, its area will be equal to that of the two round wires in parallel. A further question now arises: if wound flat-wise, the width of the rectangular wire is so great that large eddy-currents will be set up in each inductor (p. 246), while to wind it on edge will be a very arduous task, and will probably end in the insulation at the corners opening out or becoming damaged. The difficulty is, however, overcome if we again have recourse to two wires wound in parallel, each being ·085″ square d.c.c. to ·105″: the rectangular section will render it easier to wind the pair one above the other on the outside, while on the inside each pair of contiguous sections can be built up together in four layers, with one turn dropped down to form a fifth layer. Thus each of the three layers nearest the core will contain 117 wires, while the fourth and fifth layers will contain 78 and 39 respectively. To test this arrangement, let us consider the third internal layer; its diameter will be 4·73″, and its available circumference $14·8-(2·1+·6)=12·1''$, while $·105'' \times 117=12·3''$. Hence, at occasional intervals, say once in each third of the armature, it may be necessary to lower a turn from the third into the fourth layer. The distance of $\frac{1}{2}''$ dividing the innermost layer from the body of the hub may be taken as the minimum which should be allowed. Owing to the greater depth of the winding, our value for the mean length of one turn will require to be increased to $29\frac{1}{2}''=2·46$ feet; but the area of conductor is now slightly greater, and the resistance per 1,000 feet of the two in parallel is ·565; whence

$$R_a = \frac{234 \times 2{\cdot}46 \times {\cdot}565}{4 \times 1{,}000} = {\cdot}0815 \text{ ohm},$$

or when hot ·092, giving a loss of 6·05 volts and 400 watts at full current.

Our original assumption of $z_a = 4{,}370{,}000$ may still be allowed to stand, with its maximum induction of $B_a = 18{,}600$; thence from the curve of fig. 112, the joules expended in hysteresis per cubic cm. per cycle are ·00138. The cycles per second are $\frac{800}{60} = 13{\cdot}3$, and the cubic cm. of iron in the core are 6,800. The rate of loss by hysteresis in the armature core is therefore (cp. p. 409) $\cdot 00138 \times 13{\cdot}3 \times 6{,}800 = 125$ watts. The eddy-current loss can only be estimated by referring to the results obtained from previous machines, similarly constructed and wound, when tested by the method to be described subsequently. Assuming it to be 325 watts, the total loss over the armature (eq. 45) is $400 + 125 + 325 = 850$ watts. The cooling surface calculated in the manner explained on p. 410 is

$$\pi \times 9{\cdot}5 \times 14{\cdot}5 \times 2 \,.\, \frac{\pi \times 9{\cdot}5^2}{4} = 574 \text{ square inches},$$

and therefore the rise of temperature of the outside of the armature is by equation (47)

$$= 115 \times \frac{850}{574 + \frac{574 \times \sqrt{1{,}980}}{30}} = 115 \times \frac{850}{1{,}425} = 68{\cdot}5^\circ \text{ F.},$$

which is as high as is permissible. In practice it will probably be less, since at the end of the charging of the accumulators, when the highest voltage and strongest field are required, the current will fall slightly, while when lighting lamps direct with the full current, the field is weakened to reduce the voltage, and hence the eddy-current and hysteresis losses are lessened.

The insulation on the core will be barely $\frac{1}{16}''$ thick, say ·060″; upon this are the two layers of wire, $2 \times {\cdot}105'' = {\cdot}210''$, and above all the bands of binding-wire: these latter, together with their backing of mica and tape, will have a thickness of about ·035″, the wire itself being ·020″ diameter. The over-all diameter of the completed armature will thus be 9+·12+·42+·07=9·61, say $9\frac{5}{8}''$. The clearance between the binding-wire and the iron of the pole-piece should be not less than $\frac{3}{32}''$ for all diameters up to 9″, and $\frac{1}{8}''$ for diameters from 9″ to 13″; while above these sizes it should be increased to $\frac{5}{32}''$. Allowing $\frac{1}{8}''$ for clearance on either side, the bore of the pole-pieces in our present machine must be $9\frac{7}{8}''$, or $2l_g = \cdot 875'' = 2\cdot 22$ cm. With a polar angle of 130°, the area of air-gap is calculated to be 880 square cm. (p. 298), whence

$$B_g = \frac{4{,}370{,}000}{880} = 4{,}970.$$

When, however, the machine is giving 105 volts, and full current at 810 revolutions, the number of lines will be reduced to about 3,580,000, and the induction in the air-gap to 4,070; and it is under these more unfavourable conditions that we must test our design for sparklessness at the brushes. By the formula of p. 384 the permissible ampère-stream is

$$A_s \text{ max.} = \frac{144 \,.\, B_g \,.\, 2l_g}{\phi} = \frac{144 \times 4{,}070 \times 2\cdot 22}{137} = 9{,}500,$$

whereas the actual ampère-stream $= \tau \times \frac{C_a}{2} = 234 \times \frac{66}{2} = 7{,}700,$

and is therefore well within the limit. In fact, even assuming no lead of the brushes, the cross ampère-turns will be less than half X_P, and the output of the armature, as is usual in small sizes, is limited by the heating, and not by sparking.

The next point to be decided is the size of field-magnet. The length of the pole-piece parallel with the shaft should be somewhat less than the length of the armature core in order to prevent lines from curving round into the flat surfaces of the end discs of the core, and there generating eddy-currents (p. 236), and in this case it will be made $11\frac{1}{4}''$, or three-quarters of an inch less than the length of the core. On the assumption that $Z_m = 1\cdot 22 Z_a$ (cp. p. 312), the number of lines passing through the magnet will be 5,340,000; or from previous designs we may even know more definitely that P will be between 60 and 70 (cp. pp 302–305), while $X_p = X_a + X_g + X_b$ can already be calculated to be

$1{,}900+8{,}840+580=11{,}320$ (*vide* pp. 296–298); any such preliminary estimate of P or z_m may, however, require to be subsequently verified when the design is further advanced. Assuming the magnet limb to be rectangular in section, the same dimension of $11\frac{1}{4}''$ is best retained throughout the length of the limb, since then magnet-core and pole-piece are conveniently forged and machined as one slab.

Taking an average value for B_m, such as 14,000, we obtain for the wrought-iron limb an area of $\frac{5{,}340{,}000}{14{,}000}=382$ sq. cm. $=59{\cdot}5$ sq. in., and thence the thickness works out to $\frac{59{\cdot}5}{11{\cdot}25}=5{\cdot}3$, or, say, $5\frac{1}{4}''$. Such was the thickness taken on p. 298 for the simple shunt-wound field, and it might be thought that it would suffice to adopt the same value again. But here we are met by another condition with which we must comply: when working at 840 revolutions, and giving 100 volts only, the z_a required is only

$$\frac{100\times60\times10^8}{234\times840}=3{,}051{,}000.$$

Now on referring to fig. 139 it will be seen that such a value is almost on the straight part of the curve, and in consequence the magnetism will be unstable. It will be advisable, therefore, in the present case to adopt a higher induction in the field-magnet, a smaller cross-section, and a greater amount of copper in the shunt. Let us assume $B_m=14{,}700$; then the required area is $\frac{5{,}340{,}000}{14{,}700}=363$ sq. cm. $=56{\cdot}25$ sq. in., and the thickness is $\frac{56{\cdot}25}{11{\cdot}25}=5''$. Taking the length of bobbin as about equal to the diameter of armature (p. 267), say 10″, a rough sketch of the magnetic circuit such as fig. 135 shows us that the length of path in the wrought iron will be 94 cm.; the magnet limbs will not be brought nearer together, but the required polar area will be obtained by the use of wider polar strips, $c_1\ c_2,\ c_3\ c_4$. Hence the path in the cast-iron yoke will be slightly more than in fig. 135, say 18 cm. A final revision will lead us to slightly lower our value for P, bringing z_m to 5,300,000, whence $B_m=14{,}600$ and

$B_y = 8{,}220$. By referring to the curves of fig. 111 we obtain—

$$X_m = 23 \times 94 = 2{,}165$$
$$X_y = 80 \times 18 = 1{,}440$$

and total $X = 11{,}320 + 2{,}165 + 1{,}440 = 14{,}925$.

The gauge and turns of the shunt wire may now be determined exactly as in Chap. XVI. p. 329. Taking l_s as $38'' = 3·17$ ft.

$$\omega' = \frac{130 \times 1{,}000}{14{,}925 \times 3·17 \times 1·13} = 2·43 \text{ ohms},$$

and the necessary diameter is ·065″. The rise of the temperature of the coils for a given winding is obtainable from p. 402, and by a process of trial and error a certain number of complete layers is fixed upon, such that a due ratio obtains between the watts expended and the cooling surface: with 18 layers of 117 turns each, and the two bobbins in series, $T_s = 4{,}210$ and $R_s = 32·5$, or when hot $36·7$: $C_s = 3·54$ and the total watts expended in the shunt are 460. The cooling surface will be about 827 sq. inches, and therefore the rise of temperature of the outside (by eq. 45*a*)

$$= \frac{80 \times 460}{827} = 44·5^\circ \text{ F.},$$

whence (p. 396) $k = 1 + (·093 \times 1·4) = 1·13$.

The number of ampère-turns required to produce $Z_a = 3{,}051{,}000$ on no load are calculated as follows:—

$$B_a = \frac{3{,}051{,}000}{235} = 13{,}000 \qquad X_a = 11 \times 12·5 = 137$$
$$B_g = \frac{3{,}051{,}000}{880} = 3{,}467 \qquad X_g = ·8 \times 3{,}467 \times 2·22 = 6{,}157$$
$$X_b = 0$$
$$X_p = 6{,}294$$

Leakage lines $= 1·256 \times 6{,}294 \times 63 = 497{,}000$

$Z_m = 3{,}051{,}000 + 497{,}000 = 3{,}548{,}000$

$$B_m = \frac{3{,}548{,}000}{363} = 9{,}770 \qquad X_m = 5 \times 94 = 470$$
$$B_y = \frac{3{,}548{,}000}{643} = 5{,}520 \qquad X_y = 19 \times 18 = 342$$
$$\text{Total } X = 7{,}106$$

Now the shunt-winding previously determined has 4,210 turns;

therefore the shunt current in this case must be $c_{s1}=\frac{7,106}{4,210}=1\cdot69$ ampères, which is given by a resistance of $\frac{100}{1\cdot69}=59\cdot2$ ohms. The resistance of the shunt itself when warm will be $32\cdot5\times1\cdot09=35\cdot4$ ohms (an increase of 9 per cent. is here allowed, since there will not be so many watts expended in the field when working compound as when working at 130 volts). Hence the subsidiary resistance which must be inserted in the shunt when the machine is to give 100 volts is $59\cdot2-35\cdot4=23\cdot8$: in practice this is often divided up into steps, so that the voltage can be varied between 100 and 130 by means of a sliding contact.

Finally, in order to determine the series portion of the winding, it will be necessary to work out X for 105 volts, 60 ampères, 810 revolutions. Allowing a loss of two volts[1] over the series winding, $E_a=105+2+6=113$; whence

$$
\begin{array}{lll}
Z_a=3,580,000 & & \\
B_a=15,200 & X_a=26\times12\cdot5 & =\ \ 326 \\
B_g=4,070 & X_g=\cdot8\times4,070\times2\cdot22 & =7,230 \\
 & X_b= & \ \ \ 580. \\
\hline
 & & X_p=8,136
\end{array}
$$

$$\zeta=1\cdot256\times8,136\times63=642,000$$

$$
\begin{array}{lll}
Z_m=3,580,000+642,000=4,222,000 & & \\
B_m=11,650 & X_m=7\times94 & =658 \\
B_y=6,550 & X_y=32\times18 & =576 \\
\hline
 & & X=9,370
\end{array}
$$

The shunt will be arranged as a 'short shunt' (p. 344) across the brushes, and as the auxiliary resistance of 23·8 ohms will now be in the shunt circuit, $c_{s2}=\frac{107}{59\cdot2}=1\cdot81$ ampères, giving $1\cdot81\times4,210=7,610$ ampère-turns. Thus $9,370-7,610=1,760$ ampère-turns are left to be supplied by the series winding, and $T_m=\frac{1,760}{60}=$ say

---

[1] An arbitrary allowance, which, however, gives a suitable loss of watts in the series winding; it could therefore be slightly modified if any necessity for so doing arose.

29 turns. Assuming half the turns to be wound on each limb, and the available length of coil within the end-flanges to be 9$\frac{3}{8}$″, the insulated conductor may be $\frac{9{\cdot}375}{14{\cdot}5}={\cdot}645''$ wide; if this be considered rather too great a width to handle conveniently on a small former, we can wind 29 turns on each limb and then connect the two coils in parallel. The length of each turn will be 43″ $=3{\cdot}58$ ft. The watts expended in the shunt$=1{\cdot}81^2\times35{\cdot}4=116$, and those in the series winding at 60 ampères$=C_eR_m\times C_e=2\times60=120$, making a total loss of 236. The rise of temperature of the outside will now be $\frac{80\times236}{827}=23^\circ$ F., and the external winding may be taken as only rising 24° F., giving an increase of 5 per cent. in its resistance. Hence from eq. 32 the area of the series wire

$$=\frac{{\cdot}00813\times1{,}760\times3{\cdot}58\times1{\cdot}05}{2\times1{,}000}={\cdot}0269 \text{ sq. in.};$$

and since there are to be two windings in parallel, the area of each is ·01345 sq. in. In order to allow the winder a little room, let us take the available length of coil as 9$\frac{1}{4}$″; then $\frac{9{\cdot}25}{29}={\cdot}320''$, and after deducting ·020″ for insulation we obtain a flat strip ·300″ × ·045″ wound in one layer of 29 turns on each limb. Or, if more convenient, an open spiral of round wire ·134″ diameter (No. 10 B.W.G.) may be wound on each limb, and the two windings connected in parallel.

The question of whether a self-exciting machine worked low down on the no-current curve of magnetisation will prove unstable or not is a very difficult one, and at present can only be empirically decided by referring to tests on previous machines and noting approximately at what portion of their no-current curves they become unstable. A good method of doing this is to continue the lower straight portion of the curve o a upwards (fig. 183); mark c, the point at which the machine is considered unstable, and note the value of $\frac{cd}{ce}$; then a new machine will be unstable at approximately a point $c_{,}$ such that $\frac{c_{,}d}{c_{,}e_{,}}=\frac{cd}{ce}$. Of the two consequences of working low down on the curve of magnetisation (alluded to on p. 336), one, viz. failure to excite itself when run at the normal speed, need not trouble us in our 9″ × 12″ dynamo, since it can always be excited by short-circuiting the resistance in the shunt: when once excited, the resistance can be again inserted. As regards the second consequence, viz. large alterations in voltage when the speed of the prime mover alters slightly, or when the current changes, it can only be said that the exact point at which this effect becomes too marked for satisfactory running is a matter of opinion, and must be decided by circumstances.

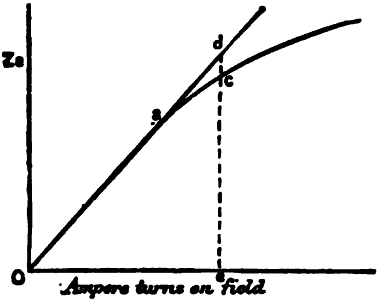

FIG. 183.

The design of an alternator has still to be considered, and for this purpose it will first be necessary to further examine the question of its E.M.F. So far, no equation has been given for the effective E.M.F. of an alternator; yet it is the value of this which must primarily determine its output.

In a bi- or multi-polar alternator, let z_a = the number of lines which issue out from one pole, and, passing into or through the armature core, form one field; then each inductor cuts z_a twice for each pair of poles (the number of pairs being reckoned as explained on p. 145, and symbolised by p), and therefore in one revolution cuts in all $2z_a.p$ lines. If N = the number of revolutions per minute, the time taken by one revolution is $\frac{60}{N}$ seconds. Hence the average rate at which each inductor cuts lines throughout an entire revolution is $2z_a.p \div \frac{60}{N} = 2z_a.p.\frac{N}{60}$.

If in each coil there are c inductors in series, and these are regarded as being bunched together on the top of each other, so that the winding has no appreciable width along the pitch-line, each inductor simultaneously generates an E.M.F. of the same value and in the same direction round the coil, and the average E.M.F. of the whole coil is therefore $2z_a . p . \frac{N}{60} . c$. Such a case is, however, impracticable, since the winding always has some width: the effect of this has been discussed at length in Chap. VIII., and it was there shown that if the width of the winding exceeds the width of the gap between two neighbouring poles of opposite sign, for a certain portion of the periodic time differential action will be set up between the two ends of the coil. Now the average E.M.F. which the coil would have on the supposition that there is no differential action, and that the E.M.F. of each inductor assists the E.M.F. of every other, is $2z_a . p . \frac{N}{60} . c$, but the actual average E.M.F. of the coil is less, since the E.M.F. of some inductors at times tends to neutralise that of others. The shape of the curve of E.M.F. due to the combined action of the several inductors is thus considerably modified, its height at either end of a half-period being lowered by the differential action; and the mean value of the curve or the actual average E.M.F. of the coil during a revolution is less than the sum of the average E.M.F.'s of the inductors. Let k' be the ratio between the average E.M.F. of the coil and the sum of the average E.M.F.'s of the inductors, both averages being taken, not merely over the time during which they are actively cutting lines, but over an entire period or any number of complete periods. Then the actual average E.M.F. of the coil $= k'$ Y (Y being the sum of the average E.M.F.'s of the inductors on the supposition that there is no differential action) $= k' . 2z_a . p . \frac{N}{60} . c$. In other words, k' is a constant which discounts the effect of differential action: its value depends on the relative proportions which the widths of the winding and field bear to the pitch, and while it cannot exceed unity, it may be less.

Now, given the curve of combined E.M.F. as affected by differential action, if any such be present, and squaring its ordinates for a complete period, we can thence obtain the effective E.M.F. which is equal to the square root of the mean square (p. 103). Let $k'' =$ the ratio which this square root of the mean square bears to the mean ordinate of the simple curve ; then the effective E.M.F. of the coil is

$$e = k'' \times \text{the actual average E.M.F.}$$
$$= k'' \,.\, k' \,.\, 2z_a \,.\, p \,.\, \frac{N}{60} \,.\, c\,;$$

or bracketing the numerical coefficients to form one joint coefficient, $k = k''\,.\,k'\,.\,2$, the effective E.M.F. of a coil $e = k\,.\,z_a\,.\,p\,.\,\frac{N}{60}\,.\,c$, and each coil that is in series with it will give the same result. Hence, if $s =$ the number of coils that are in series, and $q =$ the number of parallels into which the entire winding is divided, $s \times c = \frac{\tau}{q}$, where $\tau =$ the total number of inductors on the armature, and the effective E.M.F. of the alternator as a whole is

$$E_a = k\,.\,z_a\,.\,p\,.\,\frac{N}{60}\,.\,\frac{\tau}{q}\,.\,10^{-8} \text{ volts. . . . . } \quad (48)$$

We have thus obtained for the effective E.M.F. an equation analogous to our previous equation for the E.M.F. of a continuous-current machine, and expressed in terms of the number of lines forming one field, the total number of inductors, and the speed in revolutions per minute. It involves, however, a constant k, the numerical value of which must be further determined. Although this varies with the width of the windings and fields, and with the shape of the poles as affecting the distribution of the lines (cp. p. 154), yet it will in the main depend on the *relative* proportions of the coils and fields as compared with the pitch ; and even here it will suffice to determine certain general cases to serve as samples for our guidance in actual practice. To illustrate the principle of the method, the simplest case will first be taken; the single inductor of fig. 24 or fig. 41 rotated in a uniform field has no differential action, and $k' = 1$: its curve of E.M.F. is a simple sine curve of which the average value is $2zn$, and the maximum value πzn (pp. 58, 59), while, further, we know that for a sine curve the square root of the mean square is equal to the square root of half the square of its maximum ordinate. Hence the effective E.M.F. is

$$\sqrt{\frac{(\pi Zn)^2}{2}}=\frac{\pi Zn}{\sqrt{2}}$$

and the ratio k''

$$=\frac{\frac{\pi Zn}{\sqrt{2}}}{2Zn}=\frac{\pi}{2\sqrt{2}}.$$

The coefficient k is therefore

$$k''.k'.2=\frac{\pi}{2\sqrt{2}}\times 1\times 2=\frac{\pi}{\sqrt{2}}=2\cdot 22.$$

From this case we turn to the single inductor of fig. 43; again there is no differential action, but the curve of E.M.F. is no longer a sine curve. If the fields were of perfectly uniform density, and of width equal to the pitch, so that they changed sign abruptly (cp. fig. 64), we should obtain two rectangles, each with length of base=half a period, and height$=2z_a.\frac{N}{60}$. The square root of the mean square would be also $2z_a.\frac{N}{60}$, so that the average and effective E.M.F.'s would coincide, or $k''=1$. Hence $k=k''.k'.2=1\times 1\times 2=2$. Owing, however, to the gradual shading off of the lines at the edges of the poles, the corners of the curve of E.M.F. would in practice be rounded (cp. fig. 44), and the effective E.M.F. actually obtained would be somewhat less than the above theoretical value.

Let us now pass to the consideration of a coil of many turns, and in the first place let us suppose the width of winding and the width of field to be both equal to half the pitch (fig. 70). There is still no differential action, but the combined E.M.F. of the coil gradually rises and falls as the inductors pass under or recede from a pole (fig. 67). A simple approximate method of treating such a case will be to divide each half-period into a number of portions

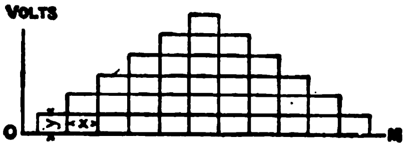

FIG. 184.—Width of winding and field each =half the pitch.

so small that the E.M.F. may be regarded as constant in value during each small portion. Let the length O M (fig. 184) represent half a period divided inte $2c$ portions, each of length $=x$, and let the height, y, of each small rectangle represent the E.M.F. due to one inductor when cutting the lines of the field. The number of inductors actually cutting the field rises from nought at end O to a maximum c at the centre when the whole coil is under a pole, and thence falls to nought at M. The curve of instantaneous E.M.F. may thus be divided into a series of steps, the number of small rectangles piled one above the other at each portion of time being equal to the number of inductors that are in action. The total number of rectangles, each of area xy, in the half-period $=c^2$, and the average E.M.F. of the coil

$$=\frac{\text{total area}}{\text{length O M}}=\frac{c^2xy}{2c\,.\,x}=y\,.\,\frac{c}{2}.$$

In order to determine the effective E.M.F. it will be necessary to obtain a second curve by squaring the ordinates of the first, and thence to find the square root of the mean square. This second curve will be similarly divided into steps, but the height of each step will be equal to the square of the height of the former steps. The total area will therefore be

$$\begin{aligned}&xy^2+x\,.\,4y^2+x\,.\,9y^2+\ldots+x\,.\,c^2y^2\\&\qquad\qquad+x\,(c-1)^2y^2+\ldots+x\,.\,9y^2+x\,.\,4y^2+xy^2\\&=xy^2\left[1+4+9+\ldots+c^2+\{1+4+9+\ldots+(c-1)^2\}\right].\end{aligned}$$

The two series are respectively the sums of the first c natural numbers and the first $(c-1)$ natural numbers; hence, by the ordinary algebraical formulæ for the summation of these series, the total area

$$\begin{aligned}&=xy^2\left\{\frac{c\,(c+1)\,(2c+1)}{6}+\frac{(c-1)\,c\,.\,(2c-1)}{6}\right\}\\&=xy^2\,.\,\frac{c\,(2c^2+1)}{3}.\end{aligned}$$

The mean square

$$=\frac{xy^2\,.\,\frac{c\,(2c^2+1)}{3}}{2cx},$$

and thence the square root of the mean square

$$=\frac{y\sqrt{2c^2+1}}{\sqrt{2}\cdot\sqrt{3}}.$$

The ratio $\frac{\text{effective E.M.F.}}{\text{average E.M.F.}}$ is therefore

$$k''=\frac{\dfrac{y\sqrt{2c^2+1}}{\sqrt{2}\cdot\sqrt{3}}}{y\cdot\dfrac{c}{2}}=\frac{\sqrt{2}\cdot\sqrt{2c^2+1}}{c\cdot\sqrt{3}}.$$

Now when c is made large, the term $\sqrt{2c^2+1}$ becomes sensibly equal to $\sqrt{2c^2}$, or the larger c, the more nearly the approximation of k'' to $\frac{2}{\sqrt{3}}=1\cdot155$. Hence the coefficient $k=k''\cdot k'\cdot 2=\frac{2}{\sqrt{3}}\times 1\times 2=\frac{4}{\sqrt{3}}$, and, with a close degree of accuracy, the effective E.M.F. of the coil is $2\cdot3\,Z_a\cdot\frac{N}{60}\cdot c$. It will be seen that in the above case, which is the one of most frequent occurrence in practice, the value of k is not very different from that for a simple inductor giving a true sine curve, and the accuracy of this may be verified by examining the combined curve of fig. 67, which bears a close resemblance to a sine curve.

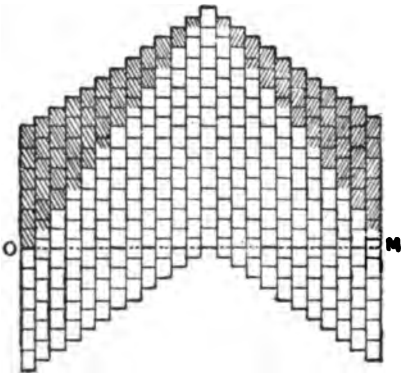

FIG. 185.—Width of winding and field each = pitch.

It remains to consider a case in which differential action comes into play; and to illustrate this let us assume a width of field and winding each equal to the pitch (fig. 62). In the ring-wound coil A let there be c inductors, and let the length O M, representing half a period (fig. 185), be divided into $2c$ portions, each of length x. If the coil be supposed at O to be moving half under the one pole and half under the other, the E.M.F. induced in the two halves is in opposite directions, which may be represented by plotting $\frac{c}{2}$ rectangles above, and the same number of rectangles below the horizontal axis; as the coil moves forward, the number of inductors under the one pole increases to a maximum, c, and thence falls to $\frac{c}{2}$. The total number of rectangles, each of area xy, in the half-period is $c \times 2c = 2c^2$, and the average E.M.F. of the coil *on the supposition that there is no differential action* is $\frac{xy \, . \, 2c^2}{2cx} = y \, . \, c$. But the actual average E.M.F. of the coil is less than this, and its value may be obtained by deducting from the heights of each series of rectangles the height of the rectangles below the horizontal axis, as in fig. 185, where the amount to be subtracted is indicated by the shading. It will now be seen that the sum of the areas of the rectangles free from shading is

$$\begin{aligned} &xy\,(1+2+3+\ldots+c+(c-1)+\ldots+3+2+1) \\ &= xy\left\{\frac{c}{2}(c+1)+\frac{c}{2}(c-1)\right\} \\ &= xyc^2 \end{aligned}$$

and the actual average E.M.F. $= \frac{xy \, . \, c^2}{2cx} = y\frac{c}{2}$.

Hence the ratio between the average E.M.F. of the coil and the sum of the average E.M.F.'s of the inductors is

$$\frac{y \, . \, \frac{c}{2}}{yc}, \text{ or } k' = \frac{1}{2}.$$

The effective E.M.F. is again obtained by squaring the ordinates of the curve of actual average E.M.F., and taking the square root of the mean square. The total area of the derived curve will be, as in the last case,

$$xy^2\,(1+4+9+\ldots+c^2+(c-1)^2+\ldots+9+4+1);$$

thence the effective E.M.F. is

$$\frac{y \, . \, \sqrt{2c^2+1}}{\sqrt{2} \, . \, \sqrt{3}},$$

and by the same steps as before we arrive at $k'' = \frac{2}{\sqrt{3}}$. The joint coefficient is therefore $k=\frac{2}{\sqrt{3}}\times\frac{1}{2}\times 2=\frac{2}{\sqrt{3}}=1{\cdot}155$, or exactly half the value that it had in the last case.

The same approximate method is easily applied to other cases in which there is differential action. Thus, if a coil of c inductors, having a width of winding equal to half the pitch, be rotated between fields of width equal to the pitch, and O M be divided into $4c$ portions, of the same length, x, as in the last case, we obtain the curve of fig. 186.

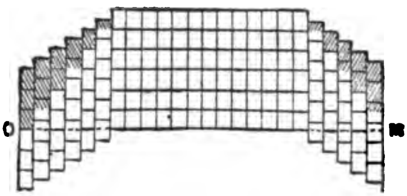

FIG. 186.—Width of winding = half the pitch, width of field = pitch.

The average E.M.F., if differential action be supposed absent, is $y\,.\,c$, while the actual average E.M.F. is

$$\frac{xy}{4c\,.\,x}\left\{1+2+3+\ldots+c+2c^2+(c-1)+\ldots+3+2+1\right\}$$

$$=y\,.\,\frac{3c}{4},$$

whence $$k'=\frac{3}{4}.$$

The area of the curve obtained from the actual E.M.F. curve when its ordinates are squared is

$$xy^2\left[1+4+9+\ldots+c^2+2c^3+(c-1)^2+\ldots+9+4+1\right]$$

$$=xy^2\,.\,\frac{c\,(8c^2+1)}{3},$$

and the effective E.M.F. is therefore

$$y\,\frac{\sqrt{8c^2+1}}{2\sqrt{3}},$$

which may be taken as very approximately equal to

$$y\,.\,\frac{2c\sqrt{2}}{2\sqrt{3}}=y\,.\,\frac{c\sqrt{2}}{\sqrt{3}}.$$

Thence

$$k''=\frac{y\dfrac{c\sqrt{2}}{\sqrt{3}}}{y\,.\,\dfrac{3c}{4}}=\frac{4\sqrt{2}}{3\sqrt{3}},$$

and $$k = \frac{4\sqrt{2}}{3\sqrt{3}} \times \frac{3}{4} \times 2 = \frac{2\sqrt{2}}{\sqrt{3}} = 1{\cdot}635.$$

The results thus obtained, together with other values of k under different conditions,[1] may be tabulated as below :—

(1) Width of each pole equal to pitch, toothed armature and winding concentrated in the recesses . . . . $k = 2{\cdot}00$
(2) Width of each pole equal to pitch, smooth armature and width of winding in each coil equal to pitch . . . . $k = 1{\cdot}155$
(3) Width of each pole equal to pitch, smooth armature and width of winding in each coil equal to half the pitch . . . . . . $k = 1{\cdot}635$
(4) Width of each pole equal to half the pitch, smooth armature and width of winding in each coil equal to pitch . . . . . . $k = 1{\cdot}635$
(5) Width of each pole equal to half the pitch, smooth armature and width of winding in each coil equal to half the pitch . . . . $k = 2{\cdot}3$
(6) Width of each pole equal to one-third of the pitch, smooth armature with width of winding in each coil equal to one-third of pitch . $k = 2{\cdot}83$

In all cases the coefficients may require in practice to be slightly modified, owing to the effect of the cross-ampère-turns of the armature in altering the actual distribution of the lines.[2]

The above equation for the E.M.F. of an alternator and the above values of k are equally applicable to unipolar ring or discoidal alternators, p being reckoned as explained on p. 159 : the ring or discoidal armature of fig. 77 will give a curve of E.M.F. exactly analogous to the curves of the corresponding armatures of fig. 71, and the average and effective E.M.F.'s will bear the same relation to each other in the two cases. In deducing the average E.M.F. of the unipolar ring, it is indifferent whether we make τ equal to the total number of wires that act as inductors at different times, i.e. to twice the number of loops (since both sides of each loop cut lines at different times), or equal to the number of loops ; in the former case, each wire cuts $z_a . p$ lines in each revolution, in the latter case, each loop cuts $2z_a . p$ lines : by either method we arrive at the same average E.M.F. It is, however, simplest to make

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[1] Extracted from Mr. Kapp's paper on 'Alternate-current Machinery,' *Proc. Inst. C.E.* February 19, 1889, vol. xcvii. part iii., to which the authors are largely indebted for the substance of the preceding pages.

[2] Cp. the remarks of Prof. Elihu Thomson read in the discussion of the above-mentioned paper of Mr. Kapp.

$\tau =$ the number of loops just as in the multipolar ring, so that the average E.M.F., when differential action is supposed absent, is $2z_a . p . \frac{N}{60} . \frac{\tau}{q}$, and the effective E.M.F. is $k . z_a . p . \frac{N}{60} . \frac{\tau}{q} . 10^{-8}$ volts. In the case of discoidal armatures, it might seem at first sight as if the unipolar alternator would only give half the voltage of the corresponding multipolar machine, since the area of pole-face is halved : this difference of area, however, is to a considerable extent counterbalanced by the lesser magnetic leakage in the unipolar machine, so that for the same exciting power z_a need not be very greatly different in the two cases. Lastly, in the unipolar disc machine, if $\tau =$ the number of radial wires which act as inductors at different times, then, as in the multipolar machine, each wire in one revolution cuts $z_a . p$ lines, where p is the number of fields or of poles on one side of the armature (p. 164), and the average E.M.F. is only $z_a . p . \frac{N}{60} . \frac{\tau}{q}$: the two disc armatures of figs. 77 and 71 will give exactly analogous curves, and therefore the effective E.M.F. of the unipolar disc is $k'' . k' . z_a . p . \frac{N}{60} . \frac{\tau}{q}$, or $k . z_a . p . \frac{N}{60} . \frac{\tau}{2q}$: k has here the same values as before for similar relative widths of field and winding, but it must be remembered that in two similar machines, the one unipolar and the other multipolar, the pitch of the unipolar disc is twice as great as that of the multipolar (pp. 162–164), and therefore the analogous case to, *e.g.* (5) in our table, is for the unipolar machine, width of field and of winding each equal to one *quarter* of the pitch. Thus for the same values of p, τ, and z_a, the E.M.F. of the unipolar alternator is only half that of the multipolar. Again, however, owing to the absence of leakage, z_a may be considerably greater, for the same area of pole, in the unipolar than in the multipolar machine, so that their effective E.M.F.'s remain more nearly the same (cp. p. 164).

Suppose that it is required to design a multipolar machine of the discoidal-ring type (fig. 74) for an output of 2,100 volts and 30 ampères with a frequency of 80 ~ per sec. when running at 600 revolutions per minute. To give the required frequency the number of poles on each side of the armature must be $P=\frac{80 \times 120}{600}=16$ (p. 145). An even number of poles is in all cases a necessity, and hence if the equation does not immediately work out to an even number, as above, either the speed or the frequency must be altered to suit the case. The core will be of soft band-iron wound upon a central wheel with thin paper insulation between the layers. The diameter of the rim of the wheel must be fixed by the judgment of the designer, and in default of other machines as a guide, the best proportions can only be determined by designing several machines for the same output but differing in their dimensions, and thence selecting the one that is most economical. In the present case we assume the wheel to be 34″ in external diameter, and the core to be 8½″ in radial depth, and 2½″ wide. The circumferential speed of the finished armature will then be slightly over 8,000 feet per minute (p. 257). The gross sectional area of the core = 137 sq. cm., and allowing 15 per cent. for insulation between the layers, the net area of iron = 116 sq. cm. Each pair of opposing poles forms in reality one polar surface, and the lines Z_a which enter into the core from them bifurcate into two groups, which pass in opposite directions on to the neighbouring poles; hence, just as in a bipolar machine, $Z_a = B_a \times 2ab$, and allowing 5,100 lines per sq. cm. as a good working induction for the armature core, we obtain $Z_a = 5{,}100 \times 2 \times 116 = 1{,}183{,}000$.

From eq. 47, if all the coils are in series, $\tau=\frac{E_a \times 10^8 \times 60}{k \,.\, Z_a \,.\, N \,.\, p}$ The loss of volts in the armature should not be more than about 3 per cent., so that $E_a = 2{,}163$; the width of each field and each armature coil will be made equal to half the pitch, whence by the table of p. 491, $k = 2{\cdot}3$, while $p=\frac{P}{2}=8$;

$$\therefore \tau=\frac{2{,}163 \times 10^8 \times 60}{2{\cdot}3 \times 1{,}183{,}000 \times 600 \times 8}=993,$$

say 992, giving 62 turns per coil. The diameter of the pitch-line is $34'' + 8\frac{1}{2}'' = 42\frac{1}{2}''$, and the pitch $=\frac{\pi \times 42{\cdot}5''}{16}=8{\cdot}3''$, so that the width of the coil should be 4·15″. Allowing ·15″ for a little play in the winding, the diameter of the insulated wire, if the coil is in one layer, may be $\frac{4}{62}=\cdot064''$, or ·044″ round wire d.c.c. with a thickness of 20 mils. The length of each turn round the armature core, allowing for easy bends on the outer and inner rims, may be estimated as 26″; whence the total length of the 992 turns = 2,150 ft. The resistance of 1,000 feet of ·044″ diameter wire of 100 per cent. conductivity being 5·33 ohms at 60° F., and the whole number of turns being in series, $R_a=11\cdot5$ at 60° F. In machines of the present type, owing to the high circumferential speed and excellent cooling action of the currents of air set up by the revolving armature (p. 404), the heating is very much less than in 2-pole continuous-current machines, having the same ratio of watts wasted to cooling surface of armature, and it will be sufficient to allow 6 per cent. for the rise in resistance due to heating: hence R_a hot $=11\cdot5\times1\cdot06=12\cdot2$ ohms, and the loss of volts is $12\cdot2\times30=366$. This is far too great a loss, and two layers of armature wire must be adopted; $\frac{4''}{31}=\cdot129''$ will permit us to use ·109″ diameter wire d.c.c. to ·129″; R_a hot now works out to 1·98 ohms, and the loss of volts to, say, 60.

The next step must be the determination of the ampère-turns required on the field-magnet, and, as explained on p. 314, it is convenient to consider a single pair of poles on one side of the ring, and then to multiply our result by the total number of pairs.

The length of path in the armature core is about 7″, say 18 cm., and the induction being 5,100, we have, from the curve of fig. 111, $X_a=2\times18=36$.

In calculating l_g, it will be well to allow a clearance of ·25″ between the winding and the pole-face, and for the insulation between the core and the winding on each side of the armature a thickness of about 115 mils.; hence $l_g=\cdot25''+(2\times\cdot129'')+\cdot115''=\cdot625''$. Since the pole-faces are small, the fringe of lines surrounding them forms a greater proportion of the area of the field than it does in the case of the continuous-current machine, and therefore, to be on the safe side, it will be advisable to take its width[1] as being only equal to ·4l_g all round the pole-face, i.e. ·4 × ·625 = ·25″. The required width of pole-face is thus equal to half the pitch — twice the width of fringe = 4·15 − ·5 = 3·65″, say 3⅝″, and its length to 8·5″ − ·5″ = 8″. The joint area of the two polar gaps is thus 2 × 8·5 × 4·15 sq. in. = 453 sq. cm., and $B_g = \frac{1{,}183{,}000}{453} = 2{,}610$, whence $X_g = {\cdot}8 \,.\, B_g \,.\, 2l_g = {\cdot}8 \times 2{,}610 \times 3{\cdot}18 = 6{,}640$.

For predetermining the value of the back ampère-turns in the case of an alternator, a convenient and exact formula is still wanting; in the present case, however, they will be about 1,000.

$$\therefore X_p = 36 + 6{,}640 + 1{,}000 = 7{,}676.$$

The leakage permeance for a pair of magnet-cores may be calculated by the methods of pp. 300, 304, and will be found to be about 35. Therefore

$$\zeta = 1{\cdot}256 \times 7{,}676 \times 35 = 336{,}000$$

and $$\frac{Z_a}{2} + \zeta = 591{,}500 + 336{,}000 = 927{,}500.$$

The alternator will be separately excited, and consequently the question of the stability of the magnetism does not arise. Hence, to determine the best value for the induction in the magnet-cores and yoke-ring, and the best length of coil, recourse must be had to the method of trial and error, so as to discover what proportions lead to a minimum cost of magnet iron and field copper. Suppose that in this case we take $B_m = 14{,}000$; then the area of each core $= \frac{927{,}500}{14{,}000} = 66{\cdot}2$ sq. cm.: this on the assumption of round magnet-cores gives a diameter of almost exactly 3⅝″, a convenient dimension, since it coincides with the width of the pole-piece. Taking the length of coil as 6½″, and adding ½″ at either end for insulation, the length of path in the pair of cores is 2 × 7″ = 35·5 cm., and therefore, by fig. 111,

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[1] Strictly speaking, the width of fringe depends on the exact shape of the pole-piece, &c., and can only roughly be fixed as a certain fraction of l_g. It is here assumed to be less than in the continuous-current machine of p. 297; in the latter any fringe is fully effective in producing E.M.F., but in an alternator, after a certain width, the extremely weak field beyond will probably give rise to differential action between the two ends of the coil. Further, if the exciting power be slightly overestimated, the error is easily remedied by slightly increasing the resistance of the rheostat which forms a necessary adjunct to the field-magnet circuit.

$$X_m = 17 \times 35{\cdot}5 = 602.$$

The length of path in the cast-iron yoke-ring will be about 7¾″ = 19·6 cm., and allowing a density of 5,500, the sectional area will be $\frac{927{,}500}{2} \div 5{,}500 = 84{\cdot}4$ sq. cm., say 2½″ × 5¼″; whence

$$X_y = 19 \times 19{\cdot}6 = 372,$$

and AT for one pair of magnet cores = 7,676 + 602 + 372 = 8,650. The exciting E.M.F. may be chosen arbitrarily, say 100 volts, but the loss in the field should not exceed 2½ per cent. of the output = 1,500 watts. The field winding may then be worked out by the formulæ of p. 319, the values of E_x for one pair of coils being $\frac{100}{16} = 6{\cdot}25$ (since each pair has to provide 8,650 ampère-turns) of the heating constant 1·09, and of the mean length of a turn, $l_x = 16'' = 1{\cdot}33$ ft. The resistance of 1,000 ft. of the required wire is

$$\omega' = \frac{6{\cdot}25 \times 1{,}000}{8{,}650 \times 1{\cdot}33 \times 1{\cdot}09} = {\cdot}496,$$

whence its diameter must be ·144″ d.c.c. to ·159″.

Assuming eight layers of forty turns each, we arrive at 640 turns on each pair of coils, giving a total resistance when warm of 7·4 ohms, and an exciting current through all the coils in series of $\frac{100}{7{\cdot}4} = 13{\cdot}5$ ampères. The rate of expenditure of energy in the field, viz. 1,350 watts, is therefore within our limit, and ample cooling surface is provided. It is important to make sure that the depth of winding is not so great as to block the space between two adjacent cores, and in the present machine there will be found to be plenty of clearance.

As far as the authors are aware, no reliable formulæ for predetermining the rise of temperature of the armature have been published for any class of alternator. In the discoidal machine designed above, the rise would probably not exceed 35° F.; hence the output is not limited by the heating, but by the questions of efficiency and reaction of the armature current on the field. The latter can be balanced by increasing the field-excitation, but if with constant excitation the drop of the characteristic curve, due to the loss of volts in the armature and the demagnetising effect of the back ampère-turns, becomes too marked, the machine becomes unsatisfactory from the point of view of the central-station engineer.

The method of testing the eddy-current loss in a continuous-current machine, alluded to on p. 408, may here be introduced as bearing directly on the design of dynamos. It consists in running the machine as a motor without load at various speeds, and noting the current through its armature, the E.M.F. applied to it, and the speed, the excitation being kept constant at the desired value throughout the test. The motor is not considered in the present work,[1] but it is here necessary to state that the power absorbed by a motor (exclusive of field-excitation and loss over armature resistance) $=(E_e - C_a R_a) C_a$, where $E_e =$ E.M.F. impressed on the motor armature, $C_a =$ the current through it, and $R_a =$ its resistance. When the motor is running light, C_a is very small, and with a fairly large armature of low resistance, $C_a R_a$ is practically negligible as compared with E_e; i.e. the back E.M.F. of the motor is closely equal to the E.M.F. impressed on it. In a motor, as in a dynamo, the E.M.F. induced in the armature $= Z_a \cdot \tau \cdot \frac{N}{60} \times 10^{-8}$ volts; and since Z_a is in our case constant, it follows that the speed is proportional to the E.M.F. impressed on the armature.

Now the losses in hysteresis and mechanical friction are simply proportional to the speed, while the eddy-current loss is proportional to the square of the speed (p. 408); hence the total power absorbed by the motor may be divided into two portions, viz. the watts spent in hysteresis and friction $= HN$, and those spent in eddies $= F \cdot N^2$, where $N =$ speed in revolutions per minute, and H and F are two constants for the machine with a given excitation. If, therefore, the speed of the motor is increased, so also are the watts lost in hysteresis and friction; but, as we have said, the back E.M.F. of the armature is increased in equal proportion. Hence, if we mentally regard C_a as split up into two portions, one of which is employed in overcoming the hysteresis and friction, and the other in overcoming the eddy-currents, the value of the former portion must remain constant in order that its product with the back E.M.F. may vary simply as the speed, and the curve showing its relation to the speed will be a horizontal straight line at some height above the horizontal axis, as shown at A C in fig. 187. On the other hand, as the speed is increased, and the watts lost in eddy-currents increase as the square of the speed, the portion of C_a which is spent in overcoming the eddies must itself increase equally with the speed, in order that its product with the back E.M.F. may vary as the square of the speed. Hence the curve showing the relation of this second portion of C_a to the speed must be a straight line inclined at some angle to the horizontal axis, as O D. Adding together the ordinates of the two curves A C and O D for each abscissa, we obtain a third curve, A B, which represents the total current through the armature: this will start from A and rise in an inclined straight line; and such is the actual form of curve obtained from an ordinarily good machine when tested in the manner described. If, therefore, we experimentally determine A B and draw A C parallel to the horizontal axis, we can deduce the required values of the constants F and H. To obtain the power in watts absorbed at any point of the curve, it is only necessary to multiply the abscissa (in volts) by the corresponding ordinate: thus for the point E the total watts absorbed =O F × F E, and of these the watts absorbed by hysteresis and

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[1] See G. Kapp, *Electric Transmission of Energy* (Whittaker & Co.).

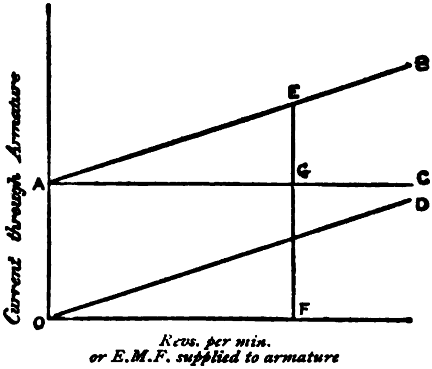

FIG. 187.—Separation of hysteresis and friction from eddy-current losses.

friction are $W_H = OF \times FG$, and the watts absorbed by eddy-currents, $W_F = OF \times GE$. Thence $H = \frac{W_H}{N_1}$, and $F = \frac{W_F}{N_1^2}$, where N_1 is the speed corresponding to OF volts.[1]

The above method, as it stands, is not applicable to alternators, since when an alternator is running as a motor without load the current and E.M.F. are not in the same phase, and consequently their product does not by itself represent the true power supplied to the motor.

The 'commercial efficiency' of a dynamo, or the ratio—

$$\frac{\text{power supplied from the terminals of the dynamo to the external circuit}}{\text{power given to the shaft of dynamo by the prime mover (engine or belt \&c.)}}$$

may be determined by adding the various losses in field, armature, and friction to the output, and dividing the output by the sum of output + losses. In the case of continuous-current machines, since the constants F and H may be somewhat greater at full load than at no load, the efficiency thus found may prove slightly too high, the true commercial efficiency at full load being perhaps 1 or 2 per cent. less. There are, however, various ways of measuring the commercial efficiency more directly. In cases where the dynamo is coupled directly to the engine, the power indicated in the engine cylinders, less the portion of this power which is wasted in the engine itself, gives the brake horse-power supplied to the dynamo; the results, however, of such a method of calculation cannot be regarded as absolutely accurate, owing to the difficulty of ascertaining the exact value of the waste in the engine. Another method is to transmit the power from the prime mover to the dynamo through a transmission dynamometer which registers the power passing through it; unfortunately, such a dynamometer when of large size is both costly and difficult to manage, and, further, does not admit of very accurate readings being obtained from it. A much more exact method is that due to Dr. Hopkinson,[2] by which two similar machines are so coupled together that the one acts as a motor driving the other as a dynamo. The output from the latter, being returned to the armature of the motor, supplies the greater part of the power required to drive it; and it is thus only necessary to supply to the motor from some external source the amount of power expended in the losses within the two machines. As this is but a small fraction of the total power developed, it is more easily measured on a transmission dynamometer, and even a large error in its determination, since it only affects the comparatively small item of the waste power, produces but slight error in the result. The only objection to the method is that it requires two machines of exactly similar size and output. These are driven at their normal speed and approximately at their normal voltage; the field of one, M, is, however, slightly weakened by a rheostat in its magnet circuit, so that its internal E.M.F. (E_2) is less than the terminal E.M.F. (E_1) of the other machine (D); hence D sends a current through M as a motor, and by means of the rheostat the amount of this current is regulated until it corresponds to the normal armature current of either machine. Let W = the total power in watts supplied from an external source to the motor armature, and C = the dynamo armature current which is also passed through the motor armature, the fields of both being separately excited. Neglecting the loss over the connecting leads, which can be made as small as we please, if we deduct from W the losses in the armature resistances of D and M, the remainder, $W - C^2(R_{a\,D} + R_{a\,M})$ is the total loss by eddy-currents, hysteresis, and friction in the two armatures. The field of the one is slightly stronger than that of the other, but if the E.M.F. of D is made slightly greater, and the E.M.F. of M is slightly less than the normal voltage of either machine, the error will be very slight if we assume this loss to be equally divided between the two. Let

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[1] For certain cautions as to the application of these values in designing armatures, *vide* pp. 408, 409.

[2] A description of this method is to be found in Dr. Hopkinson's paper on 'Dynamo electric Machinery,' *Phil. Trans.* 1886, reprinted among his *Original Papers on Dynamo Machinery* (Whittaker and Co.), p. 112, ff.

$$W \quad \frac{C^2(R_{a\,D}+R_{a\,M})}{2} = \frac{L}{2};$$

then the commercial efficiency of the dynamo is

$$\frac{E_1C}{E_1C + C^2R_{a\,D} + \frac{L}{2} + E_{f\,1}C_{f\,1}} \quad . \quad . \quad . \quad (49)$$

$E_{f1}C_{f1}$ being the watts expended in magnetising the field of the dynamo. Where it is not necessary to obtain such great accuracy, part of the armature current of the dynamo, D_1, may also be used to excite the field-magnets of both machines; in this case, assuming the machines to be shunt-wound, if C_1 = the sum of the currents C_{aM} and c_{sM} supplied respectively to the armature and field of the motor, the efficiency of the dynamo is

$$\frac{E_1 C_1}{E_1 C_1 + C_{aD}^2 R_{aD} + \frac{L}{2} + E_1 c_{sD}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

where $$L = W - C_{aD}^2 R_{aD} - C_{aM}^2 R_{aM} - E_1 (c_{sD} + c_{sM}).$$

It is convenient in carrying out these tests to couple the two shafts of motor and dynamo rigidly together, in one line; but it should be remarked that when this is done in the case of belt-driven machines, the loss by friction in the bearings does not reach its normal amount, since the pull of the belt corresponds to the transmission of but a small fraction of the normal power. On the other hand, when the motor and dynamo are coupled together by belt, an extraneous loss of power in bending the belt is introduced.

A further improvement of the above method consists in the use of a third dynamo as the external source whence the waste of energy in the system is supplied. This auxiliary dynamo need be of but small size, and may be coupled either in series with the two machines which are to be tested, or preferably in parallel with the dynamo.[1] When the method is thus modified, all the measurements can be made electrically by one voltmeter and one ammeter, and further, as regards the efficiency of the two armatures, great accuracy in the calibration of the instruments is not a matter of vital importance.

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[1] For a full description of these two methods, see a paper by G. Kapp on 'The Determination of the Efficiency of Dynamos,' *Electrical Engineer*, January 22 and 29, 1892. For an extension of the method to alternators, see Mordey, 'On Testing and Working Alternators,' *Journal Inst. Elec. Eng.* February 23, 1893.

CHAPTER XXI

THE WORKING AND MANAGEMENT OF DYNAMOS

In many cases it is required to connect two or more dynamos to the same pair of mains in order to increase the amount of electrical energy that can be supplied to their common circuit. If the dynamos are coupled together in series, although the maximum current that may be passed through them is no more than the current permissible in the smaller of the two, yet the available voltage is increased. On the other hand, if two dynamos of the same voltage are coupled in parallel, the total amount of current can be increased, although the terminal voltage remains unchanged. Any such coupling together of dynamos must necessarily be so arranged that the working of one dynamo does not interfere with the due working of the other or others; and as this requires certain precautions in their interconnections, the more usual cases which occur in practice will here be shortly considered.

The simplest case is the coupling of two series-wound continuous-current machines in series. To effect this, it is only necessary to connect the + terminal of one machine with the − terminal of the other, the external circuit being then connected to the remaining terminals of the pair. Such an arrangement is not unusual in cases of transmission of power over long distances, where it is necessary to work with high pressures in order to combine economy in the first cost of the copper leads with a high efficiency of transmission. It has been already mentioned that the delicate nature of the commutator hardly permits of more than 2,500 volts being generated in any one continuous-current dynamo; but by the use of two or more similar dynamos of the closed-coil class coupled in series, and each giving, say, 2,000 volts, a combined E.M.F. of 4,000 or more volts is obtained on the external circuit.

The coupling of two or more dynamos in parallel is even more frequent. In all large installations, such as central supply stations, as the load increases, more dynamos have to be brought into use, while the original machines are still kept running, and in such cases, in order to obtain the greatest economy, each dynamo should be worked as long and as much as possible at its maximum output : this is best attained if they are all capable of being worked in parallel, an additional machine being switched on to the same mains as soon as the load exceeds the combined output of those already running. The simplest case is that of two continuous-current shunt-wound dynamos, connected in parallel by joining their like terminals to form a common + and a common — terminal. If, as is usually the case, one machine is already running and excited, care must be taken that it is not joined in parallel with a second machine while the latter is at rest or is unexcited. The armature of the second machine would then form a short-circuit to the first, and would present no E.M.F. opposing an excessive rush of current through its low resistance. Hence the second machine, B, must be run up to its normal speed, and before it is thrown into parallel it must either be allowed to excite itself to approximately the same voltage as that of machine A, or B's shunt circuit must be closed on the first machine so as to excite B's field before its armature circuit is closed : the machines may then be safely thrown into parallel, and each will supply a certain part of the load, the exact proportion in which the total current is divided between them depending on their respective internal E.M.F.'s and armature resistances. The condition which determines this division is that after deducting the volts lost by the passage of the current over the armature resistance of either machine from its internal E.M.F., the remainder or the terminal voltage must be alike in both machines. Thus, if two similar machines, similarly excited and run at the same speed, be coupled in parallel, each will take half of the total current. If the speed of one be now lowered or its field weakened, its current will gradually pass over to the other machine, until when its internal E.M.F. falls to equality with the terminal voltage of the second machine it supplies no current at all, the whole of the load being thrown on to the one machine. To take a numerical example, suppose that each machine runs normally at 1,000 revolutions, and is then excited with 13,000 ampère-turns giving z_a=4,300,000, and E_a=103 volts, further that the loss of volts over the armature at the full load of 100 ampères reduces the terminal voltage E_b to 100 volts. If the speed of machine B falls, a larger portion of the current passes over to machine A. The increased loss over the armature resistance of A and the increased reaction of the armature current on its field combine to reduce both its internal and terminal voltage. When it takes the whole of the current, let z_a=4,100,000 be the number of lines that are produced by the 12,000 ampère-turns due to the terminal E.M.F. of 92 volts; in other words, C_e=200 ampères, and E_b=92 volts, give a point on its characteristic curve for a constant speed of 1,000 revolutions. This same terminal voltage will, however, in the case of machine B, which is carrying no current, give z_a=4,250,000, and these lines will give an internal E.M.F. of 92 volts when B is running at 900 revolutions. Thus the speed of B may be reduced by 10 per cent. before the whole of its load passes on to machine A, provided that the speed of the latter is kept strictly constant. On a further reduction in the speed or strength of field of B, its internal E.M.F. falls below the terminal voltage of A, and the latter then drives a reverse current through B's armature: the internal E.M.F. of B + the volts lost over its armature resistance due to the reverse current are then equal to the terminal voltage of A. The effect of the reverse current through B's armature is to assist in turning it as a motor with the same direction of rotation as before, without mechanically damaging the brushes, and thereby it tends to keep up B's speed. The interaction of the two machines thus exerts a considerable influence, tending to equalise their speeds and loads, and rendering it easy to work shunt-wound dynamos in parallel; while the share of the current which each machine takes is easily regulated by altering its speed or its field-excitation. In practice, when, as is most often the case, the machines are driven by separate prime movers, the equalising tendency is much assisted by the action of the prime movers themselves. Thus, when the load of one machine increases, the speed of its engine or other prime mover falls and tends to check the increase, while, for a similar reason, when a machine fails to maintain its fair share of the load, the speed of its prime mover rises, and tends to keep up its E.M.F.

The use of shunt-wound dynamos in parallel is therefore common in central supply stations: as soon as the load becomes too great for a single machine, a second is run up to speed, excited and switched into parallel with the first, the same process being repeated as often as required with other dynamos. When so connected, they all supply current to a common pair of 'omnibus bars,' whence the feeders are run to the network of mains. As a precautionary measure, a magnetic switch is frequently inserted between each dynamo and the omnibus bars: this automatically flies off and breaks contact if the armature current falls below a certain minimum; and hence, if for any reason a dynamo begins to slow down, it is cut out of circuit before a reverse current passes through it, while its load is taken up and divided among the other dynamos at work, without interruption to the general supply of current.

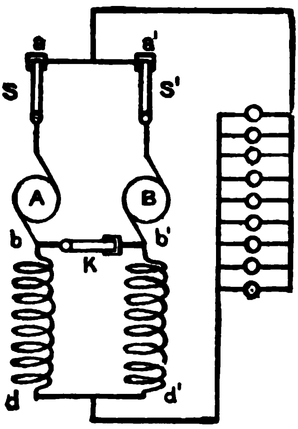

Fig. 188.—Two series-wound dynamos coupled in parallel.

Passing to series-wound dynamos connected together in parallel after the same fashion by joining their like terminals, we are met by the difficulty that if, for any reason, the E.M.F. produced by one machine, B, falls considerably below that of the other machine, A, then the current through B is reversed; and this, since the machine is series-wound, reverses its polarity. The direction of B's E.M.F. is consequently reversed, with the result that both dynamos simply act in series round their own internal resistances, and in a short time would be burnt up by the excessive current. This difficulty is, however, at once overcome by the addition of an extra wire connecting together the two brushes, as shown in fig. 188. It is especially important that this connecting lead should be of such large cross-section that its resistance is practically negligible as compared with the resistance of either of the series windings. By it the extremities of the two field-windings are joined in parallel, and the current supplied to the external circuit flows in the same direction through both and magnetises each equally. Should the internal E.M.F. of machine B fall below the voltage at the brushes of A, a reverse current passes through B driving it as a motor, but as this motor current flows through the lead *b b'*, and does not pass in a reversed direction through the series winding of B, it does not reverse B's polarity. If the fall of volts over *b b'* is considerable, a certain proportion of the motor current may pass through the alternative path of *b d d' b'*, but this is prevented by making *b b'* of sufficiently low resistance, as mentioned above. As in the case of shunt machines, if it be required to switch B into parallel with A while the latter is running, it is necessary to excite B to approximately the same voltage: this is effected by closing the switch K while B is running, and immediately afterwards switch S' may be closed.

Series-wound dynamos are, however, but seldom required to work in parallel, and the above remarks are introduced inasmuch as they apply equally well to the series windings of compound machines when worked in parallel. The arrangement usually adopted for these is shown in fig. 189, from which it will be seen that when the switches are closed, not only are like terminals connected together, but also like brushes, exactly as in the case of series machines and for the same reason; while, again, the connecting wire *b b'* must be of negligible resistance as compared with the series windings. In order to throw the one machine, say B, into parallel with the other, A, when the latter is running, it must be run up to approximately the same voltage as A, the excitation being provided

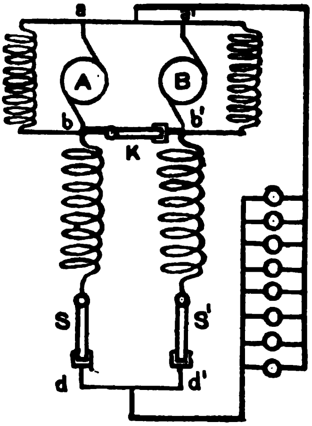

FIG. 189.—Two compound-wound dynamos coupled in parallel.

by the shunt alone; switch K is then closed, and finally switch S′, while if it be required to withdraw machine B from parallel connection, its speed should be slightly reduced until it is supplying little or no current, after which switch S′ is opened, and lastly switch K. The opening or closing of the two connections in due order may be conveniently effected by a composite switch by which the contact at K is made first on closing the switch, and broken last on opening it.

Alternate-current dynamos, with some exceptions, can also be run in parallel; the whole question of their parallel working is, however, so complicated that space will not permit of more than the merest outline of the subject being here given.[1]

If two similar alternators be running at exactly the same speed with E.M.F.'s rising and falling together, that is to say, if they synchronise in phase, and, further, if they are giving equal voltages, then so long as this condition of affairs lasts the armature circuits of the two machines may be coupled together, and they will run in parallel, dividing the load between them in proportion to the power supplied to each. If, however, the phase of the E.M.F. of one machine does not synchronise with that of the other, *i. e.* if, as it is termed, the two machines are not 'in step,' then, even though their speeds and effective voltages are identical, the E.M.F. of one may be rising when the E.M.F. of the other is falling; hence a fluctuating current will pass between the two armatures, and this may reach such intensity as to render the working difficult or positively unsafe. Now in practice the above conditions for successful parallel working, viz. perfect synchronism and equality of voltage, will seldom, if ever, be exactly fulfilled; yet, provided that the differences in the speed, phase, or voltage of the two machines be not excessive, they may be switched into parallel; and from the mutual control which they exert upon each other, they will fall into step, and continue to run together satisfactorily. In order that the difference of phase may not be too great at the moment when they are connected together, an instrument called a 'synchroniser' or 'phase indicator' is employed, of which fig. 190 is a diagrammatic sketch. On a laminated iron core three coils of insulated wire are wound as shown : of these C_1 is connected to the terminals T_{a1} and T_{a2} of alternator A, C_2 to the terminals T_{b1} and T_{b2} of alternator B, and C_3 to the terminals of an incandescent lamp, L. The whole, therefore, forms a small transformer, having two primary coils, C_1 and C_2, and one common secondary, C_3. The primaries are so arranged that when the two machines are in phase, sufficient E.M.F. is induced in the secondary to render the lamp fully incandescent, while if the machines are in different phases the lamp does not burn so brightly—the greater the difference of phase, the less being the light. Assuming, then, that one machine is already at work, and that it is desired to connect another in parallel with it, the procedure is as follows :—The second machine is run up to its full speed, and its field excited to give a voltage slightly higher than the working voltage of the first machine; the synchroniser is switched on to the two alternators, and if they are not in step, the light of the lamp pulsates, intervals of darkness rapidly succeeding intervals of brightness ; the speed of the second machine is then adjusted so as to lengthen the intervals of brightness, and at the instant when the two are coming into phase, as shown by the lamp burning steadily, the switch K connecting the armatures together is closed ; the operation is then completed, and the synchroniser may be switched off. When it is desired to shut down one of the machines, it is only necessary to reduce its exciting current until

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[1] For further information see Hopkinson's *Original Papers on Dynamo Machinery*, p. 149, ff.; Mordey, *Journal Inst. Elec. Eng.* vol. xviii. pt. 81, 1889; Swinburne, *ibid.* vol. xx. 1891; Fleming's *Alternate-current Transformer*, vol. ii. pp. 351–364; and *Electrical Engineer*, March 13, 20, 1896.

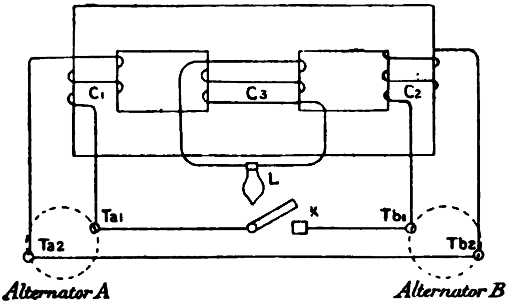

FIG. 190.—Synchroniser for coupling alternators in parallel.

all the load is taken by the other machine (or machines), and then to switch it off the mains.

Alternators cannot be run in series, for as soon as one falls the least behind another in phase, the lagging machine has the greater part of the load thrown upon it, which causes it to lag more and more, until ultimately, when the two settle down into exactly opposite phases, their E.M.F.'s neutralise each other, and no current flows.

It remains to add a few remarks on the fixing and working of dynamos in general. In the first place, good foundations are as much an essential for successful working as a good dynamo, inasmuch as any vibration is most detrimental to the life of the armature, commutator, and brushes, and if transmitted to the surrounding floor and walls will prove a constant source of annoyance. The ground should be excavated until a sound and solid bottom is reached, on which may be built up a foundation of concrete, or concrete and brickwork, sufficiently massive to absorb and damp the vibration of a fast-running dynamo. The interposition of a few thick sheets of felt between the base of the concrete and its seating assists in deadening the transmission of vibrations. With small dynamos, Lewis bolts having a tapering shank of rectangular section with jagged edges are used to hold down the bedplate or slide-rails : the square holes required for these bolts are cut into the concrete, their position being marked off usually from a wooden template of the slide-rails, and when these latter are laid down, the bolts are dropped in and fixed by lead or sulphur cement run in round them. With larger dynamos, long holding-down bolts are used, passing right through to the bottom of the concrete and terminating at their lower end in large square plates. The holes for such bolts are formed in the concrete by inserting long tapering wooden boxes of square section ; when the bedplate or rails are in position, thin cement is run into the holes thus formed, until they are full, and is then allowed to set without disturbance. In other cases, large blocks of timber are embedded in and bolted down to the concrete, and on these the rails are fixed by coach-screws. In all cases, the upper surface of the foundation should be carefully levelled with a straight-edge and spirit-level.

When driven by belt, it is essential that the dynamo should be set square with the driving engine or shafting, in order that the belt may run fairly on the centres of the two pulleys. This is ascertained by taking a length of string and stretching it tightly from the far edge of the driving pulley or fly-wheel to the dynamo pulley; when the string just touches the near edge of the driving pulley, its transverse distance from a centre line drawn round the dynamo pulley should be the same both on the side nearer to the driving pulley and on the side farther from it.

The belt itself should be soft and flexible, and joined into an endless band either by a cemented and riveted joint, or by belt-fasteners with a butt joint; a lapped or laced joint causes a jerk each time that it passes on to the dynamo pulley, which gives rise to vibration and fluctuation of the E.M.F., together with sparking. Wherever possible, the direction of rotation should be such that the under side of the belt is the tight or driving side.

Perfect steadiness of driving is of great importance, especially for direct incandescent lighting; a very slight fluctuation in the speed of the dynamo, even though it be not great enough to cause sparking, is immediately discernible as a pulsation in the light of the lamps, owing to the slight change of E.M.F. to which it gives rise. On this account most of the present types of gas- or oil-engines are inadmissible for direct lighting owing to their great fluctuation of speed during each revolution, even when fitted with heavy fly-wheels. When such prime movers are used, the dynamo is usually worked in conjunction with an accumulator battery, and even then it is advisable for the dynamo to be itself fitted with a heavy disc fly-wheel. In such cases, the secondary cells should never be placed in the same room with the dynamo, since the cotton or other fibrous insulation of the dynamo wires is rapidly attacked and eaten away by the acid spray given off during the process of charging.

After the machine has been fixed in position, the armature should be turned round by hand to see that it revolves freely and that nothing is loose; it should then be run for some time with the brushes raised from the commutator, in order to test the alignment of the bearings and of the machine generally. The adjustment of the brushes when lowered on to the commutator requires careful attention: usually two lines are cut on the gun-metal collar of the commutator next to the outer bearing (these lines being in a 2-pole machine diametrically opposite to each other), and to them the brushes must be set, the tips of the brushes carried by one arm of the rocker being in line with the one setting mark, and those of the brushes on the other arm in line with the opposite mark.

All screw contacts should be firmly screwed up, and any dirt or lacquer (if such there be) on the points of binding-screws should be cleaned off. Want of contact of the brushes on the commutator or elsewhere may cause a failure of the dynamo to excite. The electrical connections should be carefully examined and verified; and on starting, it should be borne in mind that a shunt-wound dynamo will not excite on a very low resistance, or a series-wound dynamo on a very high resistance. If, therefore, a shunt-wound dynamo fails to excite even when the main switch is open, a short-circuit in the leads may be suspected. Or if a series-wound dynamo will not excite with its normal external circuit closed by the main switch, there may be a disconnection either within or outside the machine. Any such difficulty will usually be solved by testing with an ordinary linesman's detector; or in default of a solution by this means, the connection of the brush leads to the field-winding should be transposed.

As the brushes wear, they must be fed forward through their holders; and if they do not then bed properly on the commutator over their entire thickness, their tips must be filed to the correct angle. For such trimming the brush should be clamped in a vice, with a plate to support it at the back, and filed with a smooth file, the strokes being made towards the extreme tip. When a new set of brushes requires to be bedded, it may be conveniently done by tying a piece of emery paper accurately round the commutator, and running the armature slowly while the brushes are pressed on to the emery paper. After any such operation the tips of the brushes must be cleaned from any adherent copper and emery dust.

For the filling of the lubricators, copper oil-cans should invariably be used, since iron cans are liable to be drawn to the magnet, and thereby, perhaps, cause damage by catching in the armature.

All oil-pipes and waste oil-chambers require occasional attention to see that they are not clogged. If from any neglect in this respect, or from original defective construction, oil creeps from the bearings on to the surface of the commutator, it becomes carbonised by any sparking at the brush-tips, and forms a thin conducting film which bridges across the strips of insulation between the segments of the commutator, the result being a loss of E.M.F., due to the local leakage which ensues between neighbouring coils; while if oil further makes its way on to the armature winding, it has a deteriorating effect on the protecting varnish, and causes adherent deposits of dirt and copper dust. The dynamo should therefore be kept clean, and in especial its commutator requires scrupulous care on this point. The use of any lubricant on the surface of the commutator is to be avoided—except at occasional intervals, when a little vaseline smeared on a piece of clean rag may be rubbed on while the armature is rotating.[1] The dark burnished lustre which may be seen on the commutator of a good non-sparking dynamo is the sure evidence of a careful attendant. Occasionally, before stopping, fine emery cloth may be applied if the surface shows signs of wearing into grooves and becoming uneven. If a flat begins to develop on one or more segments (p. 391) it should be filed out, the segments on each side being carefully rounded so as to retain as far as possible the cylindrical surface of the commutator. But if the disease has gone too far for such remedy, the armature must be put in a lathe, and the commutator surface turned up true; the tool should be sharp and fine-pointed, and the feed should be light, so as not to drag the copper over the insulating strips of mica: after turning, it should be lightly filed with a smooth file, and finally examined to see that no particles of copper are embedded in the mica, and bridging adjacent segments.

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[1] In high-tension open-coil machines with air-insulation dividing the segments of the commutator, oil may be more freely used.

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